Cross-Phenomenon Restrictions:

Unemployment Effects of Layoff Costs and Quit Turbulence

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Online Appendix

A Equilibrium computation

A.1 General algorithm structure

Here we outline the structure of the algorithm we used to compute equilibria.[1] It centers around approximating the joint continuation values $g_i(z)$ using linear projections on a productivity grid. It employs the following steps:

1. Fix a parameterization and construct productivity distributions over a grid of size N_z.
2. Guess initial values for:
   - ζ_i^k: coefficients for linear approximations $\hat{g}_i(z) = \zeta_i^0 + \zeta_i^1 z$ to $g_i(z)$
   - b_j: unemployment benefits
   - ω_{ij}^w: workers' outside values, not including current payment of benefit
   - ω^f: firms' outside value (in the benchmark model, $\omega^f = 0$)
   - τ: tax rate
   - u_{ij}, e_{ij}: masses of unemployed and employed workers
3. Given linear approximations $\hat{g}_i(z)$, use (2)–(5) to compute reservation productivities $\underline{z}_{ij}^o, \underline{z}_{ij}$.
4. Given cutoffs $\underline{z}_{ij}^o, \underline{z}_{ij}$, compute rejection probabilities ν_{ij}^o, ν_{ij} using (6) and compute E_{ij} using (7).
5. Compute the expected match surplus of a vacancy that encounters an unemployed worker:
$$\bar{s} \equiv \sum_{(i,j)} \frac{u_{ij}}{u} \int_{\underline{z}_{ij}^o}^{\infty} s_{ij}^o(y)\ dv_i^o(y).$$
6. Compute joint continuation values $g_i(z)$ using (8) and (9). Then update coefficients ζ_i^0, ζ_i^1 described in step 2 by regressing $g_i(z)$ on $[1\ \ z]$.
7. Update the value of posting a vacancy, market tightness, and matching probabilities:
   - under endogenous market tightness in the benchmark model,
$$w^f = 0, \qquad \theta = \left(\frac{\beta A(1-\pi)\bar{s}}{\mu}\right)^{1/\alpha}, \qquad \lambda^w(\theta) = A\theta^{1-\alpha}, \qquad \lambda_{ij}^f(\theta) = A\theta^{-\alpha}\frac{u_{ij}}{u};$$
   - under DHHR's exogenous market tightness, compute
$$\omega^f = \frac{\beta}{1-\beta} A(1-\pi)\bar{s}, \qquad \theta = 1, \qquad \lambda^w = A, \qquad \lambda_{ij}^f = A\frac{u_{ij}}{u}.$$

[1] We are grateful to Wouter den Haan, Christian Haefke, and Garey Ramey for generously sharing their computer code. That code was augmented and modified by LS and further by us.

8. Update values ω_{ij}^w of being unemployed using (10) and (11).

9. Compute net changes in worker flows (all must be zero in a steady state)

$$\begin{aligned}\Delta u_{ll} &= \rho^r + (1-\rho^r)\left\{\rho^x + (1-\rho^x)(1-\gamma^u)\gamma^s\nu_{ll}\right\} e_{ll} \\ &- \rho^r u_{ll} - (1-\rho^r)\lambda^w(\theta)(1-\nu_{ll}^o)u_{ll}\end{aligned} \tag{A.1}$$

$$\begin{aligned}\Delta u_{lh} &= (1-\rho^r)\left\{\rho^x\gamma^\ell e_{hh} + (1-\rho^x)\nu_{hh}\gamma^q(\gamma^s e_{hh} + \gamma^u e_{ll})\right\} \\ &- \rho^r u_{lh} - (1-\rho^r)\lambda^w(\theta)(1-\nu_{lh}^o)u_{lh}\end{aligned} \tag{A.2}$$

$$\begin{aligned}\Delta u_{hh} &= (1-\rho^r)\left\{\rho^x(1-\gamma^\ell)e_{hh} + (1-\rho^x)\nu_{hh}(1-\gamma^q)(\gamma^s e_{hh} + \gamma^u e_{ll})\right\} \\ &- \rho^r u_{hh} - (1-\rho^r)\lambda^w(\theta)(1-\nu_{hh}^o)u_{hh}\end{aligned} \tag{A.3}$$

$$\begin{aligned}\Delta e_{ll} &= (1-\rho^r)\lambda^w(\theta)\left\{(1-\nu_{ll}^o)u_{ll} + (1-\nu_{lh}^o)u_{lh}\right\} \\ &- \rho^r e_{ll} - (1-\rho^r)[\rho^x + (1-\rho^x)(\gamma^u + (1-\gamma^u)\gamma^s\nu_{ll}]e_{ll}\end{aligned} \tag{A.4}$$

$$\begin{aligned}\Delta e_{hh} &= (1-\rho^r)\left\{\lambda^w(\theta)(1-\nu_{hh}^o)u_{hh} + (1-\rho^x)\gamma^u(1-\nu_{hh})e_{ll}\right\} \\ &- \rho^r e_{hh} - (1-\rho^r)[\rho^x + (1-\rho^x)\gamma^s\nu_{hh}]e_{hh}\end{aligned} \tag{A.5}$$

These expressions embed the assumption of immediate realization of skill upgrades in the benchmark model. For DHHR's alternative assumption of delayed completion, see the corresponding expressions for worker flows in den Haan *et al.* (2005, Appendix A).

10. Compute average wages $\bar{p}_i$ and average productivities $\bar{z}_i$ as described in Appendix A.2, to determine government expenditures for unemployment benefits and government tax revenues using the left side and right side of (23), respectively.

11. Adjust tax rate τ in (23) to balance government budget.

12. Check convergence of a set of moments. If convergence has been achieved, stop. If convergence has not been achieved, go to 2 and use as guesses the last values computed.

A.2 Average wages and productivities

The following computations refer to the benchmark model with immediate realization of skill upgrades. For DHHR's alternative assumption of delayed completion, see den Haan *et al.* (2005, appendices A–C).

Our computation of the equilibrium measures of workers in equations (A.1)–(A.5) involve only two groups of employed workers, e_{ll} and e_{hh}, but each of these groups needs to be subdivided when we compute average wages and productivities. For employed low-skilled workers, we need to single out those who gained employment after first having belonged to group u_{lh}, i.e., low-skilled unemployed workers who received high benefits b_h. In the first period of employment, those workers will earn a higher wage $p^o_{lh}(z) > p^o_{ll}(z) \geq p_{ll}(z)$. And even afterwards, namely until their first on-the-job productivity draw, those workers will on average continue to differ from other employed low-skilled workers because of their higher reservation productivity at the time they regained employment, $\underline{z}^o_{lh} > \underline{z}^o_{ll} \geq \underline{z}_{ll}$.

Let e'_{ll} denote the measure of unemployed low-skilled workers with high benefits who gain employment in each period (they are in their first period of employment):

$$e'_{ll} = (1-\rho^r)\lambda^w(\theta)(1-\nu^o_{lh})u_{lh}.$$

Let e''_{ll} be the measure of such low-skilled workers who remain employed with job tenures greater than one period and who have not yet experienced any on-the-job productivity draw:

$$\begin{aligned} e''_{ll} &= (1-\rho^r)(1-\rho^x)(1-\gamma^u)(1-\gamma^s)\left[e'_{ll} + e''_{ll}\right] \\ &= \frac{(1-\rho^r)(1-\rho^x)(1-\gamma^u)(1-\gamma^s)}{1-(1-\rho^r)(1-\rho^x)(1-\gamma^u)(1-\gamma^s)}\, e'_{ll}. \end{aligned}$$

Given these measures of workers, we can compute the average wage of all employed low-skilled workers and also their average productivity

$$\begin{aligned} \bar{p}_l &= \int_{\underline{z}^o_{lh}}^{\infty} \left[\frac{e'_{ll}}{e_{ll}} p^o_{lh}(y) + \frac{e''_{ll}}{e_{ll}} p_{ll}(y)\right] \frac{dv^o_l(y)}{1-v^o_l(\underline{z}^o_{lh})} + \frac{e_{ll}-e'_{ll}-e''_{ll}}{e_{ll}} \int_{\underline{z}_{ll}}^{\infty} p_{ll}(y)\, \frac{dv_l(y)}{1-v_l(\underline{z}_{ll})} \\ \bar{z}_l &= \frac{e'_{ll}+e''_{ll}}{e_{ll}} \int_{\underline{z}^o_{lh}}^{\infty} y\, \frac{dv^o_l(y)}{1-v^o_l(\underline{z}^o_{lh})} + \frac{e_{ll}-e'_{ll}-e''_{ll}}{e_{ll}} \int_{\underline{z}_{ll}}^{\infty} y\, \frac{dv_l(y)}{1-v_l(\underline{z}_{ll})}. \end{aligned}$$

For employed high-skilled workers, we need to single out those just hired from the group of unemployed high-skilled workers u_{hh} who earn a higher wage in their first period of employment, $p^o_{hh}(z) > p_{hh}(z)$. This is because they do not face the risk of quit turbulence if no wage agreement is reached and hence, no employment relationship is formed. For the same reason discussed above, we also need to keep track of such workers until their first on-the-job productivity draw (or layoff or retirement, whatever comes first). Reasoning as we did earlier, let e'_{hh} and e''_{hh} denote these respective groups of employed high-skilled workers;

$$\begin{aligned} e'_{hh} &= (1-\rho^r)\lambda^w(\theta)(1-\nu^o_{hh})u_{hh} \\ e''_{hh} &= \frac{(1-\rho^r)(1-\rho^x)(1-\gamma^s)}{1-(1-\rho^r)(1-\rho^x)(1-\gamma^s)}\, e'_{hh}. \end{aligned}$$

Given these measures of workers, we can compute the average wage of all employed high-skilled workers and also their average productivity

$$\bar{p}_h = \int_{\underline{z}^o_{hh}}^{\infty} \left[\frac{e'_{hh}}{e_{hh}} p^o_{hh}(y) + \frac{e''_{hh}}{e_{hh}} p_{hh}(y) \right] \frac{dv^o_h(y)}{1 - v^o_h(\underline{z}^o_{hh})} + \frac{e_{hh} - e'_{hh} - e''_{hh}}{e_{hh}} \int_{\underline{z}_{hh}}^{\infty} p_{hh}(y) \frac{dv_h(y)}{1 - v_h(\underline{z}_{hh})}$$

$$\bar{z}_h = \frac{e'_{hh} + e''_{hh}}{e_{hh}} \int_{\underline{z}^o_{hh}}^{\infty} y \frac{dv^o_h(y)}{1 - v^o_h(\underline{z}^o_{hh})} + \frac{e_{hh} - e'_{hh} - e''_{hh}}{e_{hh}} \int_{\underline{z}_{hh}}^{\infty} y \frac{dv_h(y)}{1 - v_h(\underline{z}_{hh})}.$$

B Comparison of LS and DHHR

Our benchmark model is based on the LS model (Ljungqvist and Sargent, 2007) augmented to include quit turbulence as in the DHHR model (den Haan, Haefke and Ramey, 2005).

Besides LS having no quit turbulence, there are essentially three substantive differences between the models of LS and DHHR:

(i) how vacancies are created,

(ii) how the capital gain from a skill upgrade is split between firm and worker, and

(iii) productivity distributions.

As for vacancy creation, the LS model adopts standard assumptions of free entry of firms and an equilibrium zero-profit condition in vacancy creation, whereas DHHR assume a fixed measure of firms equal to the measure of workers so that the vacancy-unemployment ratio always equals unity under DHHR's implicit assumption of a sufficiently low vacancy posting cost that all firms without a worker post vacancies. As for skill upgrades, in the LS model an employed worker who experiences a skill upgrade can immediately choose to quit and search for employment elsewhere, whereas DHHR assume that such a worker must first work one more period with the present employer in order not to lose her skill upgrade; that has consequences for how a worker and a firm split the capital gain of a skill upgrade under Nash bargaining. Finally, the productivity distributions are assumed to be truncated normal distributions by LS and uniform distributions by DHHR, as detailed in Section 3.

Except for these differences, the remaining parameterizations of LS and DHHR are very similar.[2] A similarity that originates from an earlier exchange of views between den Haan *et al.* (2001) and Ljungqvist and Sargent (2004). Thus, Ljungqvist and Sargent (2004) advocated modifying the parameterization of den Haan *et al.* (2001) based on calibration targets in the search framework of Ljungqvist and Sargent (1998, 2008); but, as it turns out, with insufficient attention to *returns to labor mobility*. Specifically, Ljungqvist and Sargent (2004) criticized den Haan *et al.* (2001) for making low- and high-skilled workers almost indistinguishable from one another because of nearly overlapping productivity distributions for the two types of workers. As a remedy, by moving the uniform distributions apart and ending up with the disjoint supports in Figure 1b, Ljungqvist and Sargent (2004) succeeded in making low- and high-skilled workers distinct from one another; but as shown here, that fails to generate returns to labor mobility consistent with historical observations. In the subsequent analysis by LS, layoff costs were introduced, and productivity distributions had to be properly calibrated, as demonstrated in Section 3.2. Meanwhile, DHHR adopted Ljungqvist and Sargent's (2004) modification of den Haan et al.'s (2001) parameterization and proceeded to investigate quit turbulence.

[2] After taking into account DHHR's quarterly rather than semi-quarterly model period, their parameterization of sources of risk and labor market institutions are the same as in Table 1. Regarding the subjective discount factor $\hat{\beta}$ and the retirement probability ρ^r, DHHR set those to 0.995 and 0.005, respectively, at a quarterly frequency, which yield an adjusted discount factor β of 0.995 at a semi-quarterly frequency. We conducted a sensitivity analysis with respect to the different discount rates and found that adopting the DHHR discount rate in the benchmark model with LS productivity distributions does not substantively change our analysis.

C Perturbations of the benchmark model

As detailed in Appendix B, there are essentially three differences between the benchmark model with LS productivity distributions and the DHHR model in Figure 4: i) how vacancies are created, ii) how the capital gain from a skill upgrade is split between firm and worker, and iii) productivity distributions. To explain puzzling starkly different turbulence outcomes in Figure 4, our method is to start with the benchmark model with LS productivity distributions in Figure 4a and successively make perturbations one by one, with each perturbation addressing one of the three differences above.

To facilitate our perturbations, we renormalize the parameters (A, μ) in Table 1 so that equilibrium market tightness in tranquil times (no turbulence) becomes equal to one.[3] Recall that when calibrating a matching model to an aggregate unemployment rate, without any calibration targets for vacancy statistics, selecting the parameter pair (A, μ) is a matter of normalization.

C.1 First perturbation: Exogenous market tightness

The first perturbation concerns differences in the matching process. In the benchmark model, market tightness is endogenously determined by a typical free-entry-of-firms assumption. The equilibrium zero-profit condition in vacancy creation pins down market tightness. In contrast, DHHR assume fixed and equal masses of workers and firms so that market tightness is exogenously always equal to one.

Perturbation exercise As described in footnote 23, our renormalization of parameters (A, μ) in the benchmark model yields equilibrium market tightness equal to one at zero turbulence. Our first perturbation exercise is to keep market tightness constant at one as we turn up turbulence. We do that by subsidizing vacancy creation so that the value of a firm posting a vacancy is zero, $w^f = 0$, at market tightness equal to one for any given levels of layoff and quit turbulence. The vacancy subsidies are financed with lump-sum taxation so that government budget constraint (23) is unaffected.

In this exercise where subsidies are used to keep $w^f = 0$ at $\theta = 1$, let $\bar{S}^o(\gamma^\ell, \epsilon)$ denote the expected match surplus of a vacancy encountering an unemployed worker, given layoff turbulence γ^ℓ and quit turbulence $\gamma^q = \epsilon\gamma^\ell$:

$$\bar{S}^o(\gamma^\ell, \epsilon) \equiv \sum_{(i,j)} \frac{u_{ij}}{u} \int_{\underline{z}^o_{ij}}^{\infty} s^o_{ij}(y)\ dv^o_i(y) \tag{C.6}$$

where unemployment u_{ij}, reservation productivity $\underline{z}^o_{ij}$, and match surplus $s^o_{ij}(y)$ are understood to be equilibrium values under our particular perturbation exercise.

At zero turbulence, the operation of the subsidy scheme would not require any payments of subsidies

[3]Under the original parameterization $(A, \mu) = (0.45, 0.5)$ in Table 1, the equilibrium market tightness is equal to $\theta = 0.9618$ in tranquil times. We renormalize to attain an equilibrium market tightness of 1 and leave unchanged the probability that a worker encounters a vacancy. Let $(\hat{A}, \hat{\mu})$ be our new parameterization given by $\hat{A} = \kappa^{1-\alpha} A$ and $\hat{\mu} = \kappa\mu$. By setting κ equal to the market tightness under the old parameterization $\kappa = 0.9618$, the new parameterization, $(\hat{A}, \hat{\mu}) = (0.441, 0.481)$, achieves the desired outcomes.

This renormalization will be useful below when reconciling outcomes across models. Specifically, it will facilitate a perturbation exercise in which we shall replace free entry of firms in the benchmark model with the DHHR arrangement that exogenously fixes equal masses of firms and workers and a market tightness equal to one.

because we have parameterized the matching function so that equilibrium market tightness is then $\theta = 1$, a value of θ at which the zero-profit condition in vacancy creation is satisfied, $w^f = 0$, and by equation (14):

$$\mu = \beta(1-\pi)m(1)\bar{S}^o(0,0). \tag{C.7}$$

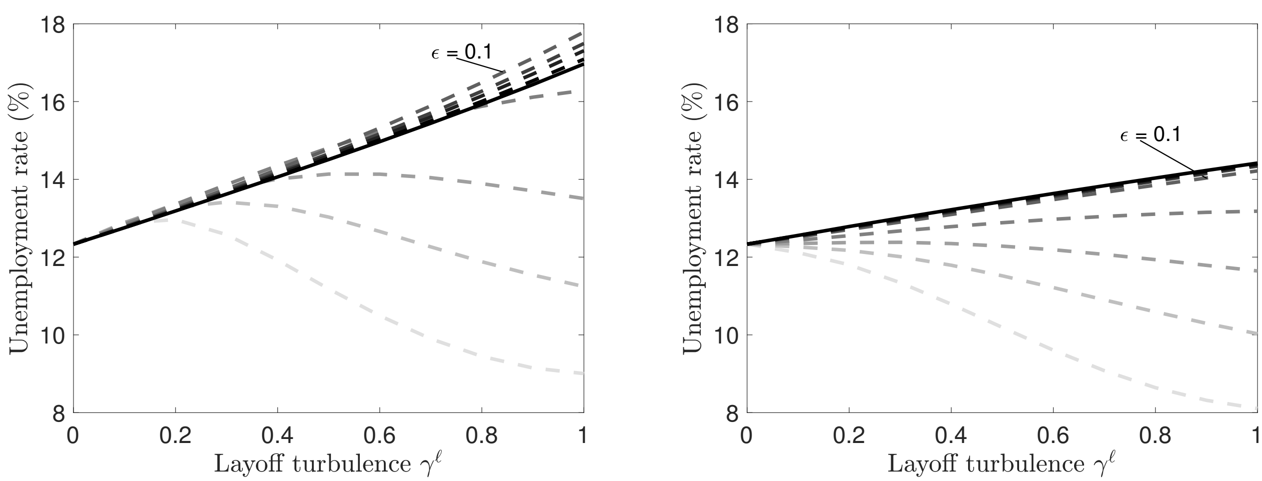
When turbulence is turned on, market tightness would have fallen if it were not for the subsidies to vacancy creation. The subsidy rate makes up for the shortfall of $\beta(1-\pi)m(1)\bar{S}^o(\gamma^\ell, \epsilon)$ when compared to the investment of incurring vacancy posting cost μ:

$$1 - subsidy(\gamma^\ell, \epsilon) = \frac{\beta(1-\pi)m(1)\bar{S}^o(\gamma^\ell, \epsilon)}{\mu} = \frac{\bar{S}^o(\gamma^\ell, \epsilon)}{\bar{S}^o(0,0)} \tag{C.8}$$

where the second equality invokes expression (C.7).

Results We observe an overall suppression of unemployment rates in Figure C.1b as compared to Figure C.1a. However, the underlying pattern of unemployment dynamics remains intact, so exogenous market tightness does not explain the puzzle.

Figure C.1: Endog. vs. exog. market tightness in benchmark with LS prod.



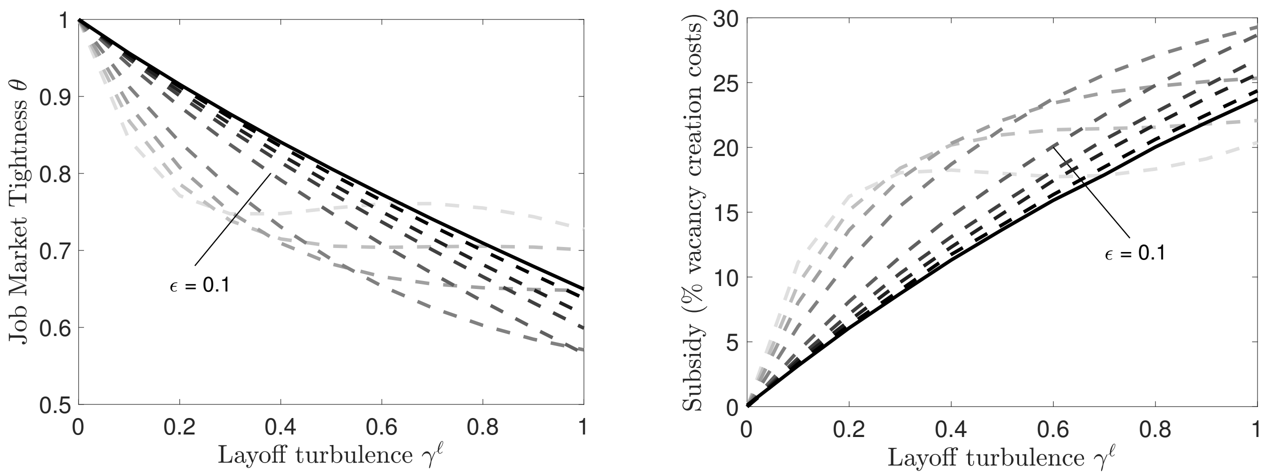
(a) Endogenous market tightness

(b) Exogenous market tightness

Discussion: Disarming the invisible hand With endogenous market tightness, there is a dramatic decline in market tightness in response to turbulence in Figure C.2a. This outcome reflects how an "invisible hand" restores firm profitability so that vacancy creation breaks even. Lower market tightness decreases the probability that a worker encounters a vacancy, which tends to increase unemployment.

Our perturbation exercise disarms those forces by exogenously freezing market tightness at one. Hence, the profitability of vacancies plummets in response to turbulence. Figure C.2b plots the subsidy rate for vacancy costs needed to incentivize firms to post enough vacancies to keep market tightness constant at one. At higher levels of turbulence, the subsidy rate becomes quite substantial. The subsidies to vacancy creation contribute to lower unemployment rates. These considerations seem to enhance a suspicion that exogenous market tightness could be the culprit behind the puzzle, so the above vindication was not a foregone conclusion.

Figure C.2: Falling market tightness vs. subsidies for vacancy creation



(a) Endogenous market tightness

(b) Exogenous market tightness

C.2 Second perturbation: Timing of completion of skill upgrades

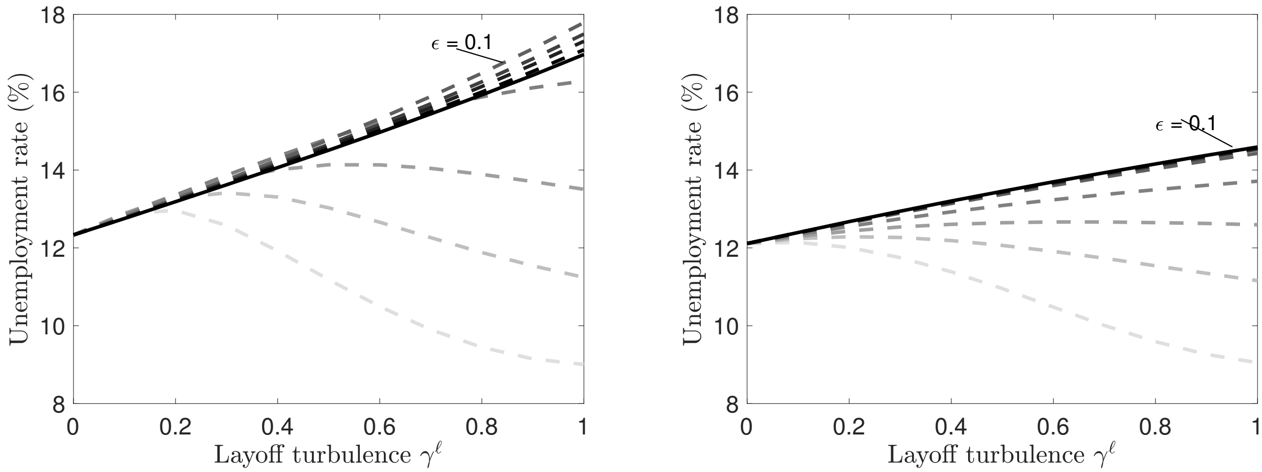
The second perturbation concerns differences in the timing of completion of skill upgrades. In the benchmark model, skill upgrades are immediately realized. In contrast, DHHR assume that a worker who receives a skill upgrade must remain with the present employer for one period in order to complete the higher skill level.

Perturbation exercise We replace immediate realization of skill upgrades in the benchmark model with delayed completion as in the DHHR model. The change in timing substantially alters the relative bargaining strengths of a worker and a firm.

Results The quantitative outcome in Figure C.3b is similar to that of the preceding perturbation exercise in Figure C.1b, i.e., it leads to an overall suppression in unemployment rates but without altering the underlying pattern of unemployment dynamics and hence, different timing of completion of skill upgrades does not explain the puzzle.

Discussion: Delayed completion requires "ransoms" Firms under DHHR's timing assumption are able to "rip off" workers whenever they transition from low to high skill at work. This is possible because the realization of that higher skill level is conditional upon a worker remaining with the present employer for at least one more period, during which the worker can be assessed a "ransom" to secure her human capital gain.

Figure C.3: Timing of completion of skill upgrade in benchmark with LS prod.

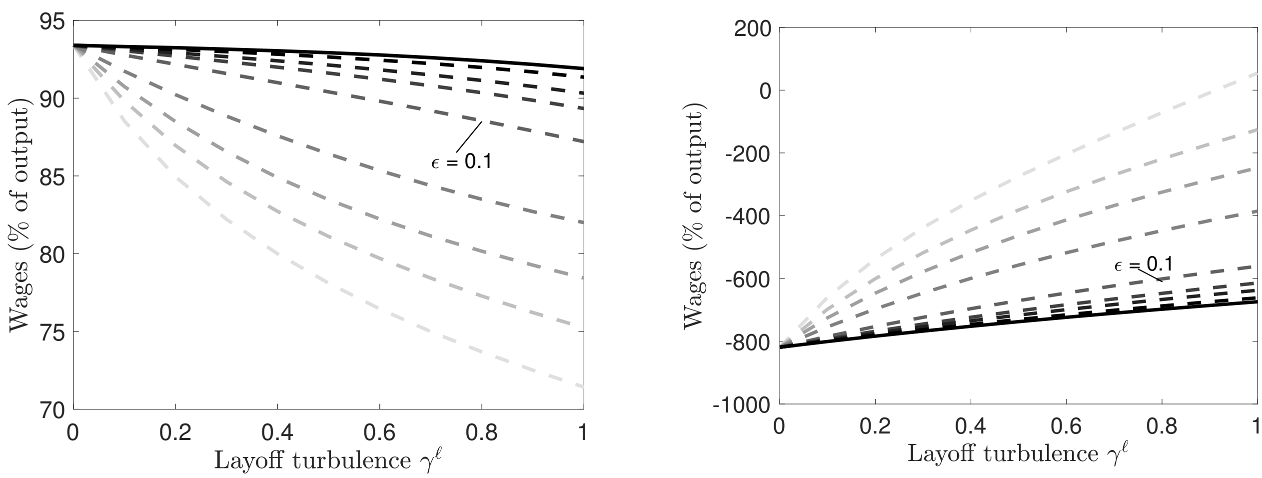


(a) Immediate upgrade

(b) Delayed upgrade

We compare average wages at skill upgrades under immediate completion (Figure C.4a) and delayed completion (Figure C.4b), expressed in terms of average output per worker in the laissez-faire economy at zero turbulence.[4] In Figure C.4b, a worker pays the "ransom" in terms of a negative semi-quarterly wage in the period of a skill upgrade, equivalent to the average annual output of a worker.

Figure C.4: Average wage in period of skill upgrade



(a) Immediate upgrade

(b) Delayed upgrade

The "ransom" becomes smaller with higher turbulence since the capital value of a skill upgrade is worth less when it is not expected to last long, as well as when quit turbulence locks high-skilled workers into employment relationships and thereby causes a less efficient allocation: fearing skill loss

[4]In the laissez-faire economy of the benchmark model with LS productivity distributions, a worker's average semi-quarterly output is 2.3 goods when $q^\ell = 0$.

at separations, high-skilled workers accept lower reservation productivities and hence, work on average at lower productivities as compared to an economy in tranquil times with higher labor mobility.

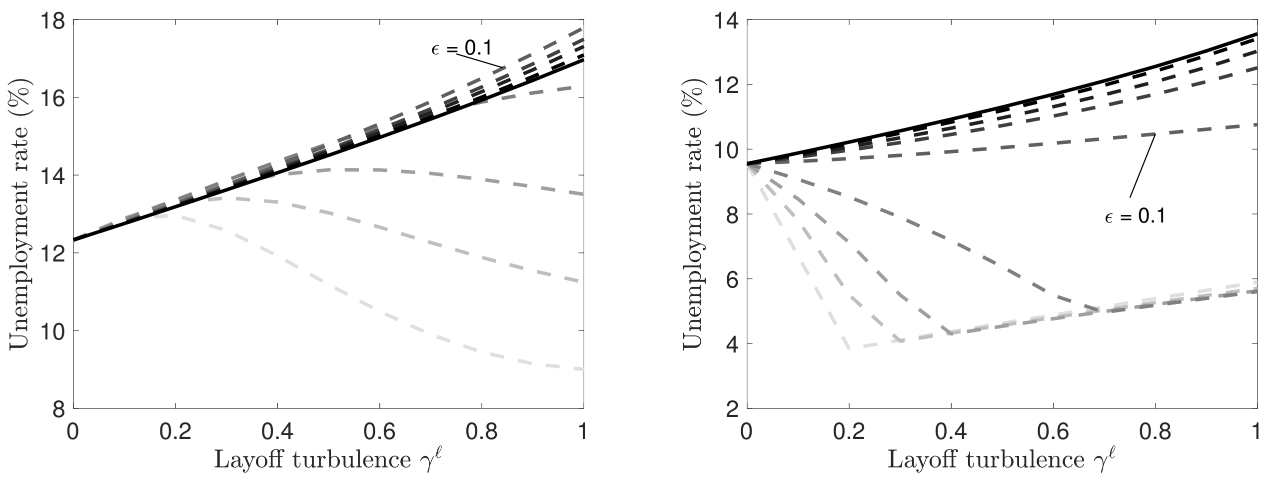
C.3 Third perturbation: Productivity distributions

The third perturbation concerns differences in productivity distributions. The benchmark model adopts the truncated normal distributions of LS with wide support. In contrast, DHHR assume uniform distributions with narrow support.

Perturbation exercise We replace the LS productivity distributions in the benchmark model with the DHHR productivity distributions.

Results The perturbation weakens the positive turbulence-unemployment relationship so much that we get DHHR-like outcomes in Figure C.5b. Thus, we conclude that differences in productivity distributions explain the different outcomes with respect to quit turbulence in Figure 4.

Figure C.5: LS vs. DHHR productivity distributions in benchmark model



(a) LS productivity distributions

(b) DHHR productivity distributions

Discussion: Meager returns to labor mobility Productivity draws on the job bring incentives for workers to change employers in search of higher productivities. The small dispersion of productivities under DHHR's uniform distributions with narrow support make returns to labor mobility be very low. As can be seen in Figure C.5b, those low returns do not compensate for even small amounts of quit turbulence and hence the initially positive turbulence-unemployment relationship at zero quit turbulence ($\epsilon = 0$) turns negative at relatively small levels of quit turbulence.

To confirm that the small dispersion of productivities explains the different outcomes with respect to quit turbulence in Figure 4, we do an additional perturbation exercise that shrinks the support of the uniform distribution further. Figure 6b in Section 3.4 shows outcomes in the benchmark model when the support of the uniform distribution has width 0.60 instead of 1. Such a shrinkage of the support takes us very close to the outcomes in the DHHR model in Figure 4b. Hence, in Section 3.4, we refer to the representation in Figure 6b as the benchmark model version of DHHR.

D Perturbations of the DHHR model

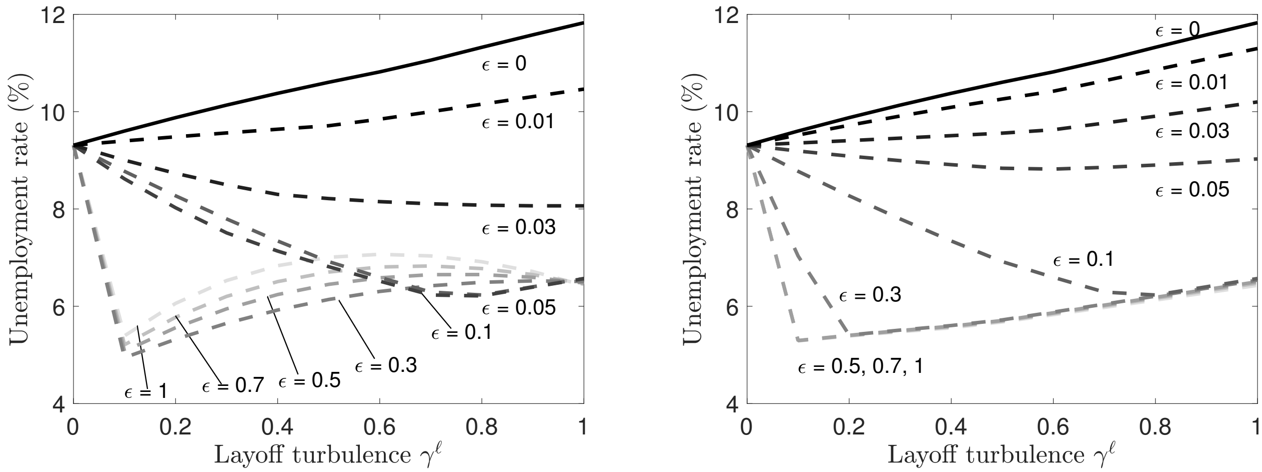
We now reverse the analysis of Appendix C by starting from the DHHR model and investigating the consequences of three perturbations. The features in the DHHR model to be perturbed are (i) exogenous labor market tightness, (ii) delayed completion of skill upgrade, and (iii) uniform productivity distributions with narrow support. But before that, we eliminate two auxiliary assumptions in the DHHR analysis.

Eliminate auxiliary assumption of zero benefits for newborn workers Instead of DHHR's assumption of no benefits during the initial unemployment spells of newborn workers, we assume that they are eligible for unemployment benefits equivalent to those of low-skilled workers. This modification reduces the number of worker types while having hardly any effect on aggregate outcomes.

Eliminate auxiliary assumption of turbulence for unemployed DHHR assume that after an encounter between a firm and an unemployed worker that does not result in an employment relationship, the worker faces the same risk of losing skills as if she had instead quit a job. DHHR describe this as an auxiliary assumption that they justify in terms of its computational tractability, but we find that it has noticeable quantitative consequences. Thus, Figure D.1 presents outcomes for the original DHHR framework with turbulence for unemployed workers and our modified DHHR model without that kind of turbulence. While the outcomes are not as stark in latter model, the underlying pattern of unemployment dynamics remains intact – it just takes some more quit turbulence to generate DHHR's key findings of a negative turbulence-unemployment relationship. From hereon, we refer to the modified model in Figure D.1b as the DHHR model.

An assumption that mere encounters between vacancies and unemployed workers are associated with risks of losing skills unless employment relationships are formed directly suppresses returns to labor mobility. But as can be inferred from Figure D.1, whether or not there is such an exposure of job seekers to skill loss does not matter much for DHHR's argumentation since, as Appendix D.3 will teach us, compressed productivity distributions in DHHR already reduce returns to labor mobility. However, the substantial incentives for labor mobility in the benchmark model with LS productivity distributions are significantly affected and suppressed by that auxiliary assumption of DHHR. Appendix E discusses this in detail.

Figure D.1: With vs. without turbulence for unemployed in DHHR



(a) With turbulence for unemployed (b) Without turbulence for unemployed

D.1 First perturbation: Exogenous market tightness

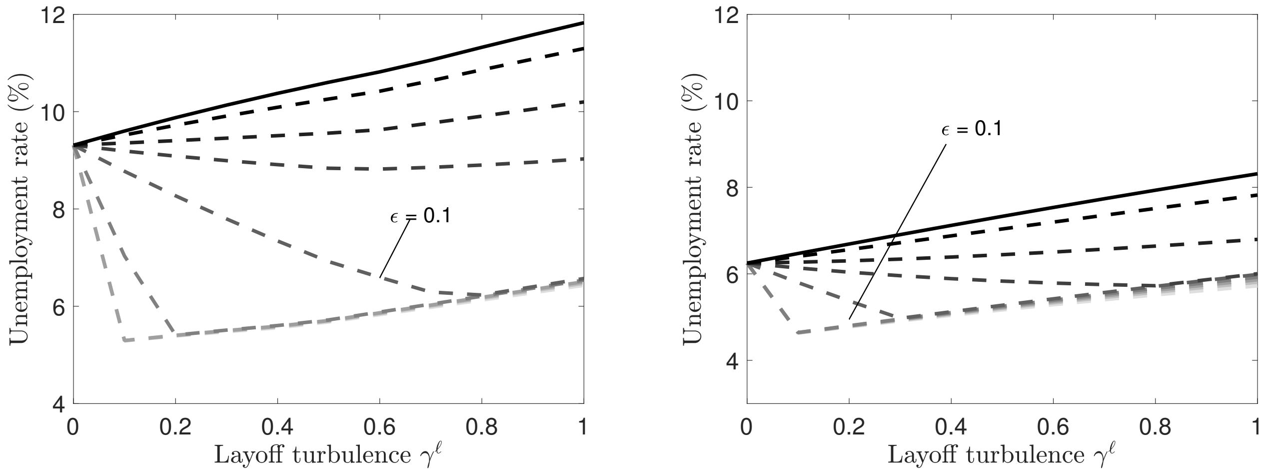
Perturbation exercise In the DHHR framework, there is an exogenous mass of firms and there are no costs for posting vacancies. Hence the value w^f of a firm posting a vacancy is trivially positive. We now perturb DHHR to feature free entry of firms, $w^f = 0$ in equilibrium, and an endogenous market tightness determined by (14). To implement that perturbation, we must introduce and assign values to two additional parameters, α and μ. Following the benchmark model, we assume that the elasticity of the matching function with respect to unemployment equals $\alpha = 0.5$, a fairly common parameterization.

Lacking an obvious way to parameterize the vacancy posting cost μ in this perturbation, we solve the model for different values of $\mu > 0$.[5] We find that for values of μ above 0.7, all voluntary quits vanish. Therefore, since DHHR's challenge to a Ljungqvist-Sargent positive turbulence-unemployment relationship is based on changes in the incidence of quits, we consider $\mu \in (0, 0.7)$ to be the permissible range. As an illustration, Figure D.2b depicts equilibrium outcomes for the midpoint of that parameter range, $\mu = 0.35$.

Results Except for the very top end of the parameter range $\mu \in (0, 0.7)$, the qualitative pattern of Figure D.2 represents the unemployment-turbulence relationship for the DHHR framework under the two alternative matching assumptions. In both cases, rather small amounts of quit turbulence reduce unemployment. Therefore, exogenous versus endogenous market tightness does not explain the puzzle.

[5] The vacancy posting cost μ must be positive to have an equilibrium with free entry of firms. The discrete model period and the Cobb-Douglas matching function call for an additional caveat. As the value of μ approaches zero, the equilibrium probability of filling a vacancy goes to zero. That creates a problem when the associated probability of a worker encountering a vacancy exceeds the permissible value of unity. Therefore, we only compute equilibria for μ greater than 0.0063. If one would like to compute equilibria for lower values of μ, it could be done by augmenting the match technology to allow for corner solutions at which the short end of the market determines the number of matches; e.g., in the present case, by freezing the job finding probability at unity while randomly allocating the unemployed across all vacancies that draw an "encounter." (See Ljungqvist and Sargent (2007, section 7.2).)

Figure D.2: Exogenous vs. endogenous market tightness in DHHR model



(a) Exogeneous market tightness

(b) Endogenous market tightness

In the vicinity of parameter value $\mu = 0.7$, the curve for $\epsilon = 0.1$ in the corresponding version of Figure D.2b (not shown here) takes on a positive slope, i.e., outcomes become LS-like with a positive turbulence-unemployment relationship. This might have been anticipated. As mentioned above, $\mu = 0.7$ is also the parameterization at which all voluntary quits vanish, which would seem to disarm the DHHR quit turbulence argument.[6]

Incidentally, as we will learn in Appendix D.3, the raw fact that voluntary quits vanish at a relatively low value of the vacancy posting cost $\mu = 0.7$ is indicative of low returns to labor mobility in the DHHR model that come from compressed productivity distributions.

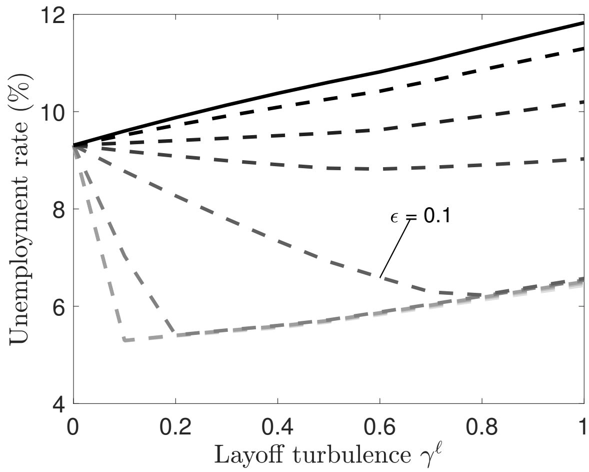
D.2 Second perturbation: Timing of completion of skill upgrades

Perturbation exercise DHHR assume that after a skill upgrade a worker must remain with the present employer for one period to complete the higher skill level. In this section, we introduce immediate completion of skill upgrades as in the benchmark model.

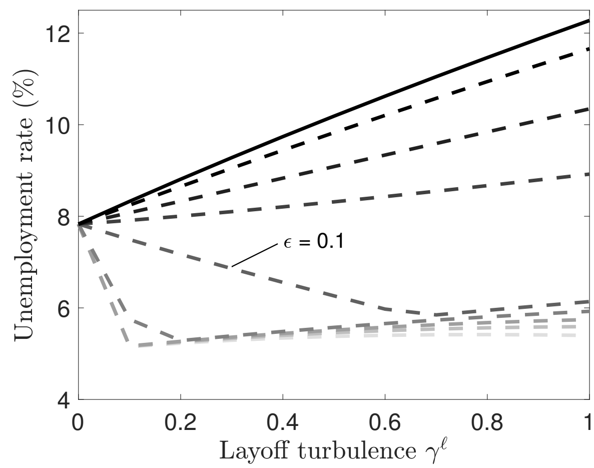
Results Figure D.3 shows that there is no substantial difference in the turbulence-unemployment relationship for the alternative timings in the DHHR model. Hence, delayed versus immediate completion of skill upgrades does not explain the puzzle.

[6]For a more nuanced reasoning about the equilibrium forces at work under the threat of losing skills in a matching model, see the discussion of an "allocation channel" and a "bargaining channel" in section E.2. While that section pertains to the introduction of turbulence facing unemployed workers in terms of a risk of losing skills after an encounter between a firm and a worker that does not result in employment, similar reasoning can be applied to quit turbulence for employed workers.

Figure D.3: Timing of completion of skill upgrade in DHHR model



(a) Delayed upgrade



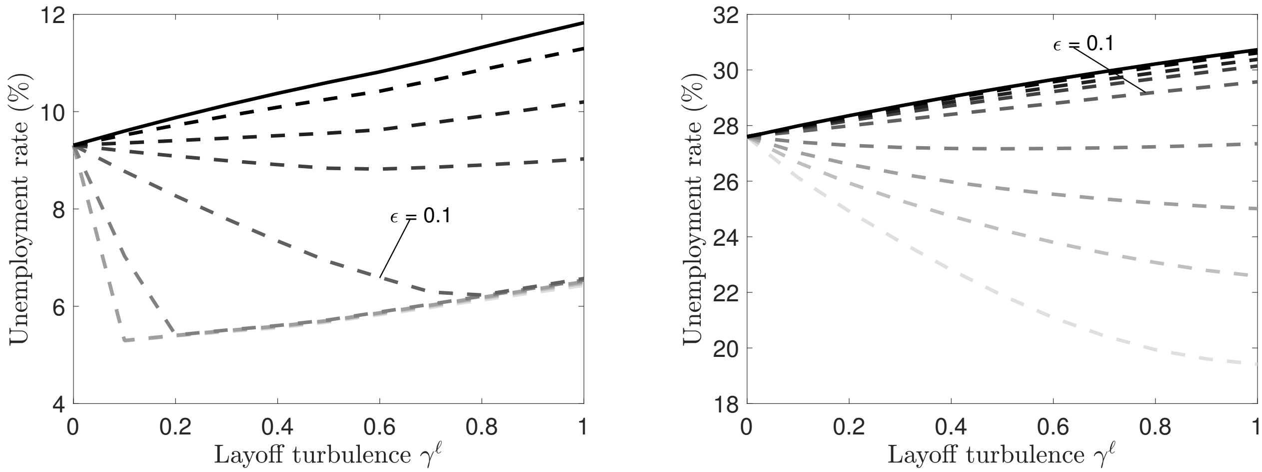
(b) Immediate upgrade

D.3 Third perturbation: Productivity distributions

Perturbation exercise DHHR assume uniform distributions with narrow support. In this section we replace those distributions in the DHHR model with the truncated normal distributions assumed by LS.

Results Figure D.4 shows how the turbulence-unemployment relationship is altered in the DHHR model when we switch from DHHR's productivity distributions to those of LS. First, the larger variances of the LS distributions exert upward pressures on reservation productivities and labor reallocation rates, but DHHR's assumption that an exogenously given market tightness equals one means that the relative number of vacancies cannot expand, so overall unemployment rates become higher. Second, and critical to our inquiry, the inference to be drawn from Figure D.4 agrees with what we inferred after studying the obverse perturbation of the benchmark model in Figure C.5; namely, differences in productivity distributions are key to explaining the puzzle. When we import the LS distributions into the DHHR model, small amounts of quit turbulence no longer unduly dissuade high-skilled workers with poor productivity draws to quit and seek better employment opportunities. Hence, the present perturbation disarms DHHR's argument for suppressed quit rates and allows the Ljungqvist-Sargent turbulence force to operate unimpeded. Figure D.4b shows how turbulence and unemployment are positively related until quit turbulence reaches about 30% of layoff turbulence after which the relationship becomes negative.

Figure D.4: DHHR vs. LS productivity distributions in DHHR model



(a) DHHR productivity distributions

(b) LS productivity distributions

E Turbulence affecting job market encounters

DHHR assume that after an encounter between a firm and an unemployed worker that does not result in employment, the worker faces the same risk of losing skills as if she had quit from a job. They justify this assumption only for its tractability in allowing them to reduce the number of worker types that they must track. In Figure D.1 of Appendix D, we confirm that the assumption does not make much of a difference for DHHR's inference about the turbulence-unemployment relationship in their model. But when we pursue a parallel analysis in the benchmark model with LS productivity distributions as we do here, we find that DHHR's simplifying assumption has a large impact. We show this in subsection E.1. To shed light on the forces at work, subsection E.2 undertakes yet another perturbation exercise that limits the exposure to such risk to the first $\bar{k}$ periods of an unemployment spell, after which there is no risk of skill loss during the rest of an unemployment spell.

To allow for a more general formulation, we assume a distinct probability γ^e of skill loss after an unsuccessful job market encounter, while γ^q continues to denote the probability of skill loss when quitting from an employment relationship.

E.1 Introducing turbulence for unemployed in benchmark model

When unemployed high-skilled workers face a probability γ^e of losing skills after unsuccessful job market encounters, the match surplus in (3) of a new job with a high-skilled worker changes to

$$s^o_{hh}(z) = (1-\tau)z + g_h(z) - [b_h + (1-\gamma^e)\omega_{hh} + \gamma^e\omega_{lh}], \tag{E.9}$$

where the outside value in brackets reflects the risk of skill loss if the firm and worker do not enter an employment relationship. The net change of the mass of low-skilled unemployed with high benefits in

(24) changes to

$$\Delta u_{lh} = (1-\rho^r)\Bigg\{ \underbrace{\rho^x \gamma^\ell e_{hh}}_{\text{1. layoff turbulence}} + \underbrace{(1-\rho^x)\gamma^q \nu_{hh}[\gamma^s e_{hh} + \gamma^u e_{ll}]}_{\text{2. quit turbulence}} - \underbrace{\lambda^w(\theta)(1-\nu^o_{lh})u_{lh}}_{\text{3. successful matches}} + \underbrace{\lambda^w(\theta)\gamma^e \nu^o_{hh} u_{hh}}_{\text{4. turbulence unempl.}} \Bigg\} - \rho^r u_{lh}, \tag{E.10}$$

where the new term numbered 4 is the inflow of unemployed high-skilled workers who have just lost their skills after job market encounters that did not lead to employment.

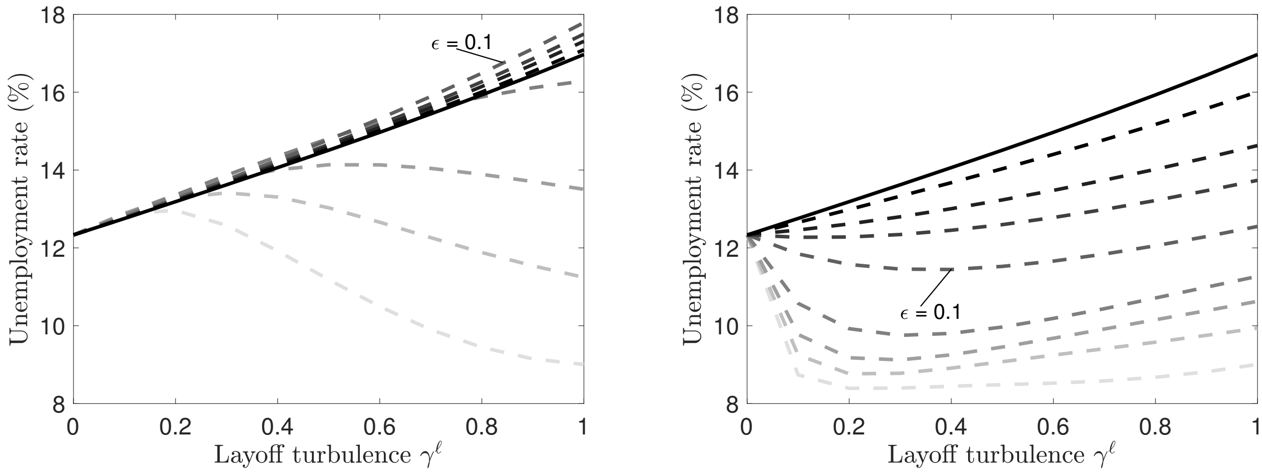
Turning to a quantitative assessment of turbulence for unemployed workers in the benchmark model with LS productivity distributions, we must take a stand on different lengths of a model period that were used in parameterizations of that model and DHHR. In the case of the exogenously given layoff risk, the probability of a layoff at the semi-quarterly frequency in the benchmark model is half of the probability at the quarterly frequency in DHHR's model, as discussed in footnote 2. Analogously, but less obviously, for the risk of skill loss after endogenously determined unsuccessful job market encounters we assume that $\gamma^e = 0.5\gamma^q$ in the semi-quarterly model as compared to DHHR's assumption that $\gamma^e = \gamma^q$ in their quarterly model. However, for the record, our conclusion from Figure E.1 remains the same with or without the latter adjustment. That is, with or without this adjustment, adding exposure of unemployed workers to risks of skill loss after unsuccessful job market encounters has sizeable effects on the turbulence-unemployment relationship in the benchmark model with LS productivity distributions.

As mentioned in footnote 11, risk of skill loss after unsuccessful job market encounters was not part of DHHR's use of quit turbulence to challenge a Ljungqvist-Sargent positive turbulence-unemployment relationship. Rather, they adopted it for computational tractability. Hence, we feel justified in discarding this auxiliary feature of DHHR's original analysis in order to focus more sharply on the key explanation to the puzzle – different productivity distributions. But it is nevertheless tempting to turn on and off their auxiliary assumption to shed further light on the mechanics of our particular matching model, and matching frameworks more generally. Therefore, we offer the following suggestive decomposition of forces at work.

E.2 Decomposition of forces at work

We seek to isolate two interrelated forces acting when job seekers are exposed to risk of skill loss after unsuccessful job market encounters in a matching model. First, the mere risk of losing skills when turning down job opportunities suppresses the return to labor mobility in many frictional models of labor markets, including the basic McCall (1970) search model where wages are drawn from an exogenous offer distribution. Such risks would render job seekers more prone to accept employment opportunities. We call this the "allocation channel." Second, the matching framework contains yet another force when risk of skill loss after an unsuccessful job market encounter weakens the bargaining position of a worker vis-à-vis a firm and accordingly affects match surpluses received by firms. That

Figure E.1: TURBULENCE FOR UNEMPLOYED IN BENCHMARK WITH LS PRODUCTIVITY



(a) Without turbulence for unemployed

(b) With turbulence for unemployed

in turn affects vacancy creation via the equilibrium condition that vacancy posting must break even. We call this the "bargaining channel."

It presents a challenge to isolate these two channels because everything is related to everything else in an equilibrium. Here we study how equilibrium outcomes change as we vary the horizon over which the risk of skill loss prevails during an unemployment spell. Thus, after an unsuccessful job market encounter, let an unemployed worker be exposed to risks of skill losses for the first $\bar{k}$ periods of being unemployed and thereafter to suffer no risk of skill loss for the remainder of that unemployment spell. To illustrate the allocation channel, consider the basic McCall search model. Starting from $\bar{k} = 0$, equilibrium unemployment would initially be significantly suppressed for each successive increase in the parameter $\bar{k}$ because workers anticipate ever longer periods of effective exposure to risk of skill loss when unemployed; but eventually, the value of $\bar{k}$ is so high that it is most unlikely that a worker remains unemployed for such an extended period of time and hence, a worker's calculation of the payoff from quitting a job would hardly be affected by any additional increase in $\bar{k}$. Thus, in a McCall search model, via the allocation channel, equilibrium unemployment would hardly change for higher values of $\bar{k}$. In contrast, we will find in the matching model that unemployment suppression effects that occur in response to increases in $\bar{k}$ don't die out beyond such high values of $\bar{k}$. We then argue that those equilibrium outcome effects can be attributed to the bargaining channel.

Notation Let u^0_{hh} denote the mass of high-skilled workers who become unemployed in each period without losing skills, and let u^k_{hh} be the mass of those workers who remain high-skilled and unemployed after an unemployment duration of $k = 1, \ldots, \bar{k} - 1$ periods. A final category $u^{\bar{k}}_{hh}$ includes all workers who remain high-skilled and unemployed after unemployment spells of *at least* $\bar{k}$ periods, i.e., $u^{\bar{k}}_{hh}$ is the mass of unemployed high-skilled workers who no longer face any risk of skill loss in their current unemployment spells.

Using the same superscript convention, let $\omega^{w,k}_{hh}$ for $k = 0, \ldots, \bar{k}$ be the future value of unemployment of an unemployed high-skilled worker in category u_{hh}^k, with $\underline{z}_{hh}^k$ and ν_{hh}^k denoting the worker's reservation productivity and rejection probability next period, and for any match accepted next period, the match surplus is $s_{hh}^k(z)$ and the initial wage is $p_{hh}^k(z)$.

Laws of motion The laws of motion for worker categories u_{hh}^k, for $k = 0, \ldots, \bar{k} - 1$, have in common that all workers leave the category next period. The inflow to the initial category u_{hh}^0 consists of employed high-skilled workers who experience non-turbulent layoffs or quits, including low-skilled employed workers who have just received a skill upgrade. Each successive category u_{hh}^k, for $k = 1, \ldots, \bar{k} - 1$, receives its inflow from not retired workers in the preceding category u_{hh}^{k-1}, those who did not match or experienced non-turbulent rejections of matches:

$$\Delta u_{hh}^k = \begin{cases} (1-\rho^r)\Big[\underbrace{\rho^x(1-\gamma^\ell)e_{hh}}_{\text{non-turbulent layoff}} + \underbrace{(1-\rho^x)\nu_{hh}(1-\gamma^q)(\gamma^s e_{hh} + \gamma^u e_{ll})}_{\text{non-turbulent quit}}\Big] - u_{hh}^k & \text{if } k = 0 \\ (1-\rho^r)\Big[\underbrace{(1-\lambda^w(\theta))}_{\text{no match}} + \underbrace{\lambda^w(\theta)\nu_{hh}^{k-1}(1-\gamma^e)}_{\text{non-turbulent rejected match}}\Big]u_{hh}^{k-1} - u_{hh}^k & \text{if } 0 < k < \bar{k}. \end{cases}$$

The final category $u_{hh}^{\bar{k}}$ also receives inflows from the preceding category $u_{hh}^{\bar{k}-1}$, but now outflows are only partial. The workers who leave are the retirees and those with accepted matches (those with rejected matches are no longer affected by turbulence and thus always remain):

$$\Delta u_{hh}^{\bar{k}} = (1-\rho^r)\Big[(1-\lambda^w(\theta)) + \lambda^w(\theta)\nu_{hh}^{\bar{k}-1}(1-\gamma^e)\Big]u_{hh}^{\bar{k}-1} - \Big[\rho^r + (1-\rho^r)\lambda^w(\theta)(1-\nu_{hh}^{\bar{k}})\Big]u_{hh}^{\bar{k}}.$$

The law of motion for u_{lh} workers is modified to receive the inflow from the different u_{hh}^k categories that suffered turbulent rejections in their first $\bar{k}$ periods of unemployment:

$$\begin{aligned} \Delta u_{lh} &= (1-\rho^r)\Big[\underbrace{\rho^x\gamma^\ell e_{hh} + (1-\rho^x)\nu_{hh}\gamma^q(\gamma^s e_{hh} + \gamma^u e_{ll})}_{\text{turbulent separations}} + \underbrace{\lambda^w(\theta)\gamma^e \sum_{k=0}^{\bar{k}-1}\nu_{hh}^k u_{hh}^k}_{\text{turbulent rejections}}\Big] \\ &- \left[\rho^r + (1-\rho^r)\lambda^w(\theta)(1-\nu_{lh}^o)\right]u_{lh}. \end{aligned}$$

The law of motion for high-skilled employed workers e_{hh} is adjusted to include those gaining employment from the different u_{hh}^k categories:

$$\begin{aligned} \Delta e_{hh} &= (1-\rho^r)\Big[\underbrace{\lambda^w(\theta)\sum_{k=0}^{\bar{k}}(1-\nu_{hh}^k)u_{hh}^k}_{\text{accepted new matches}} + \underbrace{(1-\rho^x)\gamma^u(1-\nu_{hh})e_{ll}}_{\text{accepted upgrades}}\Big] \\ &- \left[\rho^r + (1-\rho^r)(\rho^x + (1-\rho^x)\gamma^s\nu_{hh})\right]e_{hh}. \end{aligned}$$

High-skilled unemployed: match surplus, initial wage, and value of unemployment For a high-skilled worker who remains unemployed after $k < \bar{k}$ periods, the match surplus of any job opportunity next period reflects an outside option with risk γ^e of losing skills if the employment relationship is not formed; but after $\bar{k}$ periods there is no such risk:

$$s^k_{hh}(z) = \begin{cases} (1-\tau)z + g_h(z) - \Big[b_h + (1-\gamma^e)\omega^{w,k+1}_{hh} + \gamma^e \omega^w_{lh} + \omega^f\Big] & \text{if } k < \bar{k} \\ (1-\tau)z + g_h(z) - \Big[b_h + \omega^{w,k}_{hh} + \omega^f\Big] & \text{if } k = \bar{k}. \end{cases}$$

Reservation productivities and rejection probabilities satisfy

$$s^k_{hh}(\underline{z}^k_{hh}) = 0\,, \qquad \nu^k_{hh} = \int_{-\infty}^{\underline{z}^k_{hh}} dv^o_h(y)\,.$$

The wage in the first period of employment of such a high-skilled worker is

$$\begin{aligned} p^k_{hh}(z) + g^w_h(z) &= \pi s^k_{hh}(z) + b_h + (1-\gamma^e)\omega^{w,k+1}_{hh} + \gamma^e \omega^w_{lh} && \text{if } k < \bar{k} \\ p^k_{hh}(z) + g^w_h(z) &= \pi s^k_{hh}(z) + b_h + \omega^{w,k}_{hh} && \text{if } k = \bar{k}. \end{aligned}$$

The value of unemployment for a high-skilled worker in her k:th period of unemployment is equal to $b_h + \omega^{w,k}_{hh}$, where

$$\omega^{w,k}_{hh} = \begin{cases} \beta \Big[\underbrace{\lambda^w(\theta)\int_{\underline{z}^k_{hh}}^{\infty} \pi s^k_{hh}(y)\, dv^o_h(y)}_{\text{match + accept}} + \underbrace{\lambda^w(\theta)(b_h + (1-\gamma^e)\omega^{w,k+1}_{hh} + \gamma^e\omega^w_{lh})}_{\text{outside value with match}} \\ \qquad + \underbrace{(1-\lambda^w(\theta))(b_h + \omega^{w,k+1}_{hh})}_{\text{outside value without match}}\Big] & \text{if } k < \bar{k} \\ \beta \Big[\underbrace{\lambda^w(\theta)\int_{\underline{z}^k_{hh}}^{\infty} \pi s^k_{hh}(y)\, dv^o_h(y)}_{\text{match + accept}} + \underbrace{b_h + \omega^{w,k}_{hh}}_{\text{outside value}}\Big] & \text{if } k = \bar{k}. \end{cases}$$

High-skilled employed: match surplus, wage, and joint continuation value The match surplus for continuing employment of a high-skilled worker reflects the risk of layoffs and quits that can be affected by turbulence in the form of skill loss. A non-turbulent separation falls into the initial category of high-skilled unemployed, u^0_{hh}. We adjust match surpluses, wages, and joint continuation values of these workers to include the new outside value $\omega^{w,0}_{hh}$.

The match surplus of a continuing job with a high-skilled worker is

$$s_{hh}(z) = (1-\tau)z + g_h(z) - [b_h + (1-\gamma^q)\omega^{w,0}_{hh} + \gamma^q\omega^w_{lh} + \omega^f]$$

and the wage equals

$$p_{hh}(z) + g^w_h(z) = \pi s_{hh}(z) + b_h + (1-\gamma^q)\omega^{w,0}_{hh} + \gamma^q\omega^w_{lh}.$$

The joint continuation value of a job with a high-skilled worker is

$$\begin{aligned} g_h(z) &= \beta\Big[\rho^x\left(b_h + (1-\gamma^\ell)\omega_{hh}^{w,0} + \gamma^\ell\omega_{lh}^w + \omega^f\right) \\ &+ (1-\rho^x)(1-\gamma^s)((1-\tau)z + g_h(z)) \\ &+ (1-\rho^x)\gamma^s\left(E_{hh} + \nu_{hh}\left(b_h + (1-\gamma^q)\omega_{hh}^{w,0} + \gamma^q\omega_{lh}^w + \omega^f\right)\right)\Big]. \end{aligned}$$

Since a low-skilled worker faces the possibility of a skill upgrade, we also need to update the joint continue value of an employed low-skilled worker as follows:

$$\begin{aligned} g_l(z) &= \beta\Big[\rho^x(b_l + \omega_{ll}^w + \omega^f) \\ &+ (1-\rho^x)(1-\gamma^u)(1-\gamma^s)((1-\tau)z + g_l(z)) \\ &+ (1-\rho^x)(1-\gamma^u)\gamma^s\left(E_{ll} + \nu_{ll}(b_l + \omega_{ll}^w + \omega^f)\right) \\ &+ (1-\rho^x)\gamma^u\left(E_{hh} + \nu_{hh}\left(b_h + (1-\gamma^q)\omega_{hh}^{w,0} + \gamma^q\omega_{lh}^w + \omega^f\right)\right)\Big]. \end{aligned}$$

Vacancy creation Free entry of firms make a firm's value ω^f of entering the vacancy pool be zero. With more types of unemployed high-skilled workers, zero-profit condition (14) changes to become

$$\mu = \beta\frac{m(\theta)}{\theta}(1-\pi)\left[\frac{u_{ll}}{u}\int_{\underline{z}_{ll}^o}^{\infty} s_{ll}^o(y)\ dv_l^o(y) + \frac{u_{lh}}{u}\int_{\underline{z}_{lh}^o}^{\infty} s_{lh}^o(y)\ dv_l^o(y) + \sum_{k=0}^{\bar{k}}\frac{u_{hh}^k}{u}\int_{\underline{z}_{hh}^k}^{\infty} s_{hh}^k(y)\ dv_h^o(y)\right],$$

where $u = u_{ll} + u_{lh} + \sum_{k=0}^{\bar{k}} u_{hh}^k$.

High-skilled unemployment spells terminated within $\bar{k}$ periods In each period, a mass u_{hh}^0 of high-skilled workers flows into unemployment. Let $\phi^{\bar{k}}$ denote the fraction of these who will experience unemployment spells of no longer duration than $\bar{k}$ periods. To enable a recursive computation, define m_h^k as the mass of workers who remain high-skilled and unemployed after k periods, and let m_l^k be the accompanying mass that remain unemployed but who have experienced skill loss by that kth period of unemployment. Given initial conditions $m_h^0 = u_{hh}^0$ and $m_l^0 = 0$, we compute

$$\begin{aligned} m_h^k &= (1-\rho^r)\left[1 - \lambda^w(\theta) + \lambda^w(\theta)\nu_{hh}^{k-1}(1-\gamma^e)\right]m_h^{k-1} \\ m_l^k &= (1-\rho^r)\left[(1 - \lambda^w(\theta) + \lambda^w(\theta)\nu_{lh})m_l^{k-1} + \lambda^w(\theta)\nu_{hh}^{k-1}\gamma^e m_h^{k-1}\right], \end{aligned}$$

for $k = 1, \ldots, \bar{k}$;[7] and

$$\phi^{\bar{k}} = \frac{u_{hh}^0 - m_h^{\bar{k}} - m_l^{\bar{k}}}{u_{hh}^0}. \tag{E.11}$$

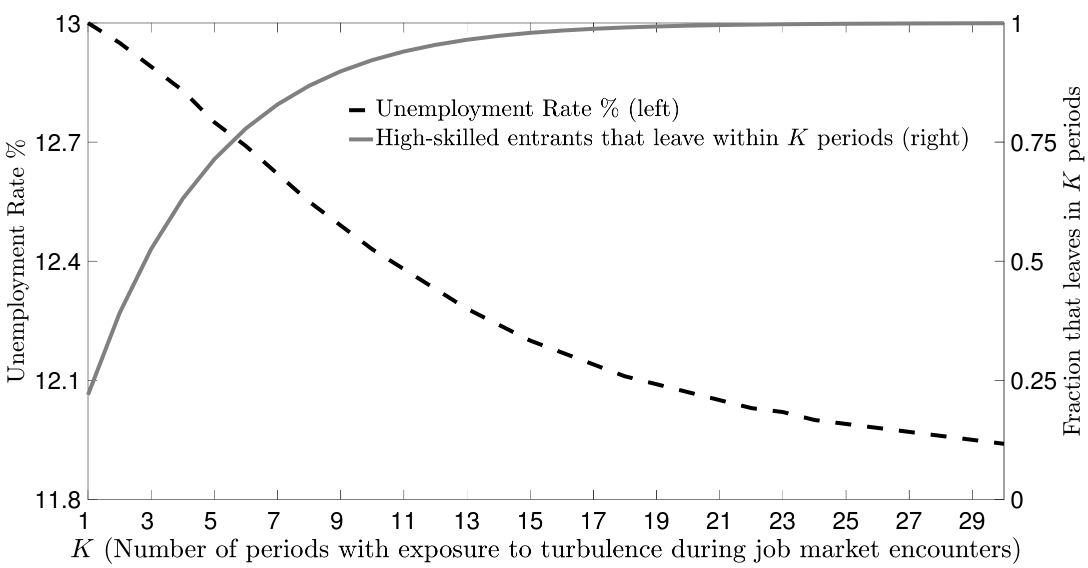
Numerical example To illustrate and decompose the forces at work, we set layoff turbulence equal to $\gamma^\ell = 0.2$ and quit turbulence to $\gamma^q = \epsilon\gamma^\ell = 0.1 \cdot \gamma^\ell = 0.02$. As discussed above, turbulence for

[7] Note that $m_h^k = u_{hh}^k$ for $k = 0, \ldots, \bar{k}-1$, while $m_h^{\bar{k}}$ is merely a subset of $u_{hh}^{\bar{k}}$.

unemployed workers in the semi-quarterly benchmark model is assumed to be half of quit turbulence, i.e., $\gamma^e = 0.5\gamma^q = 0.01$.

Figure E.2 depicts two unemployment outcomes in distinct economies that differ only with respect to the parameter value of $\bar{k}$, i.e., the length of time over which an unemployed worker is exposed to the risk of losing skills due to unsuccessful job market encounters. The two outcomes are the unemployment rate u and the fraction $\phi^{\bar{k}}$ of high-skilled entrants into unemployment who will see their unemployment spells terminated within $\bar{k}$ periods by either finding employment or retiring. For each economy indexed by $\bar{k}$, the value of u can be read off from the dashed line (in percent on the left scale), and $\phi^{\bar{k}}$ from the solid line (as a fraction on the right scale).

Figure E.2: Turbulence exposure of unemployed in benchmark with LS prod.

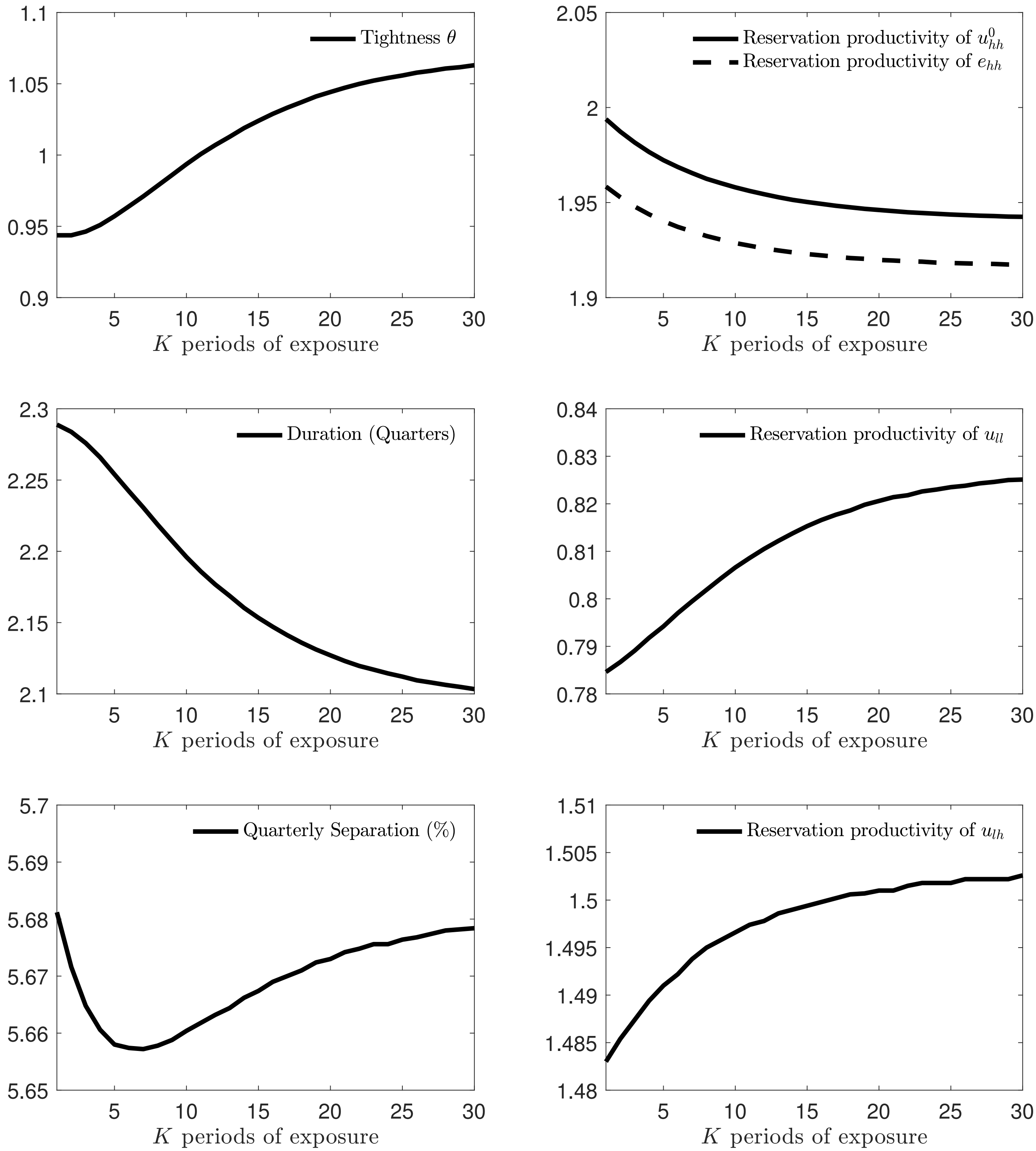


As anticipated from our above discussion of the allocation channel, the unemployment rate in Figure E.2 is lower in economies with a higher $\bar{k}$ since longer exposure to risk of skill loss reduces the return to labor mobility. Hence, fewer high-skilled workers quit their jobs, and those who do quit will on average move back into employment more quickly. For example, when $\bar{k}$ increases from 1 to 9, the unemployment rate falls by half a percentage point. As noted earlier, the allocation channel would also be operating in the basic McCall search model, and the unemployment effects of further increases in $\bar{k}$ there should become muted when the value of $\bar{k}$ is set so high that the vast majority of unemployment spells are shorter than $\bar{k}$ in durations. But, as can be seen in Figure E.2 at $\bar{k} = 9$, 90 percent of all unemployment spells by high-skilled entrants are terminated within $\bar{k}$ periods, yet the unemployment rate falls another half a percentage point after further increases in $\bar{k}$. According to our earlier discussion of the bargaining channel, there is a force in matching models that is not present in McCall models. This other force makes it possible for skill losses at unlikely long unemployment spells to have substantial effects on equilibrium outcomes through its impact on bargaining. The reason is that even though realizations of such long unemployment spells are rare, the extended risk of skill loss will weaken the bargaining position of a worker vis-à-vis a firm throughout an unemployment spell.[8]

Figure E.3 depicts additional statistics that summarize outcomes across alternative values of $\bar{k}$. The positive relationship between $\bar{k}$ and market tightness indicates how the bargaining channel tilts match surpluses to firms when the risk of skill loss after unsuccessful job market encounters weakens the bargaining position of workers. Recall that the equilibrium zero-profit condition for vacancy posting funnels expected present values of firms' match surpluses into vacancy creation. The resulting higher market tightness implies a higher probability that an unemployed worker encounters a vacancy. Evidently, a worker's higher match probability induces low-skilled unemployed workers (as well as employed ones), both those with low and those with high benefits, to choose higher reservation productivities. The net result is still a shorter average duration of unemployment spells. And with not much change in a mildly U-shaped relationship for the job separation rate, we arrive at an unemployment rate that continues to fall over most of the range in Figure E.2. From these intricacies, we conclude that the bargaining channel already operates in tandem with the allocation channel over the first range of $\bar{k}$ in that figure, but that it operates mostly on its own over the second range where most entrants of high-skilled workers into unemployment expect to terminate their unemployment spells well before $\bar{k}$ periods.

[8]For another stark example of unlikely events having large effects on equilibrium outcomes through the bargaining channel, see Ljungqvist and Sargent's (2017) analysis of alternating-offer wage bargaining as one way to make unemployment respond sensitively to movements in productivity in matching models. A general result is that the elasticity of market tightness with respect to productivity is inversely related to a model-specific "fundamental surplus" divided by worker productivity. Under alternating-offer bargaining the fundamental surplus is approximately equal to the difference between worker productivity and the sum of the value of leisure and a firm's cost of delay in bargaining. Thus, the magnitude of the latter cost is a critical determinant of the volatility of unemployment in response to productivity shocks, even though no such cost will ever be incurred because in equilibrium there will be no delay in bargaining.

Figure E.3: MORE STATISTICS POINTING TO THE "BARGAINING CHANNEL"



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