Advanced Macroeconomics II, Part I (2016)
Problem Set 4: Prudence, Habits and Stochastic Growth

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1 DARA Family

Show that a utility function in the DARA (Decreasing Absolute Risk Aversion) family exhibits prudence.

2 Habits in utility function

Consider the consumption–savings problem with uncertainty, where a household chooses consumption c_t and future assets a_{t+1} subject to a stochastic income sequence $\{y_t\}_{t=0}^{\infty}$. Her budget constraint is given by $a_{t+1} = Ra_t + y_t - c_t$. Assume the household faces the natural borrowing constraint. As a new element, suppose the household exhibits *internal* habits, and her utility depends on its consumption of two consecutive periods, as follows:

$$u(c_t - \varphi c_{t-1}), \qquad \varphi \in \mathbb{R}, \quad u' > 0, \quad u'' < 0$$

1. What is the interpretation of the parameter φ? Does your answer depend on its sign? Are c_t and c_{t-1} complements or substitutes? *(Hint: How does past consumption affect the marginal utility of consumption today?)*
2. What is the natural borrowing constraint? What conditions on utility ensures it does not bind?
3. Write the problem in recursive form, identifying the exogenous and endogenous states and controls.
4. Find the first order and envelope conditions, and use them to obtain an Euler Equation.
5. Suppose $\beta R = 1$. Is marginal utility a martingale?
6. Suppose further that utility is quadratic. Is consumption a martingale?
7. In contrast to internal habits that only depend on own behaviour, consider now *external* habits where the household's utility also depends on the average of her friends' consumption $\bar{c}_t$ as follows:

$$u(c_t - \varphi \bar{c}_t), \qquad \varphi \in \mathbb{R}$$

   This behavior is known as "Keeping up with the Joneses", as it is an idiom in many parts of the English-speaking world referring to the comparison to one's neighbour as a benchmark for social class or the accumulation of material goods. How do you interpret φ in this case? What is the Euler equation for the external habit model? If $\beta R = 1$, is u' a martingale?

3 Stochastic Growth Model

Note: For this exercise you have 2 weeks. Feel free to use Martin Ellison's note, especially Section 7, that explains step by step how to solve this exercise.

Setup Consider a very general stochastic growth model.

$$\begin{aligned}
\max_{\{c_t,k_{t+1}\}_{t=0}^{\infty}} \quad & \mathbb{E}_0\left(\sum_{t=0}^{\infty}\beta^t u(c_t)\right) \\
k_{t+1} &= (1-\delta)k_t + i_t \\
y_t &= f(\theta_t, k_t) \\
y_t &= c_t + i_t \\
k_0, \theta_0 & \quad \text{given} \\
\theta_t &\sim G(\theta_t|\theta_{t-1})
\end{aligned}$$

Preferences Assume a general CRRA utility function: $u(c_t) = \begin{cases} \frac{c_t^{1-\sigma}-1}{1-\sigma} & \text{if } \sigma \neq 1 \\ \log c_t & \text{if } \sigma = 1 \end{cases}$

Production Assume a Cobb-Douglas production function: $f(\theta_t, k_t) = \theta_t k_t^{\alpha}$

Stochastic process To start, assume the simplest process for productivity: θ_t can only take two values, high θ^H or low θ^L and it is *iid* over time[1].

$$Prob(\theta_t = \theta^H) = Prob(\theta_t = \theta^L) = p$$

1. Write a code to solve the stochastic growth model using the following values for the parameters:

Table I: **Benchmark Parameters**

Parameter	β	σ	α	δ	θ_L	θ_H	p
Value	0.9	2	0.75	0.3	0.75	1.25	0.5

2. Plot the policy functions c_t and k_{t+1} as a function of the state k_t. Draw one line for each level of productivity (low and high).

3. Simulate your economy for $T = 200$ periods and plot the following time series $y_t, c_t, i_t, k_t, u(c_t)$.

4. Compute descriptive statistics (mean, standard deviation, correlations) for the time series above. Do you observe consumption smoothing?

5. How do the volatility of the series change when you increase risk aversion σ?

6. Design an experiment to test for precautionary savings.

[1] Next week, we will introduce persistence, but let's go step by step. If you want to jump ahead, you can already code a Markov chain with 2 states.