Advanced Macroeconomics II, Part I (2016)
Problem Set 2: Pricing with Menu Costs and Cake-Eating

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1 Pricing with menu costs

Consider a profit maximizing firm that produces output with a linear technology, where one unit of labor produces one unit of output. The cost of labor, w_t, is stochastic and follows a Markov process. The firm chooses the price P_t at which to sell her product, and then commits to sell any quantity that is demanded at such price. She faces a downward sloping demand given by $P_t^{-\eta}$, where η is the demand elasticity. Every time she changes her price, she must pay a menu cost $\theta > 0$ in units of product. The timing works as follows. At the beginning of the period, the firm observes the cost of labor, then she decides whether to keep the same price (inaction) or to adjust it (action), in which case she must also choose the new reset price x_t which will be active that same period. The firm discounts the future at a rate R.

1. Write the problem of the firm recursively. Make sure to define clearly the value functions, the instantaneous profit, the state space, and other elements needed. Also state clearly any additional assumptions you make.

2. Let P_t^* be the desired price that the firm would set in the absence of any frictions. Show that this price is equal to a constant markup over the marginal cost. Therefore, P_t^* inherits the stochastic properties of w_t. *Hint: P_t^* solves a static problem.*

3. Now define the price gap $\hat{p}_t \equiv \log P_t - \log P_t^*$, which is the log difference between the current price and the frictionless price. Show that we can rewrite instantaneous *log* profits as a quadratic loss function in terms of the price gap:
$$\Pi(\hat{p}_t) = B_0 - B_1 \hat{p}_t^2$$
where $B_0, B_1 > 0$ are constants. *Hint: Use a second order Taylor approximation of the log profits around the frictionless price P_t^*.*

4. Using the quadratic approximation of the previous point, rewrite the recursive problem in terms of the price gap $\hat{p}_t$. Note how we have collapsed the problem from two states to one state.

5. Characterize the optimal pricing policy conditional on adjustment (i.e. show that the reset price gap is equal to zero). *Hint: Use the symmetry of the value function with respect to zero.*

6. Characterize the inaction region (binary policy of action/inaction) as much as you can.

2 Stochastic discrete cake-eating

Consider the stochastic discrete cake-eating problem. An agent is endowed with a cake of size C. She experiences serially correlated taste shocks z with transition probabilities Q with element (q_{ji}). In each period, after observing the taste shock, the agent decides to eat the entire cake and receive utility $u(c_t)$ or wait. We assume a CRRA utility function with multiplicative taste shocks as follows:

$$u(z_t, c_t) = \begin{cases} z_t \left[\frac{c_t^{1-\sigma}-1}{1-\sigma}\right] & \text{if } \sigma \neq 1 \\ z_t \log c_t & \text{if } \sigma = 1 \end{cases}$$

The cake shrinks by a factor $\rho \in (0,1)$ each period and the future is discounted at rate β. Taste shocks takes on five discrete values z, uniformly spaced in the interval $[1-\gamma/2, 1+\gamma/2]$. The transition probabilities are zero except as follows: $q_{1,2} = q_{4,5} = 2(1-\theta)$, $q_{2,3} = q_{3,4} = 3(1-\theta)$, with the upper triangle of the transition matrix symmetrical to the lower triangle and the diagonal elements equal to one minus the sums of the non-diagonal elements.

1. Show that the resulting serial correlation of taste shocks is θ.

2. Write the problem of the agent in recursive form. State clearly the value functions, the state, the control, the transition probability, and other assumptions that you make, for instance, the flavor of the cake.

3. Write a code (in the language and platform you prefer) that solves this cake eating problem[1]. Table 1 sets the benchmark values for the parameters to be used in the code. C denotes the size of the cake in the beginning. Choose the grid for cake size smartly and exclude cake sizes smaller than one. Explain why you should do so.

Table I: **Benchmark Parameters**

Parameter	C	σ	β	ρ	γ	θ
Value	30	2	0.999	0.8	0.5	0.9

4. Plot the value function and the policy. These objects live in a two-dimensional space (C, z).

5. Do a comparative statics analysis for each of the 5 parameters. Make sure the parameter choices satisfy the restrictions imposed by the structure of the problem (for example, probabilities between 0 and 1 impose $\theta \geq \frac{5}{6}$). For this purpose, write outer loops that solve the model, where you vary one parameter at a time and show how the policy changes. Interpret your results.

[1] In the code I shared, I had assumed that the cake shrinking rate ρ was equal to 1, and since the cake size remained constant, we can ignore it as a state. Now the size of the cake is a new state to consider.