Calibration and Cross-Phenomenon Restrictions

Layoff Costs, Turbulence, and Unemployment *

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January 15, 2023

Abstract

Cross-phenomenon restrictions involving returns to labor mobility can guide calibrations of productivity processes in macro-labor models. We exploit how returns to labor mobility determine effects on equilibrium unemployment of changes in (a) layoff costs, and (b) likelihoods of skill losses following quits ("quit turbulence"). An associated cross-phenomenon restriction together with evidence on extensive labor reallocations within market economies having different layoff costs imply that a turbulence-theoretic explanation of trans-Atlantic unemployment experiences is robust to the addition of plausible magnitudes of quit turbulence.

JEL: E24, J63, J64

Keywords: matching model, skills, turbulence, unemployment, turnover, calibration.

*We thank seminar participants at CREI, Pompeu Fabra, 7th Meeting of the European Search and Matching Network, Society of Economic Dynamics 2017, and 10th NYU Search Theory Workshop, for useful comments and suggestions. Research support is gratefully acknowledged by Baley from the Spanish Ministry of the Economy and Competitiveness, Project ECO2015-67655-P, and by Ljungqvist from the Jan Wallander and Tom Hedelius Foundation.

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"... often the most important constraint on a new theory is ... that it should agree with the whole body of past observations, as crystallized in former theories. ... New theories of course do not agree entirely with any previous theory – otherwise they would not be new – but they must not throw out all the success of former theories. This sort of thing makes the work of the theorist far more conservative than is often thought.

"The wonderful thing is that the need to preserve the successes of the past is not only a constraint, but also a guide." Steven Weinberg (2018, p. 197)

1 Introduction

This paper aims to show once again the benefits promised by a calibration approach, codified for macroeconomics by Cooley and Prescott (1995), that Tom Cooley used so creatively and insightfully. We aspire to live up to the high standards that Tom Cooley set for us when he did what he loved to call "economic science." We present a macro-labor economics variation on a theme of particle physicist Steven Weinberg in the epigraph above. Our research strategy combines essential ingredients of Tom Cooley's research. Front and center are (1) a substantive economic puzzle in the form of a structural break in the trans-Atlantic unemployment experience and (2) an apparently successful turbulence-theoretic explanation of it that (3) had been challenged as not being robust to a model perturbation that opened a channel that the turbulence model had neglected. However, direct evidence about that new channel was unavailable to help calibrate critical new parameters. At this point we apply Steven Weinberg's rules. Specifically, we study another economic phenomenon that is affected by the channel opened by the perturbed model, a phenomenon about which there is ample empirical evidence and one that earlier macro-labor models had explained. By using an implied "cross-phenomenon restriction" to calibrate critical parameters in the perturbed model, we can resolve the robustness challenge in favor of the original turbulence theory.

We work with a quantitative macroeconomic model of labor market frictions.[1] We start from two complementary studies that added forces and phenomena that had earlier been excluded from a generalized McCall (1970) search model that Ljungqvist and Sargent (1998) had constructed to quantify adverse macroeconomic consequences coming from interactions between microeconomic turbulence and generous unemployment compensation in European welfare states. Ljungqvist and Sargent modeled turbulence with risks of human capital losses coincident with *involuntary* job losses ("layoff turbulence"). While that model explained persistent and systematically higher unemployment rates in Europe than in the US since the late 1970s, it excluded any losses of human capital coincident with *voluntary* separations from jobs. Neglect of such "quit turbulence" risk is the starting point of

[1] Unlike the situation in particle physics or cosmology, there is no "standard model" of forces that shape an equilibrium unemployment rate. Each of three workable classes of models of frictional unemployment has persuasive advocates and skillful users: (1) matching models in the Diamond-Mortensen-Pissarides tradition; (2) equilibrium versions of McCall (1970) search models; and (3) search-island models in the tradition of Lucas and Prescott (1974). Calibrated versions of all three types of models have succeeded in fitting data on labor market flows and generating plausible responses of unemployment rates to government policies like generous unemployment insurance and layoff taxes.

our story because in 1998 an astute observer, Alan Greenspan (1998, p. 743), suggested that a more hazardous job market had recently suppressed mobility among employed workers and had led to less upward pressure on wages:

> "... the sense of increasing skill obsolescence has also led to an apparent willingness on the part of employees to forgo wage and benefit increases for increased job security. Thus, despite the incredible tightness of labor markets, increases in compensation per hour have continued to be relatively modest."

den Haan, Haefke and Ramey (2005, henceforth DHHR) cited Greenspan's words at the beginning of a paper that calibrated a Diamond-Mortensen-Pissarides matching model that they used to represent Greenspan's idea by including quit turbulence in the form of an immediate stochastic depreciation of a worker's human capital that in turbulent times would be triggered by a worker's decision to quit a job. DHHR reported a calibration that affirmed the quantitative importance of what they interpreted as the force Greenspan had in mind: even a small amount of quit turbulence gave workers strong enough reluctance to quit to reduce both quits and job reallocation substantially. DHHR's success in representing and quantifying Greenspan's idea had interesting quantitative ramifications that DHHR stressed: adding even a small amount of quit turbulence to their matching model reversed the unemployment-increasing interactions between layoff turbulence and welfare state generosity that Ljungqvist and Sargent (1998) had used to explain trans-Atlantic differences in unemployment rates. DDHR's representation and calibration of Greenspan's force thus cast doubt on Ljungqvist and Sargent's inference that a rise in turbulence explained the outbreak of high European unemployment.

This raises questions about what led to DHHR's reversal of Ljungqvist and Sargent's inference about the interaction of heightened turbulence and trans-Atlantic differences in unemployment outcomes. Was it DHHR's adding the Greenspan force via quit turbulence? Or was it DHHR's choice to replace Ljungqvist and Sargent's extended McCall framework with their version of a Diamond-Mortensen-Pissarides matching model? Or might it be something else, e.g., calibrations of processes exogenous to both the DHHR model and the Ljungqvist and Sargent model?

Hornstein, Krusell and Violante (2005, section 8.3) offered a tentative answer. They cited DHHR's finding as indicating lack of robustness of the earlier explanation of those trans-Atlantic unemployment rate differences, leading them to doubt its validity:

> "... once the Ljungqvist and Sargent mechanism is embedded into a model with endogenous job destruction, the comparative statics for increased turbulence are reversed, i.e., unemployment falls. The reason is that as the speed of skill obsolescence rises, workers become more reluctant to separate, and job destruction falls."

Hornstein *et al.* thus indicated that what had allowed DHHR to overturn the Ljungqvist and Sargent inference about how higher turbulence had affected Europe and America differently was Ljungqvist and Sargent's reliance on a McCall search model that largely excluded endogenous job separations.[2]

[2] Learning-by-doing human capital accumulation induces endogenous job separations in the model of Ljungqvist and Sargent (1998). But besides exogenous layoffs, there are no on-the-job shocks to productivity per unit of human capital.

To address that concern, in this paper we instead use a Diamond-Mortensen-Pissarides matching model that starts from the structure previously used by Ljungqvist and Sargent (2007, henceforth LS), to which we add quit turbulence. In contrast to DHHR's findings, when we introduce skill losses at times of voluntary quits into the LS model, we find only small effects on outcomes: quit turbulence has to be about 50% of layoff turbulence and both kinds of turbulence must be high before quit turbulence can suppress unemployment relative to tranquil zero-turbulence times. We show that the big disagreement between the matching model analysis of DHHR and a version of the LS model augmented to incorporate quit turbulence comes from differences in returns to labor mobility that are implied by parameterizations of stochastic processes that determine productivities.[3] It turns out that for quit turbulence to reverse the unemployment-increasing interactions between turbulence and welfare state generosity, equilibrium returns to labor mobility must be sufficiently small. Thus, evidence about the returns to labor mobility can shed light on the potential impact of quit turbulence. How might we assemble pertinent evidence?

To infer quantitatively plausible returns to labor mobility and hence, the potential impact of quit turbulence, we consult a cross-phenomenon restriction that involves effects on unemployment of changes in layoff costs, on the one hand, and changes in quit turbulence, on the other hand. To do this, we proceed by, first, incorporating a consensus view about the order of magnitude of the effects of layoff costs on unemployment to deduce reasonable parameter values for productivity processes, and, then, studying the associated potential impact of quit turbulence on the turbulence-unemployment relationship.

Section 2 sets forth a matching model augmented to include quit turbulence as in the DHHR model. Using the productivity processes of LS and DHHR as examples of high versus low returns to labor mobility, Section 3 studies the unemployment effects of layoff costs and quit turbulence, respectively. Magnitudes of effects of layoff costs and quit turbulence on unemployment are tied together through their common dependence on returns to labor mobility. To highlight that key link, Section 4 constructs a mapping from the parameters of the productivity process to outcome statistics for layoff costs and quit turbulence, respectively. The statistics are highly correlated. Specifically, in the relevant parameter region where voluntary separations do not shut down under levels of layoff costs inferred from welfare state observations, plausible levels of quit turbulence cannot reverse a positive turbulence-unemployment relationship. Section 5 offers some concluding remarks. Auxiliary materials appear in online Appendices.

2 A matching model with quit turbulence

Our benchmark is a standard matching model to which we add human capital dynamics that incorporate turbulence. Specifically, we adopt the LS matching model that has layoff turbulence in the form of worse skill transition probabilities for workers who suffer involuntary layoffs. We augment the model to include quit turbulence – worse skill transition probabilities for workers who experience

[3] This finding illustrates an assertion of Baley, Ljungqvist and Sargent (2022) that "returns to labor mobility have too often escaped the attention they deserve as conduits of important forces in macro-labor models." In this paper, we shall use a cross-phenomenon restriction to calibrate those returns.

voluntary quits – as in the DHHR model.[4]

2.1 Environment

Workers There is a unit mass of workers who are either employed or unemployed. Workers are risk neutral, value consumption, and have preferences ordered according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_t. \tag{1}$$

They discount future utilities at a rate $\beta \equiv \hat{\beta}(1-\rho^r)$, where $\hat{\beta} \in (0,1)$ is a subjective time discount factor and $\rho^r \in (0,1)$ is a constant probability of retirement. A retired worker exits the economy and is replaced by a newborn worker.

Worker heterogeneity Besides employment status, workers differ along two dimensions: a current skill level i that can be either low (l) or high (h) and a skill level j that determines a worker's entitlement to unemployment benefits. An employed worker has $j = i$, but for an unemployed worker, j is the skill level during her last employment spell. Workers gain or lose skills depending on their employment status and instances of layoffs and quits. We assume that all newborn workers enter the labor force with low skills and a low benefit entitlement. In this way, each worker bears two indices (i, j), the first denoting current skill and the second denoting benefit entitlement.

Firms and matching technology There is free entry of firms who can post vacancies at a cost μ per period. Aggregate numbers of unemployed u and vacancies v are inputs into an increasing, concave and linearly homogeneous matching function $M(v, u)$. Let $\theta \equiv v/u$ be the vacancy-unemployment ratio, also called market tightness. The probability $\lambda^w(\theta) = M(v, u)/u = M(\theta, 1) \equiv m(\theta)$ that an unemployed worker encounters a vacancy is increasing in market tightness. The probability $M(v, u)/v = m(\theta)/\theta$ that a vacancy encounters an unemployed worker is decreasing in market tightness.

Worker-firm relationships and productivity processes A job opportunity is a productivity draw z from a distribution $v_i^o(z)$ that is indexed by a worker's skill level i. We assume that the high-skill distribution first-order stochastically dominates the low-skill distribution: $v_h^o(z) \leq v_l^o(z)$. Wages are determined through Nash bargaining, with π and $1 - \pi$ as the bargaining weights of a worker and a firm, respectively.

Idiosyncratic shocks within a worker-firm match determine an employed worker's productivities. Productivity in an ongoing job is governed by a first-order Markov process with a transition probability matrix Q_i, also indexed by the worker's skill level i, where $Q_i(z, z')$ is the probability that next period's productivity becomes z', given current productivity z. Specifically, an employed worker retains her

[4] LS thanked Wouter den Haan, Christian Haefke, and Garey Ramey for generously sharing computer code that LS then modified. Much of our notation and mathematics follow DHHR closely. For an account of differences between the models of LS and DHHR, see Appendix A.

last period productivity with probability $1 - \gamma^s$, but with probability γ^s draws a new productivity from the distribution $v_i(z)$. As in the case of the productivity distributions for new matches, the high-skill distribution in ongoing jobs first-order stochastically dominates the low-skill distribution: $v_h(z) \leq v_l(z)$. Furthermore, an employed worker's skills may get upgraded from low to high with probability γ^u. A skill upgrade is accompanied by a new productivity drawn from the high-skill distribution $v_h(z)$. A skill upgrade is realized immediately, regardless of whether the worker remains with her present employer or quits.

We can now define our notions of layoffs and quits.

(i) **Layoffs:** At the beginning of each period, a job is exogenously terminated with probability ρ^x. We call this event a layoff. An alternative interpretation of the job-termination probability ρ^x is that productivity z becomes zero and stays zero forever. A layoff is involuntary in the sense of offering no choice.

(ii) **Quits:** As a consequence of a new productivity draw on a job and possibly a skill upgrade, a relationship can continue or be endogenously terminated. We label separation after such an event a voluntary quit because a firm and a worker agree to separate after Nash bargaining.

Turbulence We define turbulence as the risk of losing skills after a job separation. High-skilled workers might become low-skilled workers. Two types of turbulence shocks depend on the reason for a job separation, namely, a layoff or a quit. Upon a layoff, a high-skilled worker experiences a skill loss with probability $\gamma^{d,x}$. We label this risk *layoff turbulence*. Upon a quit, a high-skilled worker faces the probability γ^d of a skill loss. We label this risk *quit turbulence*.

Turbulence shocks are timed as follows. At the beginning of a period, exogenous job terminations occur and displaced workers face layoff turbulence. Continuing employed workers can experience new productivity draws on the job and skill upgrades; if workers quit, they are subject to quit turbulence. All separated workers join other unemployed workers in the matching function where they might or might not encounter vacancies next period.

Government policy The government provides unemployment compensation. An unemployed worker who was low (high) skilled in her last employment receives a benefit b_l (b_h).[5] Unemployment benefit b_i is calculated as a fraction ϕ of the average wage of employed workers with skill level i. The government imposes a layoff tax Ω on every job termination except for retirements.

The government runs a balanced budget by levying a flat-rate tax τ on production. If layoff tax revenues fully cover payments of unemployment benefits, the government sets $\tau = 0$ and returns any surplus as lump-sum transfers to workers. Since the latter will not happen in our analyses, we omit such lump-sum transfers in our expressions below.

[5] As mentioned above, newborn workers are entitled to b_l. Also, for simplicity, we assume that a worker who receives a skill upgrade and chooses to quit, is entitled to high benefits.

2.2 Match surpluses

A match between a firm and a worker with skill level i and benefit entitlement j that has drawn productivity z will form an employment relationship, or continue an existing one, if a match surplus is positive. The match surplus for a new job $s^o_{ij}(z)$ or a continuing job $s_{ij}(z)$ is given by the after-tax productivity $(1-\tau)z$ plus the future joint continuation value $g_i(z)$ minus the outside values of the match that consist of the worker's receiving unemployment benefit b_j and a future value ω^w_{ij} associated with entering the unemployment pool in the current period; and the firm's value ω^f from entering the vacancy pool in the current period, net of paying the vacancy cost μ. For notational simplicity, we define $\omega_{ij} \equiv \omega^w_{ij} + \omega^f$.

The match surplus for a new job $s^o_{lj}(z)$ or a continuing job $s_{lj}(z)$ with a low-skilled worker with benefit entitlement j is given by

$$s^o_{lj}(z) \;=\; s_{lj}(z) \;=\; (1-\tau)z + g_l(z) - [b_j + \omega_{lj}], \qquad j = l, h. \tag{2}$$

To compute the match surplus for jobs with high-skilled workers, we must distinguish between new and continuing jobs. The match surplus when forming a new job with an unemployed high-skilled worker, s^o_{hh}, involves outside values without any risk of skill loss if the match does not result in employment:

$$s^o_{hh}(z) \;=\; (1-\tau)z + g_h(z) - [b_h + \omega_{hh}]. \tag{3}$$

In contrast, the match surplus for a continuing job with a high-skilled worker or for a job with an earlier low-skilled worker who gets a skill upgrade that is immediately realized involves quit turbulence:

$$s_{hh}(z) \;=\; (1-\tau)z + g_h(z) - [b_h + \underbrace{(1-\gamma^d)\omega_{hh} + \gamma^d\omega_{lh}}_{\text{quit turbulence}}]. \tag{4}$$

Reservation productivities and rejection rates A worker and a firm split the match surplus through Nash bargaining with outside values as threat points. The splitting of match surpluses ensures mutual agreement whether to start (continue) a job. For a new (continuing) match, the reservation productivity $\underline{z}^o_{ij}$ $(\underline{z}_{ij})$ is the lowest productivity that makes a match profitable and satisfies

$$s^o_{ij}(\underline{z}^o_{ij}) \;=\; 0 \qquad \Big(s_{ij}(\underline{z}_{ij}) \;=\; -\Omega\Big). \tag{5}$$

Note that in a continuing match the surplus must fall to the negative of the layoff tax before a job is terminated.

Given the reservation productivity $\underline{z}^o_{ij}$ $(\underline{z}_{ij})$, let ν^o_{ij} (ν_{ij}) denote the rejection probability, which is given by the probability mass assigned to all draws from productivity distribution $v^o_i(y)$ $(v_i(y))$ that fall below the threshold:

$$\nu^o_{ij} \;=\; \int_{-\infty}^{\underline{z}^o_{ij}} dv^o_i(y) \qquad \left(\nu_{ij} \;=\; \int_{-\infty}^{\underline{z}_{ij}} dv_i(y)\right). \tag{6}$$

To simplify formulas below, we define

$$E_{ij} \equiv \int_{\underline{z}_{ij}}^{\infty} [(1-\tau)y + g_i(y)]\ dv_i(y). \tag{7}$$

2.3 Joint continuation values

Consider a match between a firm and a worker with skill level i. Given a current productivity z, $g_i(z)$ is the joint continuation value of the associated match. We now characterize value functions for low- and high-skilled workers.

High-skilled worker The joint continuation value of a match of a firm with a high-skilled worker with current productivity z, denoted $g_h(z)$, is affected by future layoff turbulence if the worker is laid off or by future quit turbulence if a productivity switch is rejected:

$$\begin{array}{lrcl}
\text{Exogenous separation:} & g_h(z) & = & \beta\Big[\rho^x(b_h + \underbrace{(1-\gamma^{d,x})\omega_{hh} + \gamma^{d,x}\omega_{lh}}_{\text{layoff turbulence}}) \\
\text{Productivity switch:} & & + & (1-\rho^x)\gamma^s(E_{hh} + \nu_{hh}(b_h + \underbrace{(1-\gamma^d)\omega_{hh} + \gamma^d\omega_{lh}}_{\text{quit turbulence}})) \\
\text{No changes:} & & + & (1-\rho^x)(1-\gamma^s)((1-\tau)z + g_h(z))\Big].
\end{array} \tag{8}$$

Low-skilled worker The joint continuation value of a match of a firm with a low-skilled worker takes into account the following contingencies: no changes in productivity or skills, an exogenous separation, a productivity switch, and a skill upgrade. When a skill upgrade occurs, a worker immediately become entitled to high unemployment benefits, even if the worker quits. Furthermore, a skill upgrade coincides with a new draw from the high-skill productivity distribution v_h. Thus, the joint continuation value of a match between a firm and a low-skilled worker with current productivity z is

$$\begin{array}{lrcl}
\text{Exogenous separation:} & g_l(z) & = & \beta\Big[\rho^x(b_l + \omega_{ll}) \\
\text{Immediate skill upgrade:} & & + & (1-\rho^x)\gamma^u(E_{hh} + \nu_{hh}(b_h + \underbrace{(1-\gamma^d)\omega_{hh} + \gamma^d\omega_{lh}}_{\text{quit turbulence}})) \\
\text{Productivity switch:} & & + & (1-\rho^x)(1-\gamma^u)\gamma^s(E_{ll} + \nu_{ll}(b_l + \omega_{ll})) \\
\text{No changes:} & & + & (1-\rho^x)(1-\gamma^u)(1-\gamma^s)((1-\tau)z + g_l(z))\Big].
\end{array} \tag{9}$$

2.4 Outside values

Value of unemployment An unemployed worker with current skill level i and benefit entitlement j receives benefits b_j and has a future value ω^w_{ij}. Recall that the probability that an unemployed worker becomes matched next period is $\lambda^w(\theta)$.

A low-skilled unemployed worker with benefit entitlement j obtains $b_j + \omega^w_{lj}$, where

$$\omega^w_{lj} \;=\; \beta\Big[\underbrace{\lambda^w(\theta)\int_{\underline{z}^o_{lj}}^{\infty} \pi s^o_{lj}(y)\; dv^o_l(y)}_{\text{match + accept}} \quad + \underbrace{b_j + \omega^w_{lj}}_{\text{outside value}}\Big] \qquad j = l, h. \tag{10}$$

A high-skilled unemployed worker with benefit entitlement h, obtains $b_h + \omega^w_{hh}$, where

$$\omega^w_{hh} \;=\; \beta\Big[\underbrace{\lambda^w(\theta)\int_{\underline{z}^o_{hh}}^{\infty} \pi s^o_{hh}(y)\; dv^o_h(y)}_{\text{match + accept}} \quad + \underbrace{b_h + \omega^w_{hh}}_{\text{outside value}}\Big]. \tag{11}$$

Value of a vacancy A firm that searches for a worker pays an upfront cost μ to enter the vacancy pool and thereby obtains a fraction $(1-\pi)$ of the match surplus if an employment relationship is formed next period. Let $\lambda^f_{ij}(\theta)$ be the probability of filling the vacancy with an unemployed worker of type (i, j). Then a firm's value ω^f of entering the vacancy pool is:

$$\omega^f \;=\; -\mu + \beta\Big[\underbrace{\sum_{(i,j)} \lambda^f_{ij}(\theta)\int_{\underline{z}^o_{ij}}^{\infty} (1-\pi) s^o_{ij}(y)\; dv^o_i(y)}_{\text{match + accept}} \quad + \underbrace{\omega^f}_{\text{outside value}}\Big]. \tag{12}$$

2.5 Market tightness and matching probabilities

Let u_{ij} be the number of unemployed workers with current skill i and benefit entitlement j. The total number of unemployed workers is $u = \sum_{i,j} u_{ij}$. The probability $\lambda^w(\theta)$ that an unemployed worker encounters a vacancy is function only of market tightness θ; the probability $\lambda^f_{ij}(\theta)$ that a vacancy encounters an unemployed worker with skill level i and benefit entitlement j also depends on the worker composition in the unemployment pool. Free entry of firms implies that a firm's expected value of posting a vacancy is zero. Equilibrium market tightness can be deduced from equation (12) with $w^f = 0$. We summarize these labor market outcomes as follows:

$$\omega^f \;=\; 0 \tag{13}$$

$$\mu \;=\; \beta(1-\pi)\sum_{(i,j)} \lambda^f_{ij}(\theta)\int_{\underline{z}^o_{ij}}^{\infty} s^o_{ij}(y)\; dv^o_i(y) \tag{14}$$

$$\lambda^w(\theta) \;=\; m(\theta) \tag{15}$$

$$\lambda^f_{ij}(\theta) \;=\; \frac{m(\theta)}{\theta}\frac{u_{ij}}{u}. \tag{16}$$

2.6 Wages

When computing wages, we assume standard Nash bargaining between a worker and a firm each getting their shares of the match surplus in every period.[6] Given a productivity draw z in a new

[6] An implication of the Nash bargaining assumption is that workers pay part of the layoff tax upon a job separation. An alternative assumption is that once a worker is hired, firms are the only ones liable for the layoff tax. This generates a

match with a positive match surplus, the wage $p^o_{lj}(z)$ of a low-skilled worker with benefit entitlement $j = l, h$ and the wage $p^o_{hh}(z)$ of a high-skilled worker, respectively, solve the following maximization problems:

$$\begin{aligned}
&\max_{p^o_{lj}(z)} \quad \left[(1-\tau)z - p^o_{lj}(z) + g^f_l(z) - \omega^f\right]^{1-\pi} \left[p^o_{lj}(z) + g^w_l(z) - b_j - \omega^w_{lj}\right]^{\pi} \\
&\max_{p^o_{hh}(z)} \quad \left[(1-\tau)z - p^o_{hh}(z) + g^f_h(z) - \omega^f\right]^{1-\pi} \left[p^o_{hh}(z) + g^w_h(z) - b_h - \omega^w_{hh}\right]^{\pi},
\end{aligned} \tag{17}$$

where $g^w_i(z)$ and $g^f_i(z)$ are future values obtained by the worker and the firm, respectively, from continuing the employment relationship;[7] and ω^f and $b_j + \omega^w_{ij}$ are outside values defined in (10), (11), and (12). The solution to the wage determination problems sets the sum of the worker's wage and continuation value equal to the worker's share π of the match surplus plus her outside value:

$$\begin{aligned}
p^o_{lj}(z) + g^w_l(z) &= \pi s^o_{lj}(z) + b_j + \omega^w_{lj} \qquad\qquad j = l, h \\
p^o_{hh}(z) + g^w_h(z) &= \pi s^o_{hh}(z) + b_h + \omega^w_{hh},
\end{aligned} \tag{18}$$

where the worker continuation values are

$$\begin{aligned}
g^w_l(z) &= \beta(1-\rho^x)\pi \left\{(1-\gamma^u)\left[(1-\gamma^s)s_{ll}(z) + \gamma^s \int_{\underline{z}_{ll}}^{\infty} s_{ll}(y)\; dv_l(y)\right] + \gamma^u \int_{\underline{z}_{hh}}^{\infty} s_{hh}(y)\; dv_h(y)\right\} \\
&+ \beta(\rho^x + (1-\rho^x)(1-\gamma^u))\left(b_l + \omega^w_{ll}\right) + \beta(1-\rho^x)\gamma^u\left(b_h + (1-\gamma^d)\omega^w_{hh} + \gamma^d\omega^w_{lh}\right) \\
g^w_h(z) &= \beta(1-\rho^x)\pi\left[(1-\gamma^s)s_{hh}(z) + \gamma^s\int_{\underline{z}_{hh}}^{\infty} s_{hh}(y)\; dv_h(y)\right] \\
&+ \beta\rho^x\left(b_h + (1-\gamma^{d,x})\omega^w_{hh} + \gamma^{d,x}\omega^w_{lh}\right) + \beta(1-\rho^x)\left(b_h + (1-\gamma^d)\omega^w_{hh} + \gamma^d\omega^w_{lh}\right).
\end{aligned} \tag{19}$$

For ongoing employment relationships, the wages $p_{ll}(z), p_{hh}(z)$ satisfy counterparts of the above equations that use the corresponding match surpluses $s_{ll}(z)$ and $s_{hh}(z)$:

$$\begin{aligned}
p_{ll}(z) + g^w_l(z) &= \pi s_{ll}(z) + b_l + \omega^w_{ll} \\
p_{hh}(z) + g^w_h(z) &= \pi s_{hh}(z) + b_h + \underbrace{(1-\gamma^d)\omega^w_{hh} + \gamma^d\omega^w_{lh}}_{\text{quit turbulence}},
\end{aligned} \tag{20}$$

where the latter expression for the high-skilled wage now involves quit turbulence on the right side.

two-tier wage system à la Mortensen and Pissarides (1999). Risk neutral firms and workers would be indifferent between adhering to period-by-period Nash bargaining or a two-tier wage system. As demonstrated by Ljungqvist (2002), the wage profile, not the allocation, is affected by the two-tier wage system. Match surpluses, reservation productivities, and market tightness remain the same. Under the two-tier wage system, an initial wage concession by a newly hired worker is equivalent to her posting a bond that equals her share of a future layoff tax.

[7] *Joint* continuation values defined in (8) and (9) equal sums of the individual continuation values: $g_i(z) = g^w_i(z) + g^f_i(z)$, $i = l, h$.

2.7 Government budget constraint

Unemployment benefits Benefit entitlement j awards an unemployed worker benefit b_j equal to a fraction ϕ of the average wage $\bar{p}_j$ of employed workers with skill level j. Therefore, total government expenditure on unemployment benefits amounts to

$$b_l u_{ll} + b_h(u_{lh} + u_{hh}) = \phi(\bar{p}_l u_{ll} + \bar{p}_h(u_{lh} + u_{hh})). \tag{21}$$

Layoff taxes Let Ξ be total separations excluding retirements, which are equal to

$$\Xi = (1-\rho^r)\Big[\rho^x(e_{ll} + e_{hh}) + (1-\rho^x)[(1-\gamma^u)\gamma^s \nu_{ll} + \gamma^u \nu_{hh}]e_{ll} + (1-\rho^x)\gamma^s \nu_{hh} e_{hh}\Big]. \tag{22}$$

Then government revenue from layoff taxation equals $\Omega\,\Xi$.

Income taxes Output is taxed at a constant rate τ. Let $\bar{z}_i$ be the average productivity of employed workers with skill level i. Hence, total tax revenue equals $\tau(\bar{z}_l e_{ll} + \bar{z}_h e_{hh})$, where e_{ll} (e_{hh}) is the number of employed workers with low skills and low benefit entitlement (high skills and high benefit entitlement).

Balanced budget The government runs a balanced budget. The tax rate τ on output is set to cover the expenditures on unemployment benefits described in (21) net of layoff tax revenues $\Omega\,\Xi$:

$$\phi(\bar{p}_l u_{ll} + \bar{p}_h(u_{lh} + u_{hh})) - \Omega\,\Xi = \tau(\bar{z}_l e_{ll} + \bar{z}_h e_{hh}). \tag{23}$$

For computations of average wages $\bar{p}_i$ and average productivities $\bar{z}_i$, see Appendix B.2.

2.8 Worker flows

Workers move across employment and unemployment states, skill levels, and benefit entitlement levels. Here we focus on a group of workers at the center of our analysis: low-skilled unemployed with high benefits. (Appendix B.1 describes flows for other groups of workers.)

Inflows to the low-skilled unemployed with high benefits u_{lh} occur in the following situations. Layoff turbulence affects high-skilled workers e_{hh} who get laid off; with probability $\gamma^{d,x}$, they become part of the low-skilled unemployed with high benefit entitlement. Quit turbulence affects high-skilled workers e_{hh} who reject productivity switches, as well as low-skilled workers e_{ll} who get skill upgrades and then reject their new productivity draws. All of those quitters face probability γ^d of becoming part of the low-skilled unemployed with high benefit entitlement. Outflows from unemployment occur upon successful matching function encounters and retirements. Thus, the net change of low-skilled unemployed with high benefits (equalling zero in a steady state) becomes:

$$\begin{aligned}\Delta u_{lh} \;=\; (1-\rho^r)\Bigg\{ & \underbrace{\rho^x \gamma^{d,x} e_{hh}}_{\text{1. layoff turbulence}} \;+\; \underbrace{(1-\rho^x)\gamma^d \nu_{hh}[\gamma^s e_{hh} + \gamma^u e_{ll}]}_{\text{2. quit turbulence}} \\ & - \underbrace{\lambda^w(\theta)(1-\nu^o_{lh})u_{lh}}_{\text{3. successful matches}} \Bigg\} - \rho^r u_{lh}. \end{aligned} \tag{24}$$

Terms numbered 1 and 3 in expression (24) isolate the forces behind the positive turbulence-unemployment relationship in a welfare state in the LS model. Although more layoff turbulence in term 1 – a higher probability $\gamma^{d,x}$ of losing skills after layoffs – has a small effect on equilibrium unemployment in a laissez-faire economy, it gives rise to a strong turbulence-unemployment relationship in a welfare state that offers a generous unemployment benefit replacement rate on a worker's earnings in her last employment. After a layoff with skill loss, those benefits are high relative to a worker's earnings prospects at her now diminished skill level. As a consequence, the acceptance rate $(1-\nu^o_{lh})$ in term 3 is low; because of the relatively high outside value of a low-skilled unemployed with high benefits, fewer matches have positive match surpluses, as reflected in a high reservation productivity z^o_{lh}. Moreover, given those suppressed match surpluses, equilibrium market tightness θ falls to restore firm profitability enough to make vacancy creation break even. Lower market tightness, in turn, reduces the probability $\lambda^w(\theta)$ that a worker encounters a vacancy, which further suppresses successful matches and contributes to the positive turbulence-unemployment relationship.

The assumption of quit turbulence adds the term numbered 2 in expression (24) that exerts a countervailing force against the positive turbulence-unemployment relationship described above. When higher turbulence is associated with voluntary quits that are also subject to risks of skill loss, there will be a lower incidence of voluntary quits in turbulent times because the risk of skill loss makes high-skilled workers more reluctant to quit. This makes the rejection rate ν_{hh} in term 2 become low in turbulent times. That lower rejection rate causes lower inflows into low-skilled unemployed with high benefits u_{lh} as well as into high-skilled unemployed with high benefits u_{hh}. This force might reverse the positive turbulence-unemployment relationship.

2.9 Steady state equilibrium

A steady state equilibrium consists of measures of unemployed u_{ij} and employed e_{ij}; a labor market tightness θ, probabilities $\lambda^w(\theta)$ that workers encounter vacancies and $\lambda^f_{ij}(\theta)$ that vacancies encounter workers; reservation productivities $\underline{z}^o_{ij}, \underline{z}_{ij}$, match surpluses $s^o_{ij}(z), s_{ij}(z)$, future values of an unemployed worker ω^w_{ij} and of a firm posting a vacancy ω^f; wages $p^o_{ij}(z), p_{ij}(z)$; unemployment benefits b_i and a tax rate τ; such that

a) Match surplus conditions (5) determine reservation productivities.

b) Free entry of firms implies zero-profit condition (14) in vacancy creation that pins down market tightness.

c) Nash bargaining outcomes (18) and (20) set wages.

d) The tax rate balances the government's budget (23).

e) Net worker flows, such as expression (24), are all equal to zero: $\Delta u_{ij} = \Delta e_{ij} = 0, \quad \forall\, i, j$.

2.10 Parameterization

Apart from considering alternative assumptions about the productivity process and different values of the layoff tax, the benchmark model shares the remaining parameterization with LS, in conjunction with DHHR's codification of quit turbulence, as reported in Table 1.[8] The model period is half a quarter.

Preference parameters Given a semi-quarterly model period, we specify a discount factor $\hat{\beta} = 0.99425$ and a retirement probability $\rho^r = 0.0031$, which together imply an adjusted discount of $\beta = \hat{\beta}(1 - \rho^r) = 0.991$. The retirement probability implies an average time of 40 years in the labor force.

Stochastic processes for productivity Exogenous layoffs occur with probability $\rho^x = 0.005$, on average a layoff every 25 years. We set a probability of upgrading skills $\gamma^u = 0.0125$ so that it takes on average 10 years to move from low to high skill, conditional on no job loss. The probability of a productivity switch on the job equals $\gamma^s = 0.05$, so a worker expects to retain her productivity for 2.5 years.

Layoff and quit turbulence Following DHHR, we parameterize quit turbulence as a fraction ϵ of layoff turbulence, and we vary it from zero – only layoff turbulence – to one – the two types of turbulence are equal: $\gamma^d = \epsilon \gamma^{d,x}$.

Labor market institutions We set a worker's bargaining power to be $\pi = 0.5$. We set the replacement rate in unemployment compensation at $\phi = 0.7$ and the layoff tax at $\Omega = 0$ (where the latter is to be perturbed in our investigation of returns to labor mobility).

Matching We assume a Cobb-Douglas matching function $M(v, u) = A u^{\alpha} v^{1-\alpha}$, which implies that the probability of a worker encountering a vacancy and the probability of a vacancy encountering a particular worker type, respectively, are:

$$\lambda^w(\theta) = A\theta^{1-\alpha}, \qquad \lambda^f_{ij}(\theta) = A\theta^{-\alpha}\frac{u_{ij}}{u}. \tag{25}$$

The elasticity of matches with respect to unemployment is specified to be $\alpha = 0.5$ in accordance with a consensus about plausible values falling in the mid range of the unit interval (e.g., see the survey

[8] Subject to the caveat of DHHR assuming a fixed population of firms of the same measure as that of workers and hence, an exogenous market tightness equal to 1, the remaining parameterization in Table 1 is identical or similar to that of DHHR. For a detailed account, see Appendix A.

Table 1: Parameterization of benchmark model

Parameter	Definition	Value
Preferences		
$\hat{\beta}$	discount factor	0.99425
ρ^r	retirement probability	0.0031
$\beta = \hat{\beta}(1-\rho^r)$	adjusted discount	0.991
Sources of risk		
ρ^x	exogenous breakup probability	0.005
γ^u	skill upgrade probability	0.0125
γ^s	productivity switch probability	0.05
$\gamma^{d,x}$	layoff turbulence	$[0,1]$
$\gamma^d = \epsilon\gamma^{d,x}$	quit turbulence	$\epsilon \in [0,1]$
Labor market institutions		
π	worker bargaining power	0.5
ϕ	replacement rate	0.7
Ω	layoff tax	0
Matching function		
A	matching efficiency	0.45
α	elasticity of matches w.r.t. u	0.5
μ	cost of posting a vacancy	0.5

of Petrongolo and Pissarides (2001)). We adopt LS's parameterization of the matching efficiency $A = 0.45$ and the cost of posting a vacancy $\mu = 0.5$.

3 High (LS) versus low (DHHR) returns to labor mobility

As examples of productivity distributions that imply high versus low returns to labor mobility, we use the parameterizations of LS and DHHR as reported in the first two columns of Table 2 and depicted in Figure 1. Both LS and DHHR assume productivity distributions that are the same for new and ongoing matches, $v_i^o(z) = v_i(z)$. LS parameterize truncated normal distributions in Figure 1a whereas DHHR in Figure 1b assume uniform distributions with narrow ranges.[9] The implied returns to labor mobility manifest in how equilibrium unemployment responds to layoff taxes and quit turbulence, respectively. We start by conducting those analyses in the original models of LS and DHHR, and then map each productivity process into our benchmark model under the assumption of uniform distributions. The latter mappings enable us to project the findings from LS and DHHR onto an

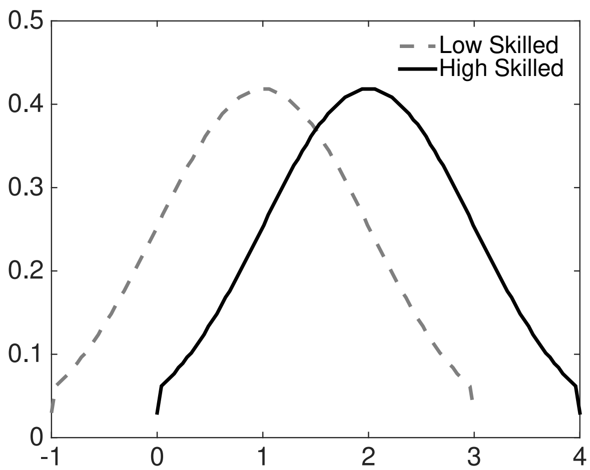
[9]LS incorrectly implemented the quadrature method at the truncation points of the normal distributions; nevertheless, the constructed distributions are still proper. Therefore, instead of recalibrating the LS model under a correct implementation of the quadrature method, we have chosen for reasons of comparability to retain the distributions presented in the published LS analysis.

entire space of productivity processes in Section 4, and thereby highlight the generality of a cross-phenomenon restriction with respect to the unemployment effects of layoff taxes on the one hand, and quit turbulence on the other hand.

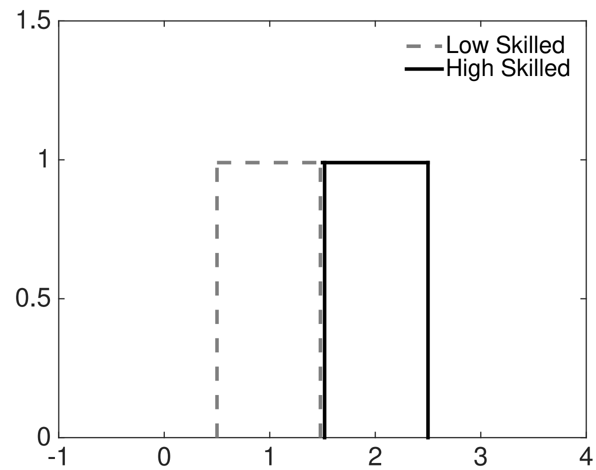
Table 2: Productivity distributions of LS and DHHR

Properties	**Original model**		**Benchmark model version**	
	LS	DHHR	LS	DHHR
Functional form, $v_i(z)$	Normal	Uniform	Uniform	Uniform
Mean, low-skilled	1	1	1	1
high-skilled	2	2	2	2
Width of support	4	1	2.25	0.6
Standard deviation	1	0.289	0.650	0.173

Figure 1: Productivity distributions of LS and DHHR



(a) LS (Normal with wide support)



(b) DHHR (Uniform with narrow support)

The original model of LS is obtained by simply importing the LS productivity distributions into our benchmark model. What we refer to as the DHHR model is their original framework except for two modifications that do not alter outcomes substantially, although they do facilitate our subsequent way of mapping DHHR into our benchmark model.[10] Regarding our analyses of the unemployment effects of layoff taxes, we de facto replicate computations by LS while the analysis in the DHHR model is new and reveals low returns to labor mobility. Regarding our analyses of the unemployment effects of quit turbulence, we mimic and expand on DHHR's finding that small levels of quit turbulence reverse the Ljungqvist-Sargent unemployment-increasing interactions between turbulence and welfare

[10] Our first modification is that instead of the zero benefits that they receive in the original DHHR setup, we assume that newborn workers are eligible for the same unemployment benefits as low-skilled workers. The second modification concerns the risk of losing skills following unsuccessful job market encounters. As a "simplifying assumption," DHHR assume that after an encounter between a firm and an unemployed worker that does not result in an employment relationship, the worker faces the same risk of losing skills as she would after quitting a job; an added risk that we omit. For an assessment of these alternative assumptions, see Appendix D.

state generosity, while showing that not to be the case in the LS model with its higher returns to labor mobility.

Next, we map the models of LS and DHHR into our benchmark model under the assumption of uniform distributions. In the case of LS, it is a question of only converting LS's truncated normal distributions into uniform distributions. For DHHR, things are more complicated because their matching framework differs from our benchmark model in two substantive but, for our purposes, inconsequential ways.[11] Hence, it turns out that the mapping of DHHR into our benchmark model also comes down to only a conversion of DHHR's productivity distributions. Specifically, we calibrate the width of support for the uniform distributions in the benchmark model to generate unemployment effects of quit turbulence similar to those in our analyses of the LS and DHHR model, respectively. As predicated by a cross-phenomenon restriction, the unemployment effects of layoff taxes for each calibration of the benchmark model will then also be in concordance with outcomes in our corresponding analyses of the LS and DHHR model, respectively.

3.1 Layoff taxes

Layoff taxes in LS In the LS model without turbulence, Figure 2 shows unemployment and rejection rates by type of worker, as well as aggregate labor flows, as functions of the layoff tax Ω. The layoff tax is expressed as a fraction of the average yearly output per worker in the laissez-faire economy.[12] The unemployment rate falls as the layoff tax increases. Employed workers, both high- and low-skilled, are especially affected by the layoff tax as their rejection rates fall significantly. Nevertheless, these workers remain mobile even with rather large layoff taxes. For example, if the layoff tax reaches the average annual output of a worker, employed high-skilled workers reject about 12% of offers.

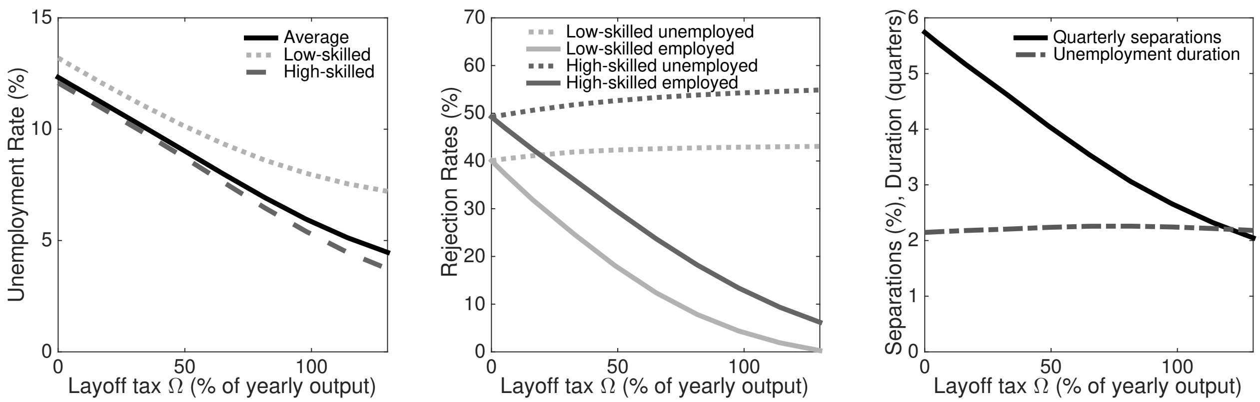
Incidentally, Figure 2 illustrates LS's explanation for a welfare state's having lower unemployment than a laissez-faire economy in tranquil times (i.e., before the onset of economic turbulence). In a matching model, countervailing forces emanating from unemployment benefits and layoff taxes can explain why the unemployment rate in a welfare state need not be high (also see Mortensen and Pissarides (1999)). Despite generous unemployment benefits with a replacement rate of $\phi = 0.7$, layoff taxes at the right end of the first panel in Figure 2 cause unemployment to fall below the laissez-fare rate of 5%.

For later use, we note that endogenous separations in the LS model shut down completely when the layoff tax reaches 184% of the average yearly output per worker. This can be discovered by extrapolating the dark solid curve in the middle panel of Figure 2; evidently, high-skilled employees are most resilient before eventually stopping to quit. The corresponding minimum layoff tax required

[11] As detailed in Appendix A, these structural differences pertain to i) how vacancies are created, and ii) how the capital gain from a skill upgrade is split between firm and worker. To show that among these two differences and the parameterization of productivity distributions it is the latter one that is the sole important source for how unemployment responds to quit turbulence, we proceed as follows. Appendix C starts with the benchmark model with LS productivity distributions in Figure 4a below and successively perturbs the three potential sources one by one, to see which one brings us closest to outcomes in the DHHR model in Figure 4b. In Appendix D, we start from the DHHR model in Figure 4b and work through the perturbations in reverse. Both procedures detect productivity distributions as being the critical source for differences in outcomes.

[12] In the LS laissez-faire economy, a worker's average semi-quarterly output is 2.3 goods in tranquil zero-turbulence times.

Figure 2: Layoff taxes in LS



to close down all endogenous separations in the laissez-faire economy with no unemployment insurance is 163%. Without unemployment compensation, the gains from quitting and searching for another job are smaller so that it requires a smaller layoff tax to shut down endogenous separations in the laissez-faire economy.

Layoff taxes in DHHR We introduce a layoff tax Ω in the DHHR model.[13] Figure 3 shows how a higher layoff tax affects equilibrium outcomes in the DHHR model without turbulence. Mobility of high-skilled employed completely shuts down at a layoff tax equivalent to 14% of the average annual output per worker in the laissez-faire economy.[14] Above this low level of layoff taxes, the rejection rate of these workers becomes zero and separation rates become constant at exogenous job termination rates. Imposing a small layoff tax eradicates the value of labor mobility. Note that for both employed and unemployed low-skilled workers, the rejection rate is zero for the DHHR parameterization at all levels of the layoff tax.

It is noteworthy that there are no endogenous separations at all in the corresponding laissez-faire economy of DHHR. So endogenous separations occur in our DHHR model only because they are encouraged by a generous replacement rate of $\phi = 0.7$.

3.2 Quit turbulence

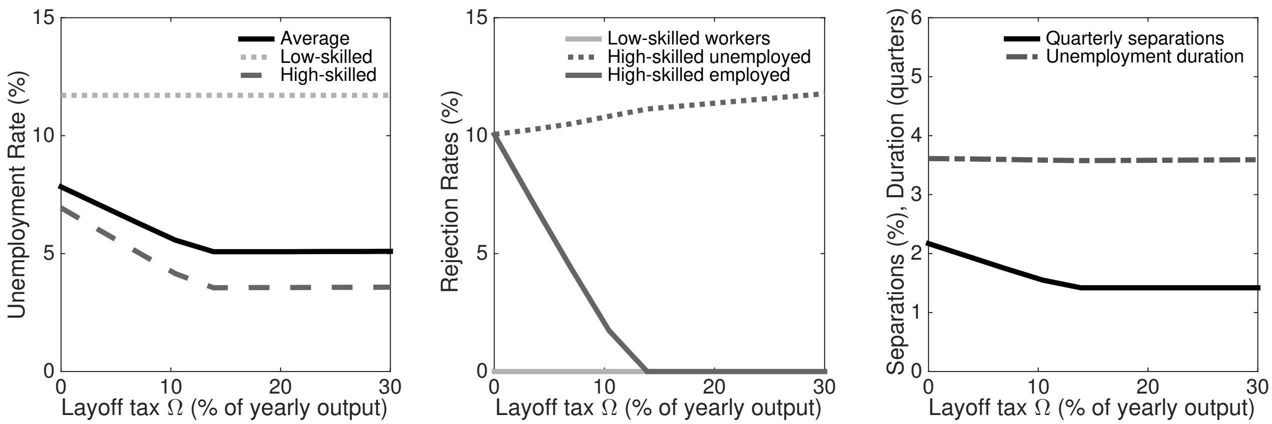
How should a model represent the uncontroversial observation that different job separators find themselves in different situations? For example, workers with valuable skills who separate in order to find better-paying jobs differ from laid-off workers whose skills are no longer in demand, e.g., due to changing technologies or their types of work moving abroad to low-wage countries.

To capture such differences, the benchmark model treats involuntary separations as earlier theories did by assuming that they lead to the most unfavorable circumstances for job separators in the sense

[13] Besides our two simplifying modifications of the original DHHR framework in footnote 10, we here assume that skill upgrades are realized immediately in the DHHR model as in the LS framework. Appendix D.2 documents a small impact on equilibrium outcomes in the DHHR model of such a change in assumptions.

[14] In the DHHR laissez-faire economy, a worker's average quarterly output is 1.8 goods in tranquil zero-turbulence times.

Figure 3: Layoff taxes in DHHR



that they present the highest risks of skill losses. In addition to such layoff turbulence, following DHHR, the benchmark model introduces quit turbulence for workers who voluntarily separate from jobs after draws of poor job-specific productivities at their current employment. Workers who voluntarily separate are ones with more favorable situations both in terms of having an opportunity to continue working after shocks to productivity at their current employment, as well as, conditional on separating, facing a lower risk of skill loss than do workers who suffer involuntary separations.

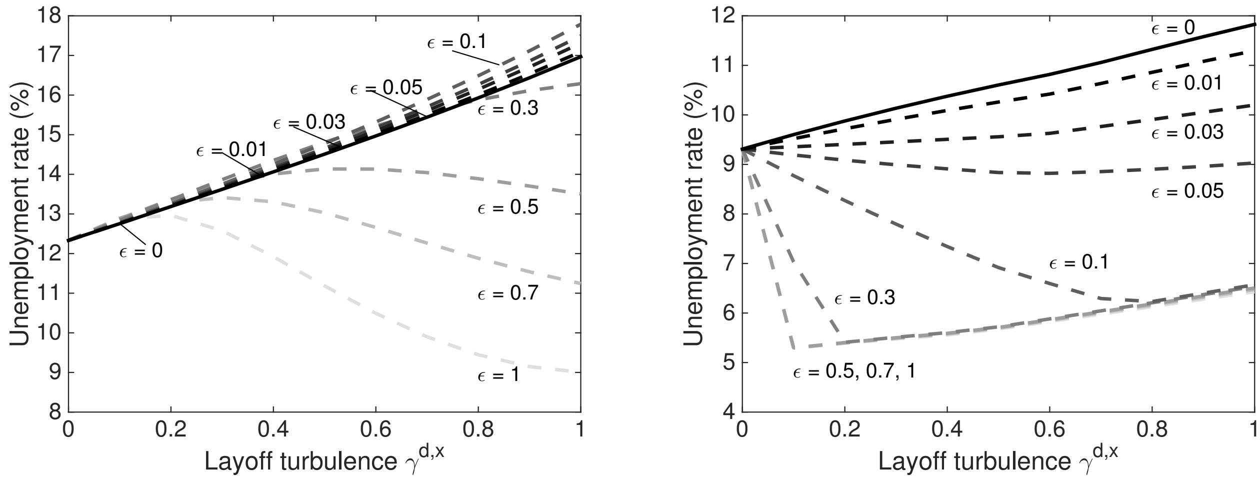
Within this setup and following DHHR, we can investigate how robust the turbulence theory's imputation of high and persistent European unemployment to interactions between microeconomic turbulence and Europe's more generous welfare states is to introducing quit turbulence. We can accomplish this by measuring how much the risk of skill loss at times of voluntary separations must be relative to the risk at times of involuntary separations in order to generate a negative rather than a positive turbulence-unemployment relationship. Because contending forces push for and against the turbulence theory, this is a quantitative issue, as described in Section 2.8.

When the productivity distributions of the benchmark model are assumed to be those of LS, the unemployment outcomes as a function of turbulence are depicted in Figure 4a. The x-axis shows layoff turbulence $\gamma^{d,x}$ and the y-axis the unemployment rate in percent. Each line has its own quit turbulence γ^d represented as a fraction ϵ of layoff turbulence, i.e., $\gamma^d = \epsilon\gamma^{d,x}$ where $\epsilon \in \{0, 0.01, 0.03, 0.05, 0.1, 0.3, 0.5, 0.7, 1\}$. In Figure 4a, we observe that quit turbulence needs to be high, about 50% of layoff turbulence, before the aggregate unemployment rate starts varying negatively with turbulence, and then only for relatively high levels of layoff turbulence.

In contrast, DHHR assert a lack of robustness of the turbulence theory because they find that the turbulence-unemployment relationship already becomes negative at very low skill loss probabilities for voluntary separators relative to those for involuntary separators:

> "... allowing for a skill loss probability following [voluntary] separation that is only 3% of the probability following [involuntary] separation eliminates the positive turbulence-unemployment relationship. Increasing this proportion to 5% gives rise to a strong *negative* relationship between turbulence and unemployment." (DHHR, p. 1362)

Figure 4: Quit turbulence in LS and DHHR



(a) LS model

(b) DHHR model

Layoff turbulence $\gamma^{d,x}$ on the x-axis. Each line represents a different quit turbulence γ^d as a fraction ϵ of layoff turbulence, i.e., $\gamma^d = \epsilon\gamma^{d,x}$. Panel a shows the benchmark model with LS productivity distributions, i.e., the LS model with no layoff tax. Panel b is the DHHR model with our two simplifying modifications in footnote 10.

Subject to our two modifications of the original DHHR model in footnote 10, we reproduce DHHR's findings in Figure 4b. Evidently, DHHR's assertion remains essentially intact – it just requires a somewhat bigger quit turbulence to generate DHHR's key findings of a negative turbulence-unemployment relationship. For example, as quoted above for the original DHHR model, the relationship becomes markedly negative at 5% of quit turbulence ($\epsilon = 0.05$), while subject to our modifications, quit turbulence needs to be 7% ($\epsilon = 0.07$).

The forces at work are as follows. Productivity draws on the job bring incentives for workers to change employers in search of higher productivities. The small dispersion of productivities under DHHR's uniform distributions with narrow support in Figure 1b make returns to labor mobility be very low. As can be seen in Figure 4b, those low returns do not compensate for even small amounts of quit turbulence and hence the initially positive turbulence-unemployment relationship at zero quit turbulence ($\epsilon = 0$) turns negative at relatively small levels of quit turbulence. In particular, high-skilled workers choose to remain on the job and accept productivities at the lower end of the support of the productivity distribution rather than quit and have to face even small probabilities of skill loss.

We also observe in Figure 4b that DHHR's negative turbulence-unemployment relationship can eventually turn positive, as starkly illustrated by a quit turbulence of $\epsilon = 0.3$ and higher. Those high levels of quit turbulence are initially characterized by a steep negative relationship that comes to an abrupt end, then a kink that is succeeded by a gentler upward-sloping turbulence-unemployment relationship. At kinks, all endogenous separations have shut down. The source of unemployment suppression – reductions in quits – has evaporated. What leads to a positive turbulence-unemployment relationship is that higher turbulence generates more low-skilled unemployed who are entitled to high benefits. For two reasons, these workers must draw relatively high productivities in order to want to join employment relationships. First, compared to low-skilled workers who are entitled to low benefits, such workers are reluctant to give up their high benefits: a stronger bargaining position comes with their high benefits. Second, a bargained wage must not only be high enough to induce those workers to surrender their high benefits; it also must be low enough to induce firms to fill vacancies. As described in footnote 8, DHHR assume a given measure of firms, and each idle firm can be thought of as being endowed with a vacancy. Hence, the opportunity cost for a firm in the above kind of encounter is the option value of waiting to fill the vacancy later because it anticipates the prospect of meeting either a low-skilled unemployed worker who has less bargaining power (i.e., one who is entitled only to low benefits) or a high-skilled unemployed worker. For these two reasons, productivities drawn for low-skilled unemployed workers with high benefits have to be relatively high in order for there to exist a bargained wage that is mutually beneficial for a worker and a firm. The resulting lower hazard rate of escaping unemployment for low-skilled workers with high benefits means that unemployment has to increase with turbulence after all endogenous separations have shut down.

3.3 Benchmark model versions of LS and DHHR

What can explain the dramatically different implications of quit turbulence in the two models analyzed in Figure 4? The answer from our investigation, as described in footnote 11, is that these differences are driven by assumptions about the productivity process. Indeed, by simply switching from the LS to DHHR productivity distributions in the benchmark model, outcomes in Figure 4a morph into those of Figure 5: the positive turbulence-unemployment relationship is weakened so much that we get DHHR-like outcomes. To arrive at what we call the benchmark model version of DHHR, we shrink the width of support of the uniform productivity distributions from DHHR's original value of 1 to 0.6. The result is Figure 6b where the responses of unemployment to layoff and quit turbulence are strikingly similar to those of the DHHR model in Figure 4b. The similarity occurs despite our having preserved the other two structural differences between the models in Figures 6b and 4b, as detailed in footnote 11.

Analogously, we seek a benchmark model version of LS, under the assumption of uniform productivity distributions. As one would expect, given the high returns to labor mobility in the LS model, the mapping requires a fairly large width of support equal to 2.25 for the uniform distributions. The resulting Figure 6a provides a good fit in terms of generating unemployment responses to turbulence similar to those of the LS model in Figure 4a. While we target only implications of turbulence to calibrate the benchmark model versions of LS and DHHR, respectively, a cross-phenomenon restriction should ensure that the corresponding implications of layoff taxes also hold up in the mappings, as will be born out in the next section.

4 Cross-phenomenon restriction

To bring out the generality of the cross-phenomenon restriction that emerged in our investigation of LS and DHHR, we now characterize the unemployment effects of layoff costs and quit turbulence, respectively, in an entire space of productivity processes. This confirms that the power of layoff costs to

Figure 5: Benchmark model with DHHR productivity distributions

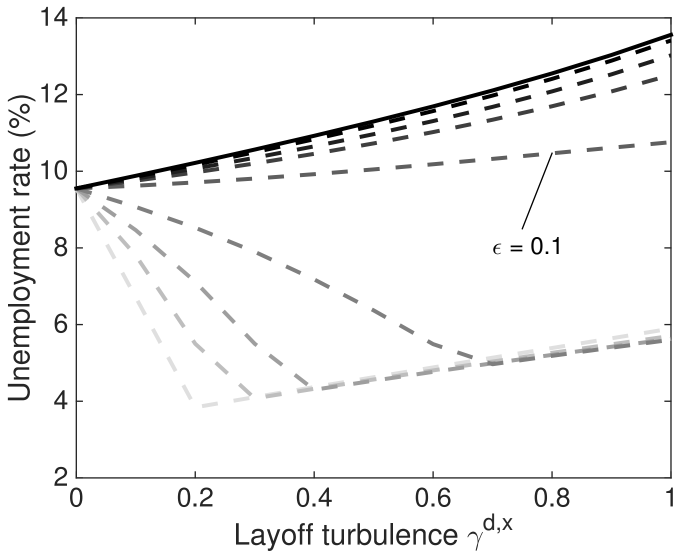
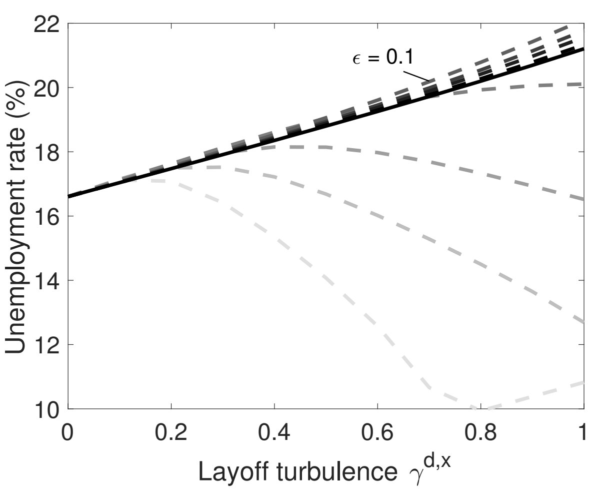
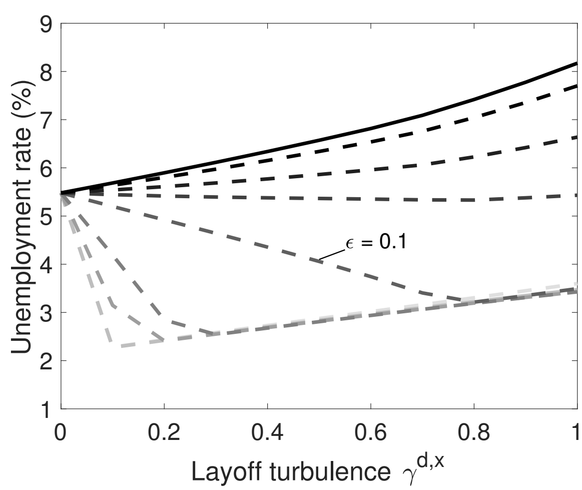


Figure 6: Benchmark model versions of LS and DHHR



(a) Benchmark version of LS



(b) Benchmark version of DHHR

reduce unemployment is correlated with how effectively quit turbulence reverses a positive turbulence-unemployment relationship. In both cases, that potency is greater the smaller are the returns to labor mobility as implied by the productivity process. Hence, outcome statistics for layoff costs and quit turbulence, respectively, are positively correlated throughout the space of productivity processes.

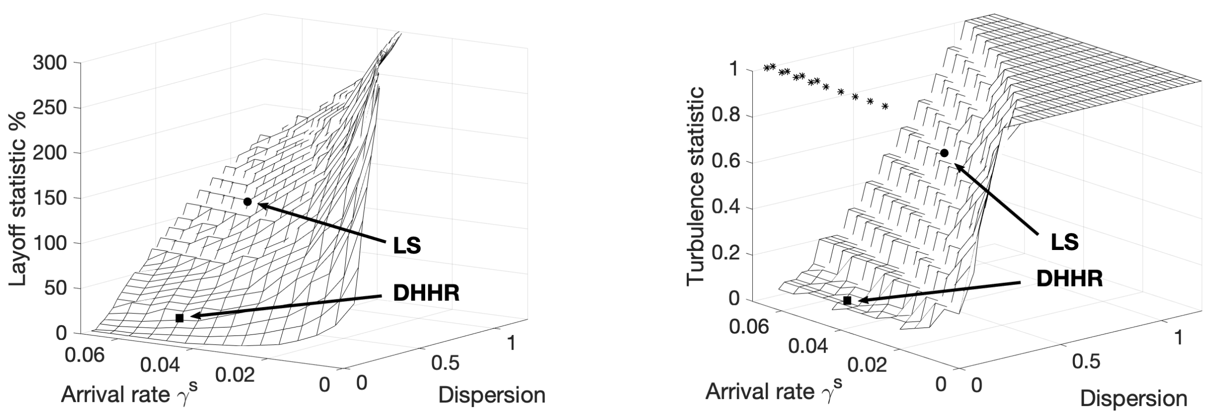
Using the benchmark model we generate this characterization in the space of productivity processes delineated by uniform distributions and indexed by the dispersion (standard deviation) as well as the arrival rate γ^s of productivity shocks in continuing matches. We choose the outcome statistic for layoff costs to be the minimum layoff cost for which all voluntary separations shut down in tranquil times ($\gamma^{d,x} = 0$). The layoff cost is expressed as a proportion of the annual output per worker in the corresponding laissez-faire economy. Conditional on a magnitude of layoff turbulence $\gamma^{d,x}$, we choose the outcome statistic for quit turbulence to be the minimum stance of quit turbulence that makes the turbulence-unemployment relationship negative. Quit turbulence is measured relative to the magnitude of layoff turbulence, namely, the fraction $\epsilon \in [0, 1]$. So, conditional on a value of $\gamma^{d,x}$, the quit turbulence statistic is the minimum value of ϵ that yields a negative turbulence-unemployment relationship, i.e., the unemployment rate falls with an incremental increase in layoff turbulence at the conditioned value of $\gamma^{d,x}$. (When the turbulence statistic equals the maximum value of 1, it is either a knife-edged case of an interior solution when the minimum value of ϵ that yields a negative turbulence-unemployment relationship occurs at 1 or, more likely, a corner solution in which there exists no $\epsilon \in [0, 1]$ that can overturn the positive turbulence-unemployment relationship.)

The two outcome statistics are shown in Figure 7.[15] Regarding the layoff cost statistic in Figure 7a, the minimum layoff tax necessary for shutting down voluntary separations is increasing in dispersion and decreasing in the arrival rate. A higher dispersion coincides with higher returns to labor mobility so that a higher layoff cost is required to shut down voluntary separations. A higher arrival rate of productivity shocks in continuing matches implies a lower expected duration of a productivity draw which suppresses returns to labor mobility for two reasons. First, a relatively low productivity draw becomes less costly to bear when it is expected to persist for a shorter period of time. Second, the prospective gain from quitting and finding another match with higher productivity becomes less attractive when the new productivity draw is expected to last for a shorter period of time. These two reasons explain why the minimum layoff tax needed to shut down voluntary separations decreases in the arrival rate. In the far right corner of Figure 7a with high dispersion and very small arrival rates, the layoff cost statistic explodes when the graph is extended. What is happening here is that the supports of the uniform productivity distributions in Figure 1b reach ever deeper into negative territory; combined with a low arrival rate, any poor productivity draw is expected to last for a long time. Consequently, firms are willing to incur very high layoff costs to terminate exceptionally poor productivity draws.[16]

[15] All outcome statistic figures are drawn for dispersion greater than 0.0722 (a support of 0.25). By omitting zero dispersion, we stay clear of economies that trivially have no endogenous separations. In such degenerate economies, the layoff cost statistic is zero and all turbulence statistics are equal to 1 since, in the absence of quits, there is no force that could reverse the positive turbulence-unemployment relationship.

[16] As a point of reference, the axis for dispersion ends at 1.2 in the outcome statistic figures, which implies a width of just above 4 for the support of the uniform distributions. Thus, at a dispersion of 1.2, the combined productivity distributions for low- and high-skilled workers cover the entire range of the x-axis in Figure 1b.

Figure 7: Cross-phenomenon restriction



(a) Layoff cost statistic

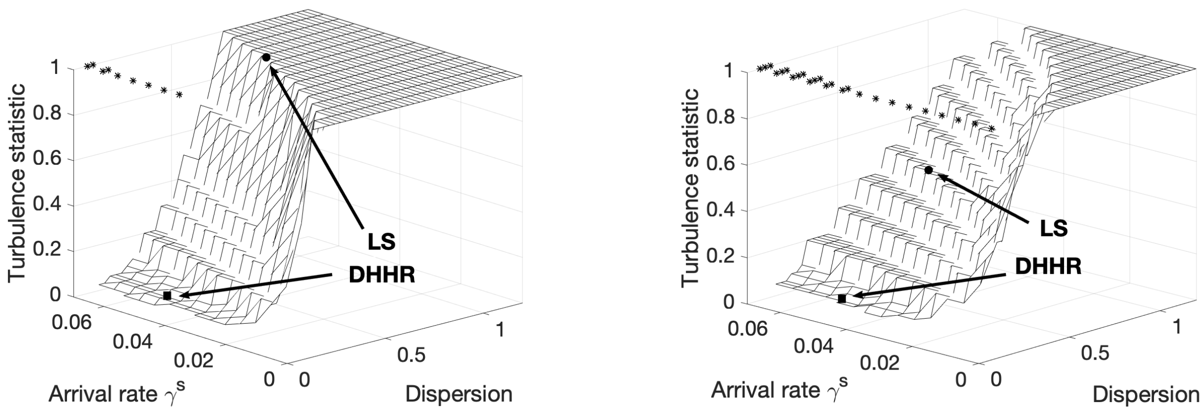
(b) Quit turbulence statistic, $\gamma^{d,x} = 0.3$

The quit turbulence statistic in Figure 7b, conditional on $\gamma^{d,x} = 0.3$, shows outcomes that clearly are correlated with those of the layoff cost statistic in Figure 7a. The reason for the similarity is that both outcome statistics are driven by the returns to labor mobility implied by the productivity process. It is this interrelatedness of the effects of layoff costs and quit turbulence that we call a cross-phenomenon restriction. A difference between the two panels in Figure 7 is that the quit turbulence statistic plateaus at a maximum value of 1 when rates of return to labor mobility are so high that there exists no amount of quit turbulence that can reverse the positive turbulence-unemployment relationship. For a different reason, there are stars at the level of $\epsilon = 1$ for very low values of dispersion in Figure 7b. Here, for a given arrival rate, sufficiently small dispersion implies rates of return to labor mobility so low that even in the absence of quit turbulence there are no voluntary separations. Thus, without any voluntary separations from the outset, there is nothing to be shut down by introducing quit turbulence and hence no force is present to reverse a positive turbulence-unemployment relationship.

The dependence of the quit turbulence statistic on the level of layoff turbulence $\gamma^{d,x}$ is conveyed in Figure 8. A lower layoff turbulence $\gamma^{d,x} = 0.1$ in Figure 8a implies a steeper slope that hastens the ascent to the plateau where there is no amount of quit turbulence that can reverse the positive turbulence-unemployment relationship, while a higher layoff turbulence $\gamma^{d,x} = 0.5$ in Figure 8b slows down the ascent. At very low dispersions, the two panels show corresponding decreases and increases in the numbers of stars, respectively.

Figures 7 and 8 include two points denoted LS and DHHR, respectively, that are benchmark model versions of those frameworks in the present space of productivity processes that were generated above in Figure 6. For both frameworks, the arrival rate is $\gamma^s = 0.05$ as reported in Table 1, while the dispersion was chosen to target turbulence-unemployment outcomes in each respective framework. Recall that the width of support for the uniform distributions in the benchmark model version of DHHR is 0.6 and hence the dispersion (standard deviation) is equal to $\sqrt{0.6^2/12} = 0.173$; the corresponding numbers

Figure 8: Quit turbulence statistic, $\gamma^{d,x} = 0.1$ versus $\gamma^{d,x} = 0.5$



(a) $\gamma^{d,x} = 0.1$

(b) $\gamma^{d,x} = 0.5$

for the benchmark model version of LS are a width of support of 2.25 and hence a dispersion equal to 0.650. In accordance with Figure 6b, the quit turbulence statistic for DHHR is very low at around 0.05 for all three values of $\gamma^{d,x}$ in Figures 7b, 8a and 8b, respectively. Likewise, the outcomes for LS are ones to be inferred from Figure 6a; specifically, the quit turbulence statistic equals 0.58 at layoff turbulence $\gamma^{d,x} = 0.3$, 1 at the lower value of $\gamma^{d,x} = 0.1$, and 0.45 at the higher value of $\gamma^{d,x} = 0.5$. These all do a good job of representing quit turbulence outcomes in Figure 4 that we set out to explain.[17]

The cross-phenomenon restriction portrayed in Figures 7 and 8 is useful when evaluating the potential for the injection of quit turbulence to undermine the turbulence-theoretic explanation of trans-Atlantic unemployment experiences. Starting with the DHHR analysis, its location in the space of productivity processes confirms our earlier assessment of a most extreme parameterization. Not only does DHHR's productivity process conflict with observations on layoff costs and unemployment, it rests perilously downstream on the border of a parameter region with no voluntary separations (marked by stars). Hence, a small parameter perturbation could paradoxically turn DHHR's feeble positive turbulence-unemployment relationship into an ironclad one. Moving upstream to the other side of DHHR's parameterization would quickly raise the quit turbulence statistic before reaching a parameter region consistent with observations on layoff costs and unemployment; considering higher values of layoff turbulence $\gamma^{d,x}$ provides little relief. Also, any candidate pair ($\gamma^{d,x}$, ϵ) for a negative turbulence-unemployment relationship, would have to be evaluated in terms of its *absolute* level of quit

[17] For the record, the layoff cost statistics in Figure 7a for the benchmark model versions of DHHR and LS are 23% and 129%, respectively, while the corresponding numbers are 14% and 186% in our layoff cost analyses in Section 3.1. The different numbers for the DHHR framework are due to the structural differences between the benchmark model version and the DHHR model described in footnote 11. In the case of LS, the difference is solely driven by the uniform productivity distributions in the benchmark model version of LS versus LS's own assumption of truncated normal distributions. Not surprisingly, it takes a higher layoff cost to shut down voluntary separations under the latter distributions with longer tails that include worse productivities than the narrower support of the uniform distributions. For our present argument, these differences are immaterial.

turbulence, $\gamma^d = \epsilon\gamma^{d,x}$. Based on Figures 7 and 8, we conclude that in the present space of productivity processes, the amount of quit turbulence required to reverse the positive turbulence-unemployment relationship is implausibly high. In contrast, the LS analysis falls within a mainstream parameter region in terms of its implied returns to labor mobility, as illustrated in the following analyses.

Another application We can harvest further insights by revisiting two celebrated macro-labor studies of layoff taxes. The first study is the Mortensen and Pissarides (1999) matching model that calibrates productivity processes to unemployment statistics and outcomes in an unemployment insurance system. The second is a search-island model of Alvarez and Veracierto (2001) that enlists establishment data on firm and worker turnover to calibrate firm size dynamics. Baley *et al.* (2022) show how high returns to labor mobility are required to accompany empirically plausible unemployment responses to variations in layoff costs in both of those frameworks and in addition how those high returns to labor mobility also sustain a positive turbulence-unemployment relationship when quit turbulence is present.[18] Thus, the cross-phenomenon restriction that prevails within the benchmark model of section 2 seems to extend beyond the theoretical framework adopted in this paper.

5 Concluding remarks

The fact that returns to labor mobility are essential contributors to several important aggregate outcomes brings informative cross-phenomenon restrictions that can guide calibrations of productivity processes. Exploiting such restrictions adheres to advice offered by Lucas (1980, pp. 696-697):

> "... we are interested in models because we believe they may help us to understand matters about which we are currently ignorant, we need to test them as useful imitations of reality by subjecting them to shocks for which we are fairly certain how actual economies, or parts of economies, would react. The more dimensions on which the model mimics the answers actual economies give to simple questions, the more we trust its answers to harder questions."

In our context, Lucas's relatively "simple question" is about cross-country observations on how layoff costs have seemed to suppress labor reallocation, while the "harder question" about which much less is known concerns unemployment effects of quit turbulence. We hope that our findings will promote further studies of the role that returns to labor mobility play in macro-labor models.

In light of our findings about consequences of recalibrating quit turbulence in order to respect a "Weinberg constraint," we rejoin the conversation with Alan Greenspan with which DHHR began their

[18]Baley *et al.* (2022) also demonstrate that for parameterizations calibrated to fit firm size dynamics, even when parameters are perturbed, high returns to labor mobility prevail in models like Alvarez and Veracierto's (2001) in which shocks to productivity are intermediated through neo-classical production functions. But other macro-labor models that rely solely on unemployment statistics to calibrate per-worker productivity processes can have returns to labor mobility that are fragile with respect to perturbations of parameters that still fit unemployment outcomes. Baley *et al.* (2022) show that this is the case for Mortensen and Pissarides's (1999) calibration. Baley *et al.* conjecture that, because they focused on employment effects of layoff taxes, equilibrium outcomes probably would have prompted Mortensen and Pissarides to explore more of their parameter space if their calibration had wandered into the region with extremely low returns to mobility.

paper. In the passage that DHHR cite, reproduced in Section 1 above, Greenspan does indeed seem to be concerned with the DHHR's quit turbulence force as well as its role in reducing job mobility that comes with DHHR's calibration. But Greenspan refrained from emphasizing such possible effects of increased turbulence more broadly. Instead, earlier in the same paragraph cited by DHHR, Greenspan (1998, p. 743) said that it was not lower but higher labor mobility (i.e., "churning") that was mainly on his mind:

> ". . . the perception of increased churning of our workforce in the 1990s has understandably increased the sense of accelerated job-skill obsolescence among a significant segment of our workforce, especially among those most closely wedded to older technologies. The pressures are reflected in a major increase in on-the-job training and a dramatic expansion of college enrollment, especially at community colleges. As a result, the average age of full-time college students has risen dramatically in recent years as large numbers of experienced workers return to school for skill upgrading."

We read Greenspan as writing here about US workers who had suffered the type of adverse human capital destruction shock that Ljungqvist and Sargent (1998, 2007, 2008) used to capture increased turbulence. Greenspan points out that such workers have ways of rebuilding their human capital in addition to the ways that are open to them in the Ljungqvist and Sargent models, thereby suggesting interesting ramifications of increased turbulence for other observations not on the table in either the DHHR or the Ljungqvist and Sargent framework. It would be interesting to add such activities to a model environment that had succeeded in explaining trans-Atlantic unemployment experiences, while adhering to Weinberg's rules.

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