Advanced Macroeconomics II, Part I (2016)
Problem Set 3: Permanent Income Hypothesis

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1 Consumption Response to Income Changes and Persistence

Consider the consumption-saving problem with uncertainty and assume the PIH holds (quadratic utility and $\beta(1+r)=1$). Income follows an AR(1) process:

$$y_{t+1} = (1-\rho)\overline{y} + \rho y_t + \varepsilon_{t+1}, \qquad \varepsilon_{t+1} \sim iid(0, \sigma_\varepsilon^2), \qquad \rho \in [0,1]$$

The goal of this exercise is to find the policy function $c(a_t, y_t)$ which is a function of the states.

1. Verify the unconditional moments: $\mathbb{E}[y_t] = \overline{y}$, $\mathbb{V}[y_t] = \frac{\sigma_\varepsilon^2}{1-\rho^2}$ and $corr[y_t, y_{t+\tau}] = \rho^\tau$.

2. Method 1: Solve for the optimal consumption and savings policies using the method of undetermined coefficients (guess and verify) applied to the Euler Equation. *Hint: Guess a linear function form* $c(a_t, y_t) = Aa_t + By_t + k$.

3. Method 2: Get an expression for permanent income as a function of current assets and current income substituting the stochastic process for y_t and using its properties. Then just set $c_t = y_t^P$.

4. Show that the marginal propensity to consume out of current income $\frac{\partial c_t}{\partial y_t}$ is increasing in the persistence of the income process ρ.

2 Identification of Shocks Through Panel Data

A very common specification used in labor economics assumes that labor income (in logs) is the sum of a permanent component y_t^P which follows a martingale, and a transitory component u_t *iid* over time:

$$\begin{aligned} y_t &= y_t^P + u_t \\ y_t^P &= y_{t-1}^P + \eta_t \end{aligned}$$

Note that η_t is the innovation to the permanent component, which is also assumed to be *iid* over time. Finally, we assume $\mathbb{E}[\eta_t] = \mathbb{E}[u_t] = \mathbb{E}[\eta_t u_t] = 0$.

1. Get an expression for the news in the income process $(\mathbb{E}_t - \mathbb{E}_{t-1})y_{t+j}$ and interpret. *Hint: Split in two cases,* $j=0$ *and* $j \geq 1$.

2. Express the change in consumption Δc_t as a function that only depends on the permanent innovation η_t and the transitory innovation u_t.

3. How do the consumption responses to permanent and transitory innovations differ?

Now suppose you have panel data on consumption and income for a sample of households $i = 1, \ldots, N$ and for periods $t = 1, \ldots, T$. All households face the same income process and preferences above, but of course they have different realizations of such process that gives rise to heterogeneity across them. Suppose the variances of (u_t^i, η_t^i) change over time (time-varying volatility) and are denoted by $\sigma_t^\eta, \sigma_t^u$.

4. Let var_t denote the cross-sectional variance at time t (i.e. the variance across individuals for a particular time). Get expressions for $var_t(\Delta c_t^i)$, $var_t(\Delta y_t^i)$ and $cov_t(\Delta c_t^i, \Delta y_t^i)$ that only depend on variances of shocks $\sigma_t^\eta, \sigma_t^u$.

5. Assume r is close to zero. Show that you can separately identify the variances of the underlying structural income shocks using the panel data on consumption and income.

3 Empirical checks of Ricardian Equivalence

Prepare the following two papers for class next week:

- Hsieh (AER, 2003). *Do Consumers React to Anticipated Income Changes? Evidence from the Alaska Permanent Fund.*
- Johnson, Parker and Souleles (AER, 2006) *Household Expenditure and the Income Tax Rebates of 2001.*

For each paper, think of the following questions (these are useful when preparing a referee report for an academic journal):

1. What is the question and why is the question important? How does the paper address the question, and what is the answer? Why is this approach different or better than other papers that have asked this question?

2. Regarding the data:
   - Is the data adequate for answering the question? Is the data a reasonable measure of what the author claims to be measuring?
   - Is the source of the data clear? Is it clear how the data was manipulated and cleaned in order to get the data used in the statistical analysis?

3. Regarding the methodology:
   - Is the methodology adequate for answering the question?
   - If there is a model, is there a clear mapping from the data to the model and viceversa?
   - Is there likely to be measurement error? If so, is this addressed or considered?
   - Is there likely to be a reverse causality or an endogeneity problem? If so, what approach is taken to achieve identification?
   - Are the conclusions tightly linked to the data? Does the author overstate the results?

4. Do you like the paper?