Can Global Uncertainty Promote International Trade?

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Online Appendix

Appendix A. Proofs and Solution Details

Appendix A.1. Preliminaries for CES model

Let p be the price of domestic goods x relative to foreign goods y and let $q \equiv 1/p$ be the price of foreign goods y relative to domestic goods x. We denote foreign variables with an asterisk *. Here, we solve the general model for any level of risk aversion σ. In the proofs for the CES case, we set $\sigma = 1 - \theta$ to obtain analytical results.

Export policies. The maximization problem for country x is given by

$$V(z_x, \mu_x, \hat{m}_y) = \max_{t_x} \mathbb{E}\left[\frac{1}{1-\sigma}\left(c_x^\theta + c_y^\theta\right)^{\frac{1-\sigma}{\theta}}\right] = \max_{t_x} \mathbb{E}\left[\frac{1}{1-\sigma}\left((z_x - t_x)^\theta + (pt_x)^\theta\right)^{\frac{1-\sigma}{\theta}}\right]. \quad \text{(A.1)}$$

The FOC of the maximization problem is given by

$$(z_x - t_x)^{\theta-1} \mathbb{E}\left[\left((z_x - t_x)^\theta + (pt_x)^\theta\right)^{\frac{1-\sigma-\theta}{\theta}}\right] = t_x^{\theta-1}\mathbb{E}\left[p^\theta\left((z_x - t_x)^\theta + (pt_x)^\theta\right)^{\frac{1-\sigma-\theta}{\theta}}\right]. \quad \text{(A.2)}$$

Rearranging yields

$$t_x = z_x\left(\frac{Q(z_x, \mu_x, \hat{m}_y)}{\bar{\lambda}(z_x, \mu_x, \hat{m}_y)}\right)^{\frac{\theta}{1-\theta}}, \quad \text{(A.3)}$$

where we define the following objects:

$$\begin{aligned} Q(z_x, \mu_x, \hat{m}_y, p) &\equiv \left(\mathbb{E}[\lambda(z_x, \mu_x, \hat{m}_y)^\theta]^{\frac{1}{\theta-1}} + \mathbb{E}\left[\left(p\lambda(z_x, \mu_x, \hat{m}_y)\right)^\theta\right]^{\frac{1}{\theta-1}}\right)^{\frac{\theta-1}{\theta}}; \\ \bar{\lambda}(z_x, \mu_x, \hat{m}_y) &\equiv \mathbb{E}[\lambda(z_x, \mu_x, \hat{m}_y)^\theta]^{\frac{1}{\theta}}; \\ \lambda(z_x, \mu_x, \hat{m}_y)^\theta &\equiv c(z_x, \mu_x, \hat{m}_y)^{1-\sigma-\theta} = \left((z_x - t_x)^\theta + (pt_x)^\theta\right)^{\frac{1-\sigma-\theta}{\theta}}. \end{aligned} \quad \text{(A.4)}$$

We guess a multiplicative solution $t(z_x, \mu_x, \hat{m}_y) = z_x\Psi(\mu_x, \hat{m}_y)$, where exports represent a share Ψ of idiosyncratic endowment z_x. Note that the share only depends on aggregate shocks. Substituting the guess, we get an expression for the consumption bundle:

$$c(z_x, \mu_x, \hat{m}_y) = \left((z_x - t_x)^\theta + (pt_x)^\theta\right)^{\frac{1}{\theta}} = z_x\Psi_2(\mu_x, \hat{m}_y; p) \quad \text{(A.5)}$$

where

$$\Psi_2(\mu_x, \hat{m}_y; p) \equiv \left((1 - \Psi(\mu_x, \hat{m}_y))^\theta + \left(p\Psi(\mu_x, \hat{m}_y)\right)^\theta\right)^{\frac{1}{\theta}}. \quad \text{(A.6)}$$

Now we have that

$$\begin{aligned} Q(z_x, \mu_x, \mu_y, p) &= z_x^{\frac{1-\sigma}{\theta}(1-\sigma-\theta)}\left(\mathbb{E}\left[\Psi_2(\mu_x, \hat{m}_y; p)^{(1-\sigma)(1-\sigma-\theta)}\right]^{\frac{1}{\theta-1}} + \mathbb{E}\left[p^\theta\Psi_2(\mu_x, \hat{m}_y; p)^{(1-\sigma)(1-\sigma-\theta)}\right]^{\frac{1}{\theta-1}}\right)^{\frac{\theta-1}{\theta}}, \\ \bar{\lambda}(z_x, \mu_x, \hat{m}_y) &= z_x^{\frac{1-\sigma}{\theta}(1-\sigma-\theta)}\mathbb{E}\left[\Psi_2(\mu_x, \hat{m}_y; p)^{(1-\sigma)(1-\sigma-\theta)}\right]^{\frac{1}{\theta}}. \end{aligned}$$

Computing the ratio of the two previous objects, we confirm the guess of a multiplicative solutions to the export policy for a domestic firm:

$$t_x = z_x \left(\frac{Q(z_x, \mu_x, \hat{m}_y, p)}{\bar{\lambda}(z_x, \mu_x, \hat{m}_y)} \right)^{\frac{\theta}{1-\theta}} = z_x \Psi(\mu_x, \hat{m}_y), \tag{A.7}$$

and obtain the domestic aggregate export share of income as

$$\Psi(\mu_x, \hat{m}_y) = \left(\frac{\left(\mathbb{E}\left[\Psi_2(\mu_x, \hat{m}_y; p)^{(1-\sigma)(1-\sigma-\theta)} \right]^{\frac{1}{\theta-1}} + \mathbb{E}\left[p^\theta \Psi_2(\mu_x, \hat{m}_y; p)^{(1-\sigma)(1-\sigma-\theta)} \right]^{\frac{1}{\theta-1}} \right)^{\frac{\theta-1}{\theta}}}{\mathbb{E}\left[\Psi_2(\mu_x, \hat{m}_y; p)^{(1-\sigma)(1-\sigma-\theta)} \right]^{\frac{1}{\theta}}} \right)^{\frac{\theta}{1-\theta}}. \tag{A.8}$$

For country y, we get the export policy for a foreign firm as

$$t_y = z_y \Psi^*(\mu_y, \hat{m}_x), \tag{A.9}$$

$$\Psi^*(\mu_y, \hat{m}_x) = \left(\frac{\left(\mathbb{E}\left[\Psi_2^*(\mu_y, \hat{m}_x; q)^{(1-\sigma)(1-\sigma-\theta)} \right]^{\frac{1}{\theta-1}} + \mathbb{E}\left[q^\theta \Psi_2^*(\mu_y, \hat{m}_x; q)^{(1-\sigma)(1-\sigma-\theta)} \right]^{\frac{1}{\theta-1}} \right)^{\frac{\theta-1}{\theta}}}{\mathbb{E}\left[\Psi_2^*(\mu_y, \hat{m}_x; q)^{(1-\sigma)(1-\sigma-\theta)} \right]^{\frac{1}{\theta}}} \right)^{\frac{\theta}{1-\theta}} \tag{A.10}$$

$$\Psi_2^*(\mu_y, \hat{m}_x; q) \equiv \left((1 - \Psi^*(\mu_y, \hat{m}_x))^\theta + \left(q \Psi^*(\mu_y, \hat{m}_x) \right)^\theta \right)^{\frac{1}{\theta}}. \tag{A.11}$$

Aggregate variables. We have all we need to find the aggregate variables. Aggregate exports are

$$T_x(\mu_x, \hat{m}_y) = \int z_x \Psi(\mu_x, \hat{m}_y) dF(z_x|\mu_x) = \exp(\mu_x + \sigma_x^2/2) \Psi(\mu_x, \hat{m}_y), \tag{A.12}$$

$$T_y(\mu_y, \hat{m}_x) = \int z_y \Psi^*(\mu_y, \hat{m}_x) dF(z_y|\mu_y) = \exp(\mu_y + \sigma_y^2/2) \Psi^*(\mu_y, \hat{m}_x). \tag{A.13}$$

Therefore, the terms of trade are given by

$$p(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) = \frac{\exp(\mu_y + \sigma_y^2/2) \Psi^*(\mu_y, \hat{m}_x)}{\exp(\mu_x + \sigma_x^2/2) \Psi(\mu_x, \hat{m}_y)} = f \frac{\Psi^*(\mu_y, \hat{m}_x)}{\Psi(\mu_x, \hat{m}_y)}, \tag{A.14}$$

where aggregate relative fundamental is defined as

$$f \equiv \exp(\mu_y - \mu_x) \exp((\sigma_y^2 - \sigma_x^2)/2). \tag{A.15}$$

Imperfect information. The equilibrium conditions of the model under imperfect information are given by $\Psi(\mu_x, \hat{m}_y)$ in (A.8), $\Psi^*(\mu_y, \hat{m}_x)$ in (A.10) and $p(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)$ in (A.14).

Perfect information. As a reference, we can solve in closed form for the equilibrium conditions of the model under perfect information:

$$\Psi^{PI}(\mu_x,\mu_y) = \frac{1}{1+p^{PI}(\mu_x,\mu_y)^{\frac{-\theta}{1-\theta}}} = \frac{1}{1+f^{-\theta}}; \tag{A.16}$$

$$\Psi^{*PI}(\mu_x,\mu_y) = \frac{1}{1+p^{PI}(\mu_x,\mu_y)^{\frac{\theta}{1-\theta}}} = \frac{1}{1+f^{\theta}}; \tag{A.17}$$

$$p^{PI}(\mu_x,\mu_y) = f\frac{\Psi^{*PI}(\mu_x,\mu_y)}{\Psi^{PI}(\mu_x,\mu_y)} = f^{1-\theta}; \tag{A.18}$$

$$f = \exp(\mu_y-\mu_x)\exp((\sigma_y^2-\sigma_x^2)/2). \tag{A.19}$$

Therefore, the larger is the fundamental of country y relative to country x, i.e., f is larger, the relative price of x goods increases ($\theta < 1$). In turn, this increases the export share of income in country x, Ψ^{PI}, and decreases the export share in country y, Ψ^{*PI}. Overall, the total export volume increases in both countries: The good shock in the foreign country has global spillovers.

Appendix A.2. Proofs

Proof of Result 1: Uncertainty Reduces the Covariance of Aggregate Exports.

Proof. We split the proof in two parts.

Part 1: From derivative to covariance. The first step is to connect the derivative dT_x/dT_y with the covariance $\mathbb{C}ov[T_x,T_y|\mathcal{I}_x]$. Note that T_y and μ_x are the only random variables for the agent in country x. A first-order approximation of the policy function $T_x(T_y,\mu_x)$ yields

$$T_x(\mu_x,T_y) \approx T_x(\mu_x,\mathbb{E}[T_y|\mathcal{I}_x]) + \beta\left(T_y - \mathbb{E}[T_y|\mathcal{I}_x]\right) + \Psi^*(\mu_x - m_x) = \alpha + \beta T_y + \Psi^*\mu_x, \tag{A.20}$$

where α gathers all constants. From an ex ante perspective, T_x is a random variable. With this approximation, the covariance of T_x with T_y is given by

$$\mathbb{C}ov[T_x,T_y] \approx \mathbb{C}ov(\alpha+\beta T_y+\Psi^*\mu_x,T_y) = \beta\mathbb{V}ar(T_y) \tag{A.21}$$

i.e., the own aggregate shock does not induce covariance with other countries' exports. Therefore, the slope is

$$\beta = \frac{\mathbb{C}ov[T_x,T_y]}{\mathbb{V}ar(T_y)} = \frac{dT_x}{dT_y}\Big|_{T_y=\mathbb{E}[T_y]}. \tag{A.22}$$

With no information, $\beta = 0$. With perfect information, $dT_x/dT_y \neq 0\ \forall T_y$, and therefore, $\beta > 0$. We have established that

$$sign\left(\frac{dT_x}{dT_y}\right) = sign\left(\mathbb{C}ov(T_x,T_y)\right) = sign(\beta). \tag{A.23}$$

Part 2: Continuity of the covariance in the amount of information. If the conditional distribution of terms of trade p is a continuous function of the signal and its precision, then the continuity of

Bayesian updating, together with the continuity of the integral operator, ensures that any conditional expectation is also continuous. Since the covariance is an expectation, it is also a continuous function of the signal precision. By (i) for no information (zero precision) the covariance is zero, and for perfect information (infinity precision) the covariance is positive. By the continuity established in (ii), there exists an interval for precision between 0 and infinity for which the covariance is increasing in precision. Therefore, more information increases the covariance of aggregate exports. □

Proof of Result 2: Uncertainty Increases Mean and Variance of Terms of Trade.

Proof. A second-order approximation of $p = T_y/T_x$ around the unconditional expectation of exports $(\mathbb{E}[T_x], \mathbb{E}[T_y])$ yields

$$\begin{aligned} \frac{T_y}{T_x} &\approx \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]} + \frac{1}{\mathbb{E}[T_x]}(T_y - \mathbb{E}[T_y]) - \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]^2}(T_x - \mathbb{E}[T_x]) \\ &+ \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]^3}(T_x - \mathbb{E}[T_x])^2 - \frac{1}{\mathbb{E}[T_x]^2}(T_x - \mathbb{E}[T_x])(T_y - \mathbb{E}[T_y]). \end{aligned} \tag{A.24}$$

Taking expectations on both sides, which makes the first-order terms equal to zero, yields

$$\mathbb{E}\left[\frac{T_y}{T_x}\right] \approx \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]} + \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]^3}\mathbb{V}ar[T_x] - \frac{1}{\mathbb{E}[T_x]^2}\mathbb{C}ov[T_x, T_y]. \tag{A.25}$$

By symmetry, $\mathbb{E}[T_y] = \mathbb{E}[T_x]$ and $\mathbb{V}ar[T_y] = \mathbb{V}ar[T_x]$, we can simplify to

$$\mathbb{E}\left[\frac{T_y}{T_x}\right] = 1 + \frac{\mathbb{V}ar[T_x]}{\mathbb{E}[T_x]^2} - \frac{\mathbb{C}ov[T_x, T_y]}{\mathbb{E}[T_x]^2}. \tag{A.26}$$

Furthermore, using the definition of coefficient of variation $\mathbb{CV}^2[z] = \mathbb{V}ar[z]/\mathbb{E}[z]^2$ and the correlation coefficient, together with symmetry across countries, we obtain:

$$\mathbb{E}[p] = 1 + \mathbb{CV}^2[T_x]\left(1 - corr([T_x, T_y])\right). \tag{A.27}$$

The proof is analogous from the foreign country's perspective, using an approximation of $1/p$.

Now for the variance, a first-order approximation of $p = T_y/T_x$ around the expectation of aggregate exports $(\mathbb{E}[T_x], \mathbb{E}[T_y])$ yields

$$\frac{T_y}{T_x} \approx \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]} + \frac{1}{\mathbb{E}[T_x]}(T_y - \mathbb{E}[T_y]) - \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]^2}(T_x - \mathbb{E}[T_x]). \tag{A.28}$$

Now take expectation on both sides and cancel the first-order terms:

$$\mathbb{E}\left[\frac{T_y}{T_x}\right] \approx \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]}. \tag{A.29}$$

Subtract the two previous expressions to compute the variance:

$$\begin{aligned} \mathbb{V}ar\left[\frac{T_y}{T_x}\right] &= \mathbb{E}\left[\left(\frac{T_y}{T_x} - \mathbb{E}\left[\frac{T_y}{T_x}\right]\right)^2\right] = \mathbb{E}\left[\left(\frac{1}{\mathbb{E}[T_x]}(T_y - \mathbb{E}[T_y]) - \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]^2}(T_x - \mathbb{E}[T_x])\right)^2\right] \\ &= \frac{1}{\mathbb{E}[T_x]^2}\left[\mathbb{V}ar[T_y] + \frac{\mathbb{E}[T_y]^2}{\mathbb{E}[T_x]^2}\mathbb{V}ar[T_x] - 2\frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]}\mathbb{C}ov[T_x, T_y]\right]. \end{aligned} \tag{A.30}$$

Keeping the variance of exports constant, the larger covariance decreases the variance of the terms of trade. By symmetry, $\mathbb{E}[T_y] = \mathbb{E}[T_x]$ and $\mathbb{V}ar[T_y] = \mathbb{V}ar[T_x]$, we can simplify to

$$\mathbb{V}ar\left[\frac{T_y}{T_x}\right] = \frac{2}{\mathbb{E}[T_x]^2}\left(\mathbb{V}ar[T_x] - \mathbb{C}ov[T_x, T_y]\right). \tag{A.31}$$

Again, using the definition of coefficient of variation and the correlation coefficient and symmetry:

$$\frac{\mathbb{V}ar\,[p]}{2} = CV^2[T_x]\left(1 - corr([T_x, T_y])\right). \tag{A.32}$$

The proof is analogous for the foreign country. □

Proof of Proposition 1. This is a special case of Proposition 3, proven below.

Proof of Lemma 1.

Lemma 1. *Let $g(\cdot)$ be the probability density function of the terms of trade and $\phi(\cdot)$ be a normal probability density. Suppose that $g(\cdot) = h(\cdot) * \phi(\cdot)$, where h is continuous. Then the function h must be somewhere convex.*

Proof. The terms of trade are a ratio of two nonnegative stochastic variables: T_x/T_y. Both T_x and T_y are proportional to a log-normal variable. As such, they can take any positive value with strictly positive probability density. Thus, the ratio of the two variables has positive density over the positive real line. Thus, the function $g(p)$ takes on value zero for all $p < 0$.

We can thus deduce three properties on the function h: (1) If $g(p)$ takes on value zero for all $p < 0$ and the normal density is positive-valued over the whole real line, then $h(p)$ must be zero for all $p < 0$. (2) For g to be a probability density, it must be that $h(p)$ does not fall below zero. (3) h cannot be a constant function. Since we know it takes value zero, it would then be zero everywhere. If that were true, g would be zero everywhere, which is not a probability density because it does not integrate to one.

From these three properties we can deduce that h must be somewhere convex. Suppose not. If the function h is nowhere convex, then it is globally, weakly concave. Since it is not a constant function, there exists some x^* such that $h'(x^*) \neq 0$. Let m be a linear function with slope $h'(x^*)$ that passes x^*. Then for all $x \in \mathbb{R}$, $h(x) \leq m(x)$. Since g is a linear function with nonzero slope, there exists p^* such that $m(p^*) < 0$. This means that $h(p^*) < 0$, which violates the assumption that $h(p)$ does not fall below zero. This contradiction proves that under the assumptions stipulated, h must be somewhere convex. □

Proof of Proposition 2: Optimal exports as a function of terms of trade moments..

Proof. Given the domestic country state—endowment μ_x and signal about foreign endowment $\tilde{m}_y$—the FOC of the maximization problem yields

$$\mathbb{E}_x[w(p)] = 0 \qquad \text{with} \qquad w(p) = pU_y(f_x - T_x(p), pT_x(p)) - U_x(f_x - T_x(p), pT_x(p)) \tag{A.33}$$

where the expectation operator is conditional on its information set $\mathbb{E}_x[\cdot] = \mathbb{E}[\cdot|\mu_x, \tilde{m}_y]$ (equal to its state), $w(p)$ is the marginal utility of exports, and $f_x = e^{\mu_x}$ is the country's aggregate endowment.

A second-order approximation of $w(p)$ around the terms of trade conditional expectation $\mathbb{E}_x[p]$ gives:

$$\begin{aligned}
0 = \mathbb{E}_x[w(p)] &\approx w(\mathbb{E}_x[p]) + w'(\mathbb{E}_x[p])\mathbb{E}_x[p - \mathbb{E}_x[p]] + \frac{1}{2}w''(\mathbb{E}_x[p])\mathbb{E}_x[p - \mathbb{E}_x[p])^2] \\
0 &= w(\mathbb{E}_x[p]) + \frac{\mathbb{V}ar_x[p]}{2}w''(\mathbb{E}_x[p]) \qquad \text{(A.34)} \\
0 &= U_y\left(\mathbb{E}_x[p]\right)\left\{\mathbb{E}_x[p] - \rho_y^{(1)}(\mathbb{E}_x[p])\left(\frac{2-\tilde{\rho}_y^{(2)}(\mathbb{E}_x[p])}{2}\right)\frac{\mathbb{V}ar_x[p]}{\mathbb{E}_x[p]}\right\} - U_x\left(\mathbb{E}_x[p]\right) \\
0 &= \varphi(T, \mathbb{E}_x[p], \mathbb{V}ar_x[p]).
\end{aligned}$$

The expression $\varphi(\cdot) = 0$ determines optimal exports as a function of conditional moments of the terms of trade. Rearranging the expression in terms of the marginal rate, we obtain the result:

$$\frac{U_x\left(\mathbb{E}_x[p]\right)}{U_y\left(\mathbb{E}_x[p]\right)} = \mathbb{E}_x[p]\left\{1 - \rho_y^{(1)}(\mathbb{E}_x[p])\left(2 - \tilde{\rho}_y^{(2)}(\mathbb{E}_x[p])\right)\frac{\mathbb{C}\mathbb{V}[p]^2}{2}\right\}. \qquad \text{(A.35)}$$

□

Proof of Proposition 3: Sign of change in exports for general case.

Proof. By Implicit Function Theorem applied to $\varphi(T, \mathbb{E}_x[p], \mathbb{V}ar_x[p]) = 0$, we have that

$$\begin{aligned}
\frac{\partial\varphi}{\partial T_x}\left(\frac{\partial T_x}{\partial\mathbb{E}_x[p]}d\mathbb{E}_x[p] + \frac{\partial T_x}{\partial\mathbb{V}ar_x[p]}d\mathbb{V}ar_x[p]\right) + \frac{\partial\varphi}{\partial\mathbb{E}_x[p]}d\mathbb{E}_x[p] + \frac{\partial\varphi}{\partial\mathbb{V}ar_x[p]}d\mathbb{V}ar_x[p] = 0 \\
\frac{\partial T_x}{\partial\mathbb{E}_x[p]}d\mathbb{E}_x[p] + \frac{\partial T_x}{\partial\mathbb{V}ar_x[p]}d\mathbb{V}ar_x[p] = -\left(\frac{\partial\varphi}{\partial\mathbb{E}_x[p]}d\mathbb{E}_x[p] + \frac{\partial\varphi}{\partial\mathbb{V}ar_x[p]}d\mathbb{V}ar_x[p]\right)\Big/\frac{\partial\varphi}{\partial T_x}.
\end{aligned}$$

Since the denominator is negative (utility is concave in exports), the sign of the derivative is given by the numerator.

$$\begin{aligned}
num &= \frac{\partial\varphi}{\partial\mathbb{E}_x[p]}d\mathbb{E}_x[p] + \frac{\partial\varphi}{\partial\mathbb{V}ar_x[p]}d\mathbb{V}ar_x[p] \\
&= \left(w'(\mathbb{E}_x[p]) + \frac{\mathbb{V}ar_x(p)}{2}w'''(\mathbb{E}_x[p])\right)d\mathbb{E}_x[p] + \frac{1}{2}w''(\mathbb{E}_x[p])d\mathbb{V}ar_x[p] \\
&= U_y(\mathbb{E}_x[p])\left[\left(\left(1-\tilde{\rho}_y^{(1)}\right) + \frac{\rho_y^{(1)}\rho_y^{(2)}}{\mathbb{E}_x[p]^2}\frac{\mathbb{V}ar_x[p]}{2}(3-\tilde{\rho}_y^{(3)})\right)d\mathbb{E}_x[p] - \frac{\rho_y^{(1)}}{\mathbb{E}_x[p]}(2-\tilde{\rho}_y^{(2)})\frac{d\mathbb{V}ar_x[p]}{2}\right]
\end{aligned}$$

where the new term $w'''(p)$ is equal to $w'''(p) = U_y\frac{\rho_y^{(1)}\rho_y^{(2)}}{p^2}(\tilde{\rho}_y^{(3)} - 3)$, where $\tilde{\rho}_y^{(3)} = \rho_y^{(3)}\left(1 - \frac{U_{xyyy}/p}{U_{yyyy}}\right)$ and $\rho_y^{(3)} \equiv -\frac{C_y U_{yyyy}}{U_{yyy}}$ is the coefficient of relative temperance. If this expression is positive, then an increase in the conditional mean and variance of the terms of trade increases exports. □

Proof of Corollary: Sign of change in exports for CES. Recall that for CES cases, all cross-derivatives are equal to zero and thus adjusted risk attitudes (denoted with tildes) are equal to the standard ones, i.e., $\tilde{\rho}_y^{(k)} = \rho_y^{(k)}$. With this preference, we have that $\rho_y^{(k)} = k - \theta$ for all k. Substituting this result into the required condition yields

$$\underbrace{\left[\frac{\theta}{1-\theta}\right]}_{\text{risk aversion}} + \frac{\mathbb{CV}[p]^2}{2}(2-\theta)\left\{\underbrace{\left[\frac{-\theta}{(2-\theta)}\right]}_{\text{prudence}} + (3-\theta)\underbrace{\left[\frac{\theta}{(3-\theta)}\right]}_{\text{temperance}}\right\} < 0. \tag{A.36}$$

Finally, we simplify the expression to obtain the condition for the CES-like case:

$$\theta\left[1 + \frac{\mathbb{CV}[p]^2}{2}(1-\theta)^2\right] < 0. \tag{A.37}$$

Appendix B. A Model with Preference Shocks

In this section, we consider a related model in which there are preference shocks instead of aggregate endowment shocks. We want to show that this related model is equivalent to the original model, once we redefine variables.

Preferences. Suppose that the utility of all agents depends on preference shocks ρ_x and ρ_y:

$$\mathbb{E}\left[(\rho_x c_x^\theta + \rho_y c_y^\theta)^{1/\theta}\right] \tag{B.1}$$

where $\ln \rho_x \sim N(0, s_x^2)$ and $\ln \rho_y \sim N(0, s_y^2)$.

Endowments. Each agent in the domestic country has an idiosyncratic endowment of $\tilde{z}_x$ units of good x, where $\ln \tilde{z}_x \sim \mathcal{N}(m_x, \sigma_x^2)$. Each agent in the foreign country has an idiosyncratic endowment $\tilde{z}_y$ units of good y, where $\ln \tilde{z}_y \sim \mathcal{N}(m_y, \sigma_y^2)$. The means of these distributions are constants (in contrast to the previous model, in which they were random variables). These constants are common knowledge.

Information. All agents in country x know ρ_x, but not ρ_y. All agents in country y know ρ_y but not ρ_x. Each set of agents can receive normally distributed signals about the unknown preference shock. They form posterior beliefs by Bayes' Law.

Solution. Under these new preferences, the new optimization problem becomes

$$\tilde{t}_x = \arg\max \mathbb{E}\left[\left(\rho_x(\tilde{z}_x - \tilde{t}_x)^\theta + \rho_y\,(\tilde{p}\tilde{t}_x)^\theta\right)^{1/\theta}\middle|\, \mathcal{I}_x\right] \tag{B.2}$$

$$\tilde{t}_y = \arg\max \mathbb{E}\left[\left(\rho_x\left(\frac{\tilde{t}_y}{\tilde{p}}\right)^\theta + \rho_y(\tilde{z}_y - \tilde{t}_y)^\theta\right)^{1/\theta}\middle|\, \mathcal{I}_y\right]. \tag{B.3}$$

Market clearing has the same form as equation (16) in the main text.

We can rewrite the country-x problem as

$$\tilde{t}_x = \arg\max \mathbb{E}\left[\left((\rho_x^{1/\theta}\tilde{z}_x - \rho_x^{1/\theta}\tilde{t}_x)^\theta + \left(\rho_y^{1/\theta}\tilde{t}_x\right)^\theta \tilde{p}^\theta\right)^{1/\theta}\middle|\, \mathcal{I}_x\right]. \tag{B.4}$$

Next, the market-clearing condition becomes

$$\tilde{p} = \frac{\tilde{T}_y(m_y, \hat{m}_x)}{\tilde{T}_x(m_x, \hat{m}_y)} = \frac{\int \tilde{t}_y(\tilde{z}_y, m_y, \hat{m}_x)dF(\tilde{z}_y|m_y)}{\int \tilde{t}_x(\tilde{z}_x, m_x, \hat{m}_y)dF(\tilde{z}_x|m_x)}. \tag{B.5}$$

Since the agent in the x country knows ρ_x, we can redefine the choice variable to be $t_x \equiv \rho_x^{1/\theta}\tilde{t}_x$ and do a simple change of variable in the optimization problem and the constraint:

$$t_x = \arg\max \mathbb{E}\left[\left((\rho_x^{1/\theta}\tilde{z}_x - t_x)^\theta + t_x^\theta\left(\frac{\rho_x^{1/\theta}}{\rho_y^{1/\theta}}\frac{1}{\tilde{p}}\right)^{-\theta}\right)^{1/\theta}\middle|\, \mathcal{I}_x\right] \tag{B.6}$$

$$\tilde{p} = \frac{\rho_x^{1/\theta}}{\rho_y^{1/\theta}} \frac{\int t_y(\tilde{z}_y, m_y, \hat{m}_x) dF(\tilde{z}_y|m_y)}{\int t_x(\tilde{z}_x, m_x, \hat{m}_y) dF(\tilde{z}_x|m_x)}. \tag{B.7}$$

Comparing the expression above to (16), we see that

$$\tilde{p} = \frac{\rho_x^{1/\theta}}{\rho_y^{1/\theta}} p. \tag{B.8}$$

Substituting in for $\tilde{p}$ in (B.10), we get

$$t_x = \arg\max \mathbb{E}\left[\left((\rho_x^{1/\theta} \tilde{z}_x - t_x)^\theta + (t_x p)^\theta \right)^{1/\theta} \middle| \mathcal{I}_x \right]. \tag{B.9}$$

The last step is to define a random variable $z_x \equiv \rho_x^{1/\theta} \tilde{z}_x$ and substitute it into our problem:

$$t_x = \arg\max \mathbb{E}\left[\left((z_x - t_x)^\theta + (t_x p)^\theta \right)^{1/\theta} \middle| \mathcal{I}_x \right]. \tag{B.10}$$

Notice that this optimization problem is identical to the benchmark problem. The expectation measures are also identical, because the distribution of $\rho_x^{1/\theta} \tilde{z}_x$ is the same as the distribution of z_x. For $\ln(\rho_x^{1/\theta} \tilde{z}_x) \sim N(m_x, s_x^2 + \sigma_x^2)$. Similarly, $\ln z_x \sim N(m_x, s_x^2 + \sigma_x^2)$. Thus, the two problems are equivalent; one is just a change of variable of the other.

Interpretation. The one difference between these two models is the interpretation of what trade volume consists of. In the second problem, T_x is $\rho_x^{1/\theta}$ times trade volume. However, the trade share is identical in both models. To see this, note that in the preference shock model, the country-x trade share is

$$\frac{\tilde{t}_x}{\tilde{c}_x} = \frac{\tilde{t}_x}{\tilde{z}_x - \tilde{t}_x}. \tag{B.11}$$

Substituting in $t_x = \rho_x^{1/\theta} \tilde{t}_x$ and $z_x = \rho_x^{1/\theta} \tilde{z}_x$, we get

$$\frac{\tilde{t}_x}{\tilde{c}_x} = \frac{\rho_x^{-1/\theta} t_x}{\rho_x^{-1/\theta} z_x - \rho_x^{-1/\theta} t_x} = \frac{t_x}{z_x - t_x} = \frac{t_x}{c_x}. \tag{B.12}$$

Thus, the trade share in the preference shock model is equal to the trade share in the original model. Following the same steps reveals that the country-y trade share is also equal in both models. Since the aggregate supply shock model and the preference shock model are equivalent problems, it follows that providing both countries more information about preference shocks must reduce the trade share, for the same reason that more information about aggregate supply does.

Appendix C. Details of Computational Algorithm

This section describes the algorithm to compute the equilibrium, the simulation strategy, and parameter choices.

Summary of computational algorithm. We solve the fixed-point problem by iterating on the export policy functions Ψ and Ψ^*, which are approximated using linear splines. For each country we define grids for their two states: aggregate endowment and posterior mean of foreign endowment. We also define grids for foreign endowment and the second-order beliefs countries use to evaluate their perceived price function (Appendix C.1). Expectations with respect to foreign endowment and second-order beliefs are computed using Gaussian quadrature (Appendix C.2). Once we have solved the fixed-point problem (Appendix C.3), we simulate the repeated economy for T=100,000 periods and compute average statistics across simulations (Appendix C.4).

Appendix C.1. Polynomial approximation to policy functions

Functional Basis. Let $\{\Phi_k\}_{k=1}^M$ be a basis of polynomials with support $x \in [a, b]$. We use linear splines and uniform nodes for the two states of each country.

1. **Grid for state 1: Own endowment**:
   - In x country it is distributed $\mu_x \sim \mathcal{N}(m_x, s_x^2)$, where m_x, s_x are parameters. We construct uniform nodes $\{\mu_x^i\}_{i=1}^N$ in the support $[m_x - 4s_x, m_x + 4s_x]$.
   - In y country it is distributed $\mu_y \sim \mathcal{N}(m_y, s_y^2)$, where m_y, s_y are parameters. We construct uniform nodes $\{\mu_y^i\}_{i=1}^N$ in the support $[m_y - 4s_y, m_y + 4s_y]$.
2. **Grid for state 2: Posterior mean of foreign endowment**:
   - In x country, the posterior mean of foreign endowment is $\hat{m}_y \sim \mathcal{N}(m_y, \bar{s}_y^2)$ where $\bar{s}_y^2 = s_y^4/(s_y^2 + \tilde{s}_y^2)$. To use a fixed grid that does not change with the precision of information, we construct the nodes $\{\hat{\mu}_y^j\}_{j=1}^N$ over the support $[m_y - 4s_y, m_y + 4s_y]$.
   - In y country, the posterior mean of foreign endowment is $\hat{m}_x \sim \mathcal{N}(m_x, \bar{s}_x^2)$ where $\bar{s}_x^2 = s_x^4/(s_x^2 + \tilde{s}_x^2)$. We construct the nodes $\{\hat{m}_x^j\}_{j=1}^N$ over the support $[m_x - 4s_x, m_x + 4s_x]$.

Approximating functions. We approximate four conditional expectations with polynomials:

$$\mathbb{E}_{\mu_y,\hat{m}_x}\left[\Psi_2(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)^{1-\theta}\Big|\mathcal{I}_x\right] \approx g^1(\mu_x, \hat{m}_y),$$

$$\mathbb{E}_{\mu_y,\hat{m}_x}\left[\Psi_2(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)^{1-\theta} p(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)^{\theta}\Big|\mathcal{I}_x\right] \approx g^2(\mu_x, \hat{m}_y),$$

$$\mathbb{E}_{\mu_x,\hat{m}_y}\left[\Psi_2^*(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y)^{1-\theta}\Big|\mathcal{I}_y\right] \approx h^1(\mu_y, \hat{m}_x),$$

$$\mathbb{E}_{\mu_x,\hat{m}_y}\left[\Psi_2^*(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y)^{1-\theta} q(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)^{\theta}\Big|\mathcal{I}_y\right] \approx h^2(\mu_y, \hat{m}_x),$$

where the polynomials are constructed using the basis for each dimension evaluated at the nodes described above:

$$\begin{aligned}
g^1(\mu_x^i, \hat{m}_y^j) &\equiv \sum_{k,k' \in K \times K'} g^1_{k,k',i,j} \Phi_k(\mu_x^i) \Phi_{k'}(\hat{m}_y^j) \\
g^2(\mu_x^i, \hat{m}_y^j) &\equiv \sum_{k,k' \in K \times K'} g^2_{k,k',i,j} \Phi_k(\mu_x^i) \Phi_{k'}(\hat{m}_y^j) \\
h^1(\mu_y^i, \hat{m}_x^j) &\equiv \sum_{k,k' \in K \times K'} h^1_{k,k',i,j} \Phi_k(\mu_y^i) \Phi_{k'}(\hat{m}_x^j), \\
h^2(\mu_y^i, \hat{m}_x^j) &\equiv \sum_{k,k' \in K \times K'} h^2_{k,k',i,j} \Phi_k(\mu_y^i) \Phi_{k'}(\hat{m}_x^j).
\end{aligned}$$

Appendix C.2. Computing expectations

For each country, we have two random variables—foreign endowment and second-order beliefs—for which we evaluate expectations using Gaussian quadrature. For this, we must define a set of nodes $\{x_a\}_{a=1}^{N_q}$ and weights $\{w_a\}_{a=1}^{N_q}$ such that $\mathbb{E}[f(X)] = \sum_{a=1}^{N_q} w_a f(x_a)$ and further moments conditions are satisfied.

Grid for random variable 1: Foreign endowment: The distribution of foreign aggregate endowment depends on the second state, the posterior mean $\hat{m}$.

- In x country, for each value of the second state (the posterior mean) we have that foreign endowment is Normal with mean equal to the posterior mean $\hat{m}_y^j$ and variance equal to the posterior variance $\hat{s}_y^2 = \left(s_y^{-2} + \tilde{s}_y^{-2}\right)^{-1} = (1/s_y^2 + 1/\tilde{s}_y^2)^{-1}$, or

$$\mu_y^j \sim \mathcal{N}(\hat{m}_y^j, \hat{s}_y^2). \tag{C.1}$$

  Then for each $j = 1, ..., N$, Gaussian quadrature constructs nodes of foreign endowment $\{\mu_y^{j,b}\}_b = 1, ..., N_q$ and corresponding weights $\{\omega^b\}_b = 1, ..., N_q$. Note that the weights do not depend on j.

- In y country, for each value of the second state (the posterior mean $\hat{m}_x^j$) we have that foreign endowment is Normal with mean equal to the posterior mean $\hat{m}_x^j$ and variance equal to the posterior variance $\hat{s}_x^2 = \left(s_x^{-2} + \tilde{s}_x^{-2}\right)^{-1} = (1/s_x^2 + 1/\tilde{s}_x^2)^{-1}$, or

$$\mu_x^j \sim \mathcal{N}(\hat{m}_x^j, \hat{s}_x^2). \tag{C.2}$$

  Then for each $j = 1, ..., N$, Gaussian quadrature constructs nodes of foreign endowment $\{\mu_x^{j,b}\}_b = 1, ..., N_q$ and corresponding weights $\{\omega^b\}_b = 1, ..., N_q$.

Extreme cases.

- Perfect Info: As $\tilde{s}_y \to 0$, $\mu_y^j \sim \mathcal{N}(\hat{m}_y^j, 0) = \mathcal{N}(\mu_y^j, 0)$. The grid degenerates to a single point for each j: $\mu_y^{j,b} = \mu_y^j$.

- No Info: As $\tilde{s}_y \to \infty$, $\mu_y^j \sim \mathcal{N}(\hat{m}_y^j, s_y^2) = \mathcal{N}(m_y, \bar{s}_y^2)$ which is equal to the distribution of the posterior mean (the second state). Clearly, $\bar{s}_y^2 = s_y^4/(s_y^2 + \tilde{s}_y^2) \to s_y^2$ as well, which makes the distribution of foreign endowment equal to the prior. However, in the code we have fixed grids for the states so that they do not depend on signal precision. Therefore, as we reduce signal precision, the grid will not converge to the prior. However, the simulations take care of this.

Grid for random variable 2: Second-order beliefs: From the perspective of the domestic country, second-order beliefs about the posterior mean (that is, what the domestic country thinks the posterior mean of the foreign country is) is a normal random variable that depends on the first state, the domestic aggregate endowment μ.

- In country x, for each value of the first state (aggregate endowment μ_x^i), we have that the second-order belief is Normal $\hat{m}_x^i \sim \mathcal{N}(\hat{\hat{m}}_x^i, \hat{\hat{s}}_x^2)$ with mean and variance as follows:

$$\hat{\hat{m}}_x^i \equiv \frac{s_x^{-2} m_x + \tilde{s}_{p_x}^{-2} \mu_x^i}{s_x^{-2} + \tilde{s}_{p_x}^{-2}}, \quad \hat{\hat{s}}_x^2 \equiv \tilde{s}_{p_x}^{-2}(s_x^{-2} + \tilde{s}_{p_x}^{-2})^{-2} = \frac{1}{(\frac{\tilde{s}_{px}}{s_x^2} + \frac{1}{\tilde{s}_{px}})^2}, \tag{C.3}$$

  where $\tilde{s}_{p_x}$ is the foreign signal noise as perceived by the domestic country. With known information structures $\tilde{s}_{p_x} = \tilde{s}_x$, but with unknown information structures $\tilde{s}_{p_x} \neq \tilde{s}_x$.

  Then for each $i = 1, ..., N$, Gaussian quadrature constructs nodes for second-order beliefs $\{\mu_x^{i,a}\}_a = 1, ..., N_q$ and corresponding weights $\{\Psi^{*a}\}_a = 1, ..., N_q$.

- In country y we have that for each value of the first state μ_y^i the second-order belief is Normally distributed $\hat{m}_y^i \sim \mathcal{N}(\hat{\hat{m}}_y^i, \hat{\hat{s}}_y^2)$ with mean and variance:

$$\hat{\hat{m}}_y^i \equiv \frac{s_y^{-2} m_y + \tilde{s}_{p_y}^{-2} \mu_y^i}{s_y^{-2} + \tilde{s}_{p_y}^{-2}}, \quad \hat{\hat{s}}_y^2 \equiv \tilde{s}_{p_y}^{-2}(s_y^{-2} + \tilde{s}_{p_y}^{-2})^{-2} = \left(\frac{\tilde{s}_{p_y}}{s_y^2} + \frac{1}{\tilde{s}_{p_y}}\right)^{-2}. \tag{C.4}$$

Extreme cases.

- Perfect Info: As $\tilde{s}_{p_x} \to 0$, then the distribution becomes degenerate at the true realizations: $\hat{m}_x^i \sim \mathcal{N}(\mu_x^i, 0) \quad \forall i$ and the grid becomes: $\hat{m}_x^{i,a} = \mu_x^i,\ a = 1, ..., N_q$.
- Imperfect Info: As $\tilde{s}_{p_x} \to \infty$, then the distribution becomes degenerate at the prior means $\hat{m}_x^i \sim \mathcal{N}(m_x, 0) \quad \forall i$ and the grid becomes $\hat{m}_x^{i,a} = m_x,\ a = 1, ..., N$.

Appendix C.3. Finding the fixed point

1. For reference, we organize the states as follows. For country x: $(\mu_x, \hat{m}_y)$ and for country y: $(\mu_y, \hat{m}_x)$. For the price and other economy-wide variables, we make the following convention: $p(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)$.
2. Guess an initial set of coefficients for polynomials $\{g^1_{k,k',i,j}, g^2_{k,k',i,j}, h^1_{k,k',i,j}, h^2_{k,k',i,j}\}$.
    - We start by solving the perfect information case and approximate the policies with the polynomials to get the first set of coefficients. Since with perfect information

$\hat{m}_y = \mu_y$ and $\hat{m}_x = \mu_x$, we have the following system of equations:

$$\begin{aligned}
\Psi^{PI}(\mu_x, \mu_y) &= \frac{1}{1 + p^{PI}(\mu_x, \mu_y)^{\frac{\theta}{\theta-1}}} \\
\Psi^{*PI}(\mu_y, \mu_x) &= \frac{1}{1 + p^{PI}(\mu_x, \mu_y)^{\frac{\theta}{1-\theta}}} \\
p^{PI}(\mu_x, \mu_y) &= f \frac{\Psi^{*PI}(\mu_y, \mu_x)}{\Psi^{PI}(\mu_x, \mu_y)}.
\end{aligned}$$

Thus the price with information frictions is solved as: $p^{PI}(\mu_x, \mu_y) = f^{1-\theta} \quad \forall(\mu_y, \mu_x)$.

3. For country x:

- For each state $(\mu_x^i, \hat{m}_y^j)$, approximate Ψ using the polynomials g^1 and g^2 evaluated at the state:

$$\Psi(\mu_x^i, \hat{m}_y^j) \approx \frac{1}{1 + \left(\frac{g^1(\mu_x^i, \hat{m}_y^j)}{g^2(\mu_x^i, \hat{m}_y^j)}\right)^{\frac{1}{1-\theta}}}. \tag{C.5}$$

- For each quadrature node $(\mu_y^a, \hat{m}_x^b)$ approximate Ψ^* using the polynomials h^1 and h^2 evaluate at the nodes $\{\mu_y^a\}_{a=1}^{N_q}$, $\{\hat{m}_x^b\}_{b=1}^{N_q}$

$$\Psi^*(\mu_y^a, \hat{m}_x^b) \approx \frac{1}{1 + \left(\frac{h^1(\mu_y^a, \hat{m}_x^b)}{h^2(\mu_y^a, \hat{m}_x^b)}\right)^{\frac{1}{1-\theta}}}. \tag{C.6}$$

- Construct price p and Ψ_2 in 4 dimensions using $\Psi(\mu_x^i, \hat{m}_y^j)$ and $\Psi^*(\mu_y^a, \hat{m}_x^b)$:

$$p(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b) \approx \exp((\mu_y^a - \mu_x^i)) \exp((\sigma_y^2 - \sigma_x^2)/2) \frac{\Psi^*(\mu_y^a, \hat{m}_x^b)}{\Psi(\mu_x^i, \hat{m}_y^j)} \tag{C.7}$$

$$\Psi_2(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b) = \left((1 - \Psi(\mu_x^i, \hat{m}_y^j))^\theta + \left(\Psi(\mu_x^i, \hat{m}_y^j) p(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b)\right)^\theta\right)^{\frac{1}{\theta}}. \tag{C.8}$$

- Compute the conditional expectations of $\Psi_2^{1-\theta}$ and $\Psi_2^{1-\theta} p^\theta$ that integrate out the two random variables $(\mu_y, \hat{m}_x)$ as the weighted sum of the functions evaluated at the quadrature nodes, using the quadrature weights $\{\omega^a\}_{a=1}^{N_q}$ and $\{\Psi^{*b}\}_{b=1}^{N_q}$:

$$\begin{aligned}
& \mathbb{E}_{\mu_y, \hat{m}_x}\left[\Psi_2^{1-\theta}(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x) \middle| \mathcal{I}_x\right] \\
= & \int_{\mu_y} \int_{\hat{m}_x} \Psi_2^{1-\theta}(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x) \quad \phi\left(\frac{\mu_y - \hat{m}_y}{\hat{s}_y}\right) \phi\left(\frac{\hat{m}_x - \hat{\hat{m}}_x}{\hat{\hat{s}}_x}\right) d\mu_y d\hat{m}_x \\
\approx & \sum_{a=1}^{N_q} \sum_{b=1}^{N_q} \omega^a \Psi^{*b} \Psi_2^{1-\theta}(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b)
\end{aligned}$$

$$
\begin{aligned}
& \mathbb{E}_{\mu_y,\hat{m}_x}\left[\Psi_2^{1-\theta}(\mu_x,\hat{m}_y,\mu_y,\hat{m}_x)p^{\theta}(\mu_x,\hat{m}_y,\mu_y,\hat{m}_x)\middle|\mathcal{I}_x\right] \\
= & \int_{\mu_y}\int_{\hat{m}_x}\Psi_2^{1-\theta}(\mu_x,\hat{m}_y,\mu_y,\hat{m}_x)p^{\theta}(\mu_x,\hat{m}_y,\mu_y,\hat{m}_x)\phi\left(\frac{\mu_y-\hat{m}_y}{\hat{s}_y}\right)\phi\left(\frac{\hat{m}_x-\hat{\hat{m}}_x}{\hat{\hat{s}}_x}\right)d\mu_y d\hat{m}_x \\
\approx & \sum_{a=1}^{N_q}\sum_{b=1}^{N_q}\omega^a\Psi^{*b}\Psi_2^{1-\theta}(\mu_x^i,\hat{m}_y^j,\mu_y^a,\hat{m}_x^b)p^{\theta}(\mu_x^i,\hat{m}_y^j,\mu_y^a,\hat{m}_x^b).
\end{aligned}
$$

4. For country Y, we do analogous calculations.
   - For each state $(\mu_y^i,\hat{m}_x^j)$, approximate Ψ^* using the polynomials h^1 and h^2 evaluated at the state:
$$
\Psi^*(\mu_y^i,\hat{m}_x^j)\approx\frac{1}{1+\left(\frac{h^1(\mu_y^i,\hat{m}_x^j)}{h^2(\mu_y^i,\hat{m}_x^j)}\right)^{\frac{1}{1-\theta}}}. \tag{C.9}
$$
   - For each quadrature node $(\mu_x^a,\hat{m}_y^b)$ approximate Ψ using the polynomials g^1 and g^2 evaluated at the nodes $\{\mu_x^a\}_{a=1}^{N_q}$, $\{\hat{m}_y^b\}_{b=1}^{N_q}$
$$
\Psi(\mu_x^a,\hat{m}_y^b)\approx\frac{1}{1+\left(\frac{g^1(\mu_x^a,\hat{m}_y^b)}{g^2(\mu_x^a,\hat{m}_y^b)}\right)^{\frac{1}{1-\theta}}}. \tag{C.10}
$$
   - Construct p and Ψ_2^* in 4 dimensions using $\Psi^*(\mu_y^i,\hat{m}_x^j)$ and $\Psi(\mu_x^a,\hat{m}_y^b)$ (note that the state for the price is in the same order as for the X-country):
$$
p(\mu_x^a,\hat{m}_y^b,\mu_y^i,\hat{m}_x^j)\approx\exp(\mu_y^i-\mu_x^a)\exp((\sigma_y^2-\sigma_x^2)/2)\frac{\Psi^*(\mu_y^i,\hat{m}_x^j)}{\Psi(\mu_x^a,\hat{m}_y^b)} \tag{C.11}
$$
$$
\Psi_2^*(\mu_y^i,\hat{m}_x^j,\mu_x^a,\hat{m}_y^b)=\left((1-\Psi^*(\mu_y^i,\hat{m}_x^j))^{\theta}+\left(\frac{\Psi^*(\mu_y^i,\hat{m}_x^j)}{p(\mu_x^a,\hat{m}_y^b,\mu_y^i,\hat{m}_x^j)}\right)^{\theta}\right)^{\frac{1}{\theta}}. \tag{C.12}
$$
   - Compute the conditional expectations of ${\Psi_2^*}^{1-\theta}$ and ${\Psi_2^*}^{1-\theta}q^{\theta}$ that integrate out the two random variables $(\mu_x,\hat{m}_y)$, where $q=1/p$. This is just the weighted sum of the functions evaluated at the quadrature nodes, using the quadrature weights $\{\omega^a\}_{a=1}^{N_q}$ and $\{\Psi^{*b}\}_{b=1}^{N_q}$:
$$
\begin{aligned}
& \mathbb{E}_{\mu_x,\hat{m}_y}\left[{\Psi_2^*}^{1-\theta}(\mu_y,\hat{m}_x,\mu_x,\hat{m}_y)\middle|\mathcal{I}_y\right] \\
= & \int_{\mu_y}\int_{\hat{m}_x}{\Psi_2^*}^{1-\theta}(\mu_y,\hat{m}_x,\mu_x,\hat{m}_y)\quad\phi\left(\frac{\mu_x-\hat{m}_x}{\hat{s}_x}\right)\phi\left(\frac{\hat{m}_y-\hat{\hat{m}}_y}{\hat{\hat{s}}_y}\right)d\mu_x d\hat{m}_y \\
\approx & \sum_{a=1}^{N_q}\sum_{b=1}^{N_q}\omega^a\Psi^{*b}{\Psi_2^*}^{1-\theta}(\mu_y^i,\hat{m}_x^j,\mu_x^a,\hat{m}_y^b)
\end{aligned}
$$

$$
\begin{aligned}
& \mathbb{E}_{\mu_x,\hat{m}_y}\left[\Psi_2^{*\,1-\theta}(\mu_y,\hat{m}_x,\mu_x,\hat{m}_y)q^{\theta}(\mu_x,\hat{m}_y,\mu_y,\hat{m}_x)\middle|\mathcal{I}_y\right] \\
= & \int_{\mu_y}\int_{\hat{m}_x}\Psi_2^{*\,1-\theta}(\mu_y,\hat{m}_x,\mu_x,\hat{m}_y)q^{\theta}(\mu_x,\hat{m}_y,\mu_y,\hat{m}_x)\phi\left(\frac{\mu_x-\hat{m}_x}{\hat{s}_x}\right)\phi\left(\frac{\hat{m}_y-\hat{\hat{m}}_y}{\hat{\hat{s}}_y}\right)d\mu_x d\hat{m}_y \\
\approx & \sum_{a=1}^{N_q}\sum_{b=1}^{N_q}\omega^a\Psi^{*b}\Psi_2^{*\,1-\theta}(\mu_x^i,\hat{m}_y^j,\mu_y^a,\hat{m}_x^b)q^{\theta}(\mu_x^a,\hat{m}_y^b,\mu_y^i,\hat{m}_x^j).
\end{aligned}
$$

5. Update coefficients by (i) fitting polynomials to approximate the conditional expectations and (ii) using a linear combination of the new coefficients with the previous guess.
6. Repeat steps until convergence of coefficients.
7. Once convergence is achieved, recover all variables at the firm level and at the aggregate level. Recall the definitions of domestic, foreign, and relative fundamentals:

$$
f_x \equiv e^{\mu_x+\frac{1}{2}\sigma_x^2}, \qquad f_y \equiv e^{\mu_y+\frac{1}{2}\sigma_y^2} \qquad f \equiv e^{(\mu_y-\mu_x)}e^{\frac{1}{2}(\sigma_y^2-\sigma_x^2)}. \tag{C.13}
$$

   (a) Price function:

$$
p(\mu_x,\mu_y,\hat{m}_x,\hat{m}_y) = f\,\frac{\Psi^*(\mu_y,\hat{m}_x)}{\Psi(\mu_x,\hat{m}_y)}. \tag{C.14}
$$

   (b) Firms' export policy and consumptions in x country:

$$
\begin{aligned}
t_x(z_x,\mu_x,\hat{m}_y) &= z_x\Psi(\mu_x,\hat{m}_y) \\
c_x(z_x,\mu_x,\hat{m}_y) &= z_x(1-\Psi(\mu_x,\hat{m}_y)) \\
c_y(z_x,\mu_x,\hat{m}_y,\mu_y,\hat{m}_x) &= p(\mu_x,\mu_y,\hat{m}_x,\hat{m}_y)t_x(z_x,\mu_x,\hat{m}_y) \\
c(z_x,\mu_x,\hat{m}_y,\mu_y,\hat{m}_x) &= z_x\Psi_2(\mu_x,\hat{m}_y,\mu_y,\hat{m}_x).
\end{aligned}
$$

   (c) Firms' export policy and consumptions in y country:

$$
\begin{aligned}
t_y^*(z_y,\mu_y,\hat{m}_x) &= z_y\Psi^*(\mu_y,\hat{m}_x) \\
c_y^*(z_y,\mu_y,\hat{m}_x) &= z_y(1-\Psi^*(\mu_y,\hat{m}_x)) \\
c_x^*(z_y,\mu_y,\hat{m}_x,\mu_x,\hat{m}_y) &= \frac{t_y(z_y,\mu_y,\hat{m}_x)}{p(\mu_x,\mu_y,\hat{m}_x,\hat{m}_y)} \\
c^*(z_x,\mu_x,\hat{m}_y,\mu_y,\hat{m}_x) &= z_y\Psi_2^*(\mu_y,\hat{m}_x,\mu_x,\hat{m}_y).
\end{aligned}
$$

   (d) Aggregate variables in x country:

$$
\begin{aligned}
T_x(\mu_x,\hat{m}_y) &= f_x\Psi(\mu_x,\hat{m}_y) \\
C_x(\mu_x,\hat{m}_y) &= f_x(1-\Psi(\mu_x,\hat{m}_y)) \\
C_y(\mu_x,\hat{m}_y,\mu_y,\hat{m}_x) &= p(\mu_x,\mu_y,\hat{m}_x,\hat{m}_y)T_x(\mu_x,\hat{m}_y) \\
C(\mu_x,\hat{m}_y,\mu_y,\hat{m}_x) &= f_x\Psi_2(\mu_x,\mu_y,\hat{m}_x,\hat{m}_y).
\end{aligned}
$$

(e) Aggregate variables in y country:

$$\begin{aligned}
T_y^*(\mu_y, \hat{m}_x) &= f_y \Psi^*(\mu_y, \hat{m}_x) \\
C_y^*(\mu_y, \hat{m}_x) &= f_y(1 - \Psi^*(\mu_y, \hat{m}_x)) \\
C_x^*(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y) &= \frac{T_y(\mu_y, \hat{m}_x)}{p(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)} \\
C^*(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y) &= f_y \Psi_2^*(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y).
\end{aligned}$$

Appendix C.4. Simulation strategy and parameter choice

We solve and simulate different versions of the model:

(i) full information, where both countries know their own endowment and the other country's endowment exactly ($\hat{m}_x = \mu_x$ and $\hat{m}_y = \mu_y$);
(ii) no information, where each country knows its own endowment, but since neither country gets any signal, their beliefs about the other country's endowment are given by the unconditional distribution ($\hat{m}_x = m_x$ and $\hat{m}_y = m_y$); and
(iii) noisy signals, where each country knows its own endowment and receives signals about the other country's endowment. In this last case we solve and simulate the model for various levels of signal precision, always keeping the precision symmetric across countries.

We draw a series of aggregate productivities and fix it across information models. Then for each information model we generate posterior means of foreign endowment by drawing signals centered on the realized aggregate productivities and with precision determined by the model at hand.

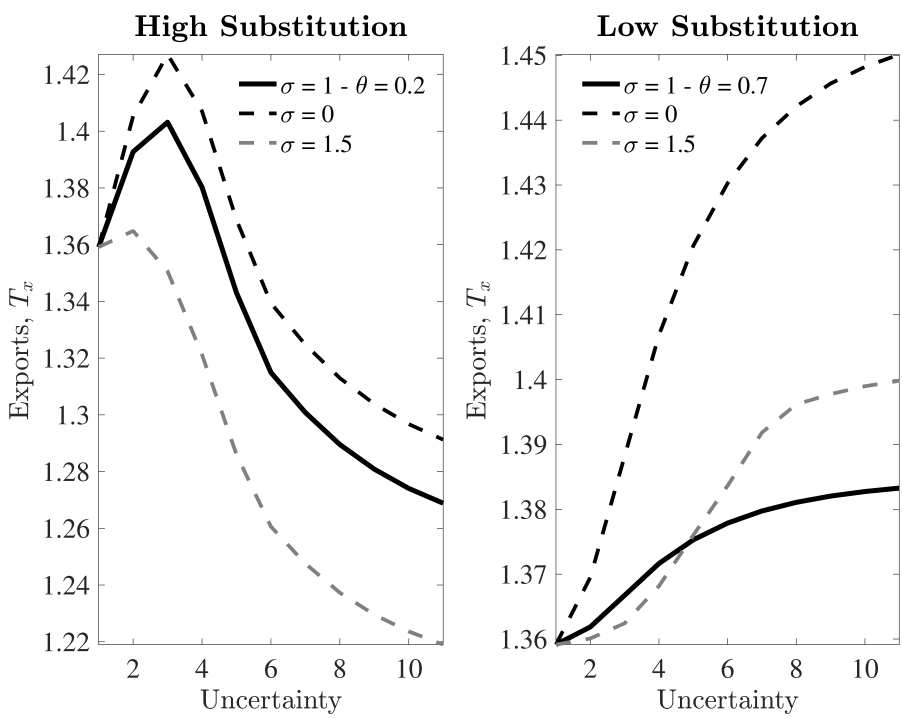
Parameters. Table C.1 describes the parameters we use to simulate the models and how they are chosen or calibrated. The unconditional means of aggregate endowment m_x, m_y and the dispersion of firm endowment σ_x, σ_y only appear as differences between home and foreign values. Only the differences $m_x - m_y$ or $\sigma_x - \sigma_y$ affect the export shares or the relative prices. The absolute quantities produced and traded do depend on these parameters, but not any relative quantities or prices. Since the question of fundamental asymmetry between two countries and its effect on trade is not the focus of this paper, we assume that there is no difference in the distribution of fundamentals by imposing symmetry: $m_x = m_y$ and $\sigma_x = \sigma_y$. We normalize $m = 0$ and $\sigma = \sqrt{2}$ so that the mean of aggregate (log) endowment is unity: $\mathbb{E}[\log f_x] = m_x + \frac{1}{2}\sigma_x^2 = 1$. To set volatility of the aggregate endowment shocks, we note that it is the ratio of signal to aggregate dispersion that matters for outcomes $\tilde{s}/s$. Since we will vary signal precision across simulations, we assume symmetry and normalize the volatility of aggregate shocks to unity ($s_x = s_y = 1$). We consider two values for $\theta \in \{0.3, 0.8\}$ that imply an elasticity of substitution across the two consumption goods of $1/(1-\theta) \in \{1.42, 5\}$, respectively. Finally, the signal noise each country observes about the other's endowment $\tilde{s}^2$ are varied across simulations in the range $[0, \infty]$ (from perfect information to imperfect information). Numerically, the highest level of uncertainty used is 3.

Table C.1: **Summary of Model Parameters**

Parameter	θ	σ	$m_x = m_y$	$s_x = s_y$	$\sigma_x = \sigma_y$	$\tilde{s}_x = \tilde{s}_y$
Value	$0.3, 0.8$	$1 - \theta$	0	1	$\sqrt{2}$	$[0, \infty]$

Appendix D. Comparative statics for risk-aversion σ

The following figure replicates Figure 4 in the main text showing the optimal export decision. The left panel uses a high elasticity of substitution $\theta = 0.8$, and the right panel uses a low elasticity of substitution $\theta = 0.3$. In each panel, we show 3 levels of risk aversion $\sigma \in \{0, 1.5, 1 - \theta\}$, where the case $\sigma = 1 - \theta$ (solid line) is our CES case from the main text. From the figure, it is clear that none of the qualitative results change.



Notes: Equilibrium exports for domestic country T_x for different levels of uncertainty (signal noise $\tilde{s}^2$). The left panel uses a high elasticity of substitution $\theta = 0.8$, and the right panel uses a low elasticity of substitution $\theta = 0.3$. In each panel, we show the export decision for three levels of risk aversion $\sigma \in \{0, 1.5, 1 - \theta\}$.

Figure D.1: Effect of Uncertainty in Trade for Different Levels of Risk Aversion