Advanced Macroeconomics II, Part I (2016)
Problem Set 5: Durables & Solving the Equity Premium Puzzle

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1 Durables

Consider the consumption-savings problem with durable consumption goods:

$$V\left(a_t, D_{t-1}, y_t\right) = \max_{e_t} \quad u\left(D_t\right) + \beta \mathbb{E}_t\left[V\left(a_{t+1}, D_t, y_{t+1}\right)\right]$$

subject to wealth and durable accumulation:

$$\begin{aligned} a_{t+1} &= R_t\left(a_t + y_t - e_t\right) \\ D_t &= \left(1-\delta\right) D_{t-1} + e_t, \qquad D_{-1} \text{ given} \end{aligned}$$

1. Show that the problem yields the standard Euler equation for the *stock* of durables.

$$u'(D_t) = \beta R \mathbb{E}_t[u'(D_{t+1})]$$

2. Assume quadratic utility and $\beta R = 1$. What is the behaviour of the stock of durables? And of purchases of durables? What happens when $\delta = 1$? (Note: You may ignore corners at zero and the bliss point).

3. Plot the impulse-response of purchases to an income shock. Does this model help to explain why the consumption of durables is so volatile?

4. Mankiw (JME, 1982): "Hall's Consumption Hypothesis and Durable Goods" estimates the process for purchases and found a lagged MA coefficient equal to 0.1 on annual data. What does this imply for δ? Is this a reasonable model?

2 Solving the Equity Premium Puzzle

Environment Consider the Lucas (1978) pure exchange model of an economy with a representative household that consumes C_t and owns a representative firm that produces output and pays stochastic dividends D_t. The model is characterized by the following three equations:

$$\begin{aligned} P_t^e &= \beta \mathbb{E}_t \left[\frac{U'(C_{t+1})}{U'(C_t)} \left(P_{t+1}^e + D_{t+1} \right) \right] \\ P_t^b &= \beta \mathbb{E}_t \left[\frac{U'(C_{t+1})}{U'(C_t)} \right] \\ C_t &= D_t \end{aligned}$$

where P_t^e denotes the price of equity in the firm and P_t^b the price of a risk–free bond that pays 1 unit of consumption in every state. The last equality comes from market clearing. Define the risk–free rate[1] as $R_{t+1}^b \equiv \frac{1}{P_t^b}$, the equity return as $R_{t+1}^e \equiv \frac{P_{t+1}^e + D_{t+1}}{P_t^e}$, and the excess return as $R_{t+1}^x \equiv R_{t+1}^e - R_{t+1}^b$.

2.1 Baseline

Mehra and Prescott (JME, 1985): "The Equity Premium: A Puzzle" take the previous model to the data and conclude that the equity premium (defined as the unconditional average of excess return across all periods $\mathbb{E}[R_{t+1}^x]$) is too high for reasonable levels of risk aversion $\gamma \in [1, 4]$. This is known as the equity premium puzzle, or its mirror, the risk–free rate puzzle.

Write a code that calculates the average risk–free rate and the equity premium. You can use two methods (it is a good exercise to try both and compare results):

- Exact: Find the ergodic distribution of the dividend process as the eigenvector associated with the unit eigenvalue and use it to compute the unconditional expectations by averaging the return in the high and low state. This method obtains the exact moments. In Matlab, you can use the command *eig* applied to the transition matrix P:

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[eigvec, eigval]=eig(P'); [row,col]=find(eigval==1); E=eigvec(:,col)/sum(eigvec(:,col));
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- Approximate: Simulate the economy for many periods to obtain time series and then compute the unconditional expectations as the sample averages. This method obtains approximate moments, unless you have an infinite sample.

Use the following benchmark parametrization:

- CRRA $U(C_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma}$, with discount rate: $\beta = 0.99$ and risk aversion parameters: $\gamma \in \{2, 4, 6, 10\}$
- Dividend growth rate takes two values[2]: $d^H = 0.054$ and $d^L = -0.018$
- Transition probabilities $P = \begin{pmatrix} p & 1-p \\ 1-p & p \end{pmatrix}$, with $p = 0.43$.

[1] Note that the payoff of the bond (one unit of consumption) is done at $t+1$, hence the notation R_{t+1}^b. However, this payoff is known and comes out of the expectation.

[2] Dividends evolve as $D_{t+1} = (1 + d^j) D_t$ for $j = H, L$.

2.2 Asymmetry

Cecchetti, Lam, and Mark (AER, 1990): "Mean Reversion in Equilibrium Asset Prices" note that the consumption process is not symmetric: booms tend to last longer than busts; busts tend to be sharper than booms. They estimate an asymmetric transition matrix: $P = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix}$, with $p = 0.987$ and $q = 0.516$. The growth rates are $d^H = 0.02252$ and $d^L = -0.06785$.

- Show how does this change affect the equity premium and the risk–free rate.

2.3 Rare Disasters

Barro (QJE, 2006): "Rare Disasters and Asset Markets in the Twentieth Century" argues that rare disasters (such as the Great Depression, World War I and II) may explain the equity premium and risk–free rate puzzles. Barro looks at GDP worldwide and argues that catastrophes occur much more frequently than in the US sample and they are often much more severe.

One might represent Barro's transition matrix as follows:

$$P = \begin{pmatrix} p - r/2 & 1 - p - r/2 & r \\ 1 - p - r/2 & p - r/2 & r \\ 1/2 & 1/2 & 0 \end{pmatrix}$$

with $r = 0.017$ and $p = 0.43$. The growth rates for high and low states are as before, and the growth rate for the disaster state is $d^{disaster} = -0.3$.

- Show how does this change affect the equity premium and the risk–free rate.

2.4 Distorted Beliefs

Cecchetti, Lam, and Mark (AER, 2000): "Asset Pricing under Distorted Beliefs: Are Equity Returns Too Good to Be True?" consider distorted beliefs: agents misperceive the mean growth rate of the endowment in the two states. Beliefs are too pessimistic about the persistence of expansions and too optimistic about the persistence of contractions. Agents do not learn about the true process over time.

The true transitions are as above: $p = 0.987$ and $q = 0.516$. Consider two types of distorted beliefs:

(i) Excessive pessimism about expansions, excessive optimism about contractions: $\tilde{p} = 0.7$ and $\tilde{q} = 0.2$.

(ii) Excessive pessimism for both states: $\tilde{p} = 0.6$ and $\tilde{q} = 0.9$.

- Show how does this change affect the equity premium and the risk–free rate. Be careful to use true probabilities when computing the average returns.
- Show that in the first case, the equity premium puzzle can be solved for an agent that is moderately risk averse and relatively impatient ($\gamma = 6.4, \beta = 0.9$).
- Show that in the second case, the equity premium puzzle can be solved for an agent that is nearly risk neutral and relatively patient ($\gamma = 0.23, \beta = 0.98$).