The Macroeconomics of Partial Irreversibility*

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Abstract

We investigate the macroeconomic effects of partially irreversible investment arising from a wedge between the buying and selling price of capital. We derive sufficient statistics that characterize the implications of irreversibility for three long-run macroeconomic outcomes—capital allocation, capital valuation, and capital fluctuations. Measuring these statistics with investment microdata, we find that irreversibility distorts the allocation of capital, lowers capital valuation, and increases the persistence of capital fluctuations. Corporate income tax cuts, by lowering the deductibility of capital losses due to the price wedge, effectively increase irreversibility and structurally change long-run capital behavior.

JEL: D30, D80, E20, E30

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1 Introduction

Investing in physical capital is far from frictionless. Building up the capital stock and setting it up for productive purposes entails adjustment costs that reflect a variety of frictions involved in the investment process (Caballero and Engel, 1999), such as searching for suppliers, disrupting production, restructuring plants, or retraining workers. Disinvesting is also costly, as resources are needed to uninstall the capital goods, search for buyers of the used capital, and conduct all necessary transactions. Besides the adjustment costs, the market for physical capital is characterized by a significant *wedge* between the buying and selling prices, which renders investment partially irreversible (Bertola and Caballero, 1994; Abel and Eberly, 1996; Lanteri, 2018).[1]

With a price wedge, firms operate with caution. In times of high productivity, firms do not scale up their capital stock as quickly because they fear an adverse shock that would force them to sell their capital at a lower price; and in times of low productivity, firms prefer to hold onto their capital or only sell modestly to avoid the price penalty. Due to the potential capital losses, investment becomes less responsive to productivity shocks, and capital reallocation slows down. What is the role of price wedges in shaping the allocation of capital? What are their implications for firms' market value and the transmission of aggregate shocks? And finally, are the macroeconomic implications of price wedges different from those of other adjustment frictions?

To answer these questions, we develop a parsimonious investment model with idiosyncratic productivity shocks and two frictions: a fixed adjustment cost and a price wedge. In this setting, (i) we analytically characterize the long-run properties of aggregate capital, including its allocation, valuation, and fluctuations; (ii) we measure the aggregate effects of irreversibility and disentangle them from the effects of fixed adjustment costs using investment microdata; and (iii) we study the effects of corporate tax reforms with the new notion of "after-tax" frictions, which reduces the complex interactions of corporate taxes and investment frictions to the simple rescaling of the frictions.

Our contribution is to demonstrate—theoretically and quantitatively—that partial irreversibility is crucial to understand firm-level investment, aggregate capital behavior, and the effects of corporate tax changes. Concretely, we show that ignoring price wedges and wrongly attributing all lumpy behavior to fixed adjustment costs results in overestimating the value of capital and underestimating the persistence of aggregate fluctuations. Additionally, to the extent that the tax code allows for deductions of capital losses, price wedges change the elasticity of aggregate outcomes to corporate income taxes. These results highlight the importance of measuring, modeling, and understanding partial irreversibility in capital markets, and its macroeconomic implications, to design growth and stabilization policies.

[1] Ramey and Shapiro (2001) measure a price wedge above 70% for the aerospace manufacturing equipment, and new evidence by Kermani and Ma (2020) shows that price wedges are quite sizeable across sectors and types of capital, with an industry-wide price wedge of 65% for plant, property, and equipment. We calculate price wedges as 1 minus the recovery rate, defined as the liquidation value over replacement cost net of depreciation.

Let us explain the key features of an environment with irreversibility and its challenges. To guide the discussion, consider a firm's capital to productivity ratio. In a frictionless world, capital tracks productivity and their ratio is constant across time and firms. With only fixed adjustment costs, this ratio moves with idiosyncratic productivity during periods of inaction, which generates heterogeneity across firms; however, upon adjustment, all firms reset to the same ratio, fully cleansing the history of shocks. The *unique reset point* significantly simplifies aggregation as firms become identical conditional on adjustment. With a price wedge, there are *two reset points*, each corresponding to the decision to upsize the capital stock at the buying price or to downsize it at the selling price. The investment policy now depends not only on the history of idiosyncratic shocks but also on the past reset point; this generates micro-history dependence—a Markovian structure for the sign of consecutive adjustments—which adds a new layer of heterogeneity across firms and significantly complicates aggregation. We solve this challenge by providing unique steady-state cross-sectional statistics, easily computed in the microdata, that fully encode the history dependence generated by irreversibility for each macroeconomic outcome.

The first outcome is capital valuation, identified with the weighted average of firm-level marginal q's—the value of an additional unit of capital relative to its replacement cost (Tobin, 1969; Abel, 1979; Hayashi, 1982; Abel and Eberly, 1996). Aggregate q summarizes an economy's benefits and costs of accumulating capital: the stream of future production, the user cost of capital, and, naturally, the capital losses accrued due to the price wedge. We characterize the effect of irreversibility on q with two insights: first, capital losses of adjusters can be "amortized" across the periods in which they were inactive and transitioning from being buyers to sellers and vice versa; and second, the time spent at a particular state during the transition is proportional to the mass of firms occupying that state. Taken together, these results imply that the economy-wide capital losses are equal to the average local drift of capital losses in the cross-section. Using Chilean microdata, we find that irreversibility reduces q in approximately 5% (from 1.10 to 1.05); but since q is close to unity, investment frictions have a moderate effect on capital valuation and primarily operate by reducing the stock of capital.

The second outcome is capital fluctuations, identified with the cumulative impulse response (CIR) of average capital to an unanticipated small shock to aggregate productivity. Along the transition, we assume that firms follow their steady-state decision rules. The CIR summarizes both the impact and the persistence of the capital response to the aggregate productivity shock. With one reset point, Álvarez, Le Bihan and Lippi (2016) characterized the CIR by keeping track of firms only until their first adjustment, and in Baley and Blanco (2021), we derived two sufficient statistics for the CIR of aggregate capital, namely, the dispersion of capital-productivity ratios and their covariance with the time elapsed since their last adjustment. With two reset points, to our surprise, these sufficient statistics remain valid but with higher values due to irreversibility. Additionally, irreversibility introduces a new term that reflects how aggregate productivity shocks change the mass of adjusters across the two reset points relative to the steady-state. We obtain a CIR of 2.6, which means that a 1% reduction in aggregate productivity generates a total deviation in capital-to-productivity ratios of 2.6% above steady state along the transition path. Our estimates show that irreversibility accounts for at least 25% of the CIR, suggesting that price wedges are an important source of persistence behind aggregate fluctuations.

Our characterization of aggregate outcomes highlights the importance of price wedges. A natural question arises: Can we precisely identify the aggregate effects of price wedges vis-à-vis other investment frictions? In particular, both price wedges and fixed adjustment costs may appear observationally equivalent under the lens of a particular set of data moments, for instance, the rate of inaction and the dispersion of the investment rates; nevertheless, these two frictions have very different implications for aggregate outcomes, as explained above. Addressing the identification challenge is fundamental to correctly measure and assess the implications of investment frictions.

We proceed in two steps, which require investment microdata and a price wedge as inputs. In the first stage, we recover the two reset points from the firm's Euler equations constructed with the first-order conditions and the envelope theorem for arbitrary choice sets (Milgrom and Segal, 2002). In the second stage, with the reset points at hand, we derive mappings from the joint distribution of investment and duration of inaction *conditional on the sign of the last adjustment* to the cross-sectional steady-state moments that characterize capital allocation, valuation, and fluctuations, including the new sufficient statistics that capture the role of irreversibility. The premise behind these mappings is that, conditional on the sign of the last adjustment, the only remaining source of heterogeneity is the history of productivity shocks received during periods of inaction, which can be summarized by the size and the timing of investment. Our estimates suggest that half of the difference between reset points comes from the endogenous response to the exogenous price wedge, which confirms irreversibility's large effects for firm-level investment.

As a concrete application of our framework, we examine the macroeconomic effects of corporate income tax cuts. The novelty of our analysis is a parsimonious modeling of the intricacies of the tax code regarding the deductions of capital losses and the capitalization of adjustments costs. Specifically, because capital losses are deducted, the corporate income tax effectively reduces the price wedge. To our knowledge, this is the first analysis of the macroeconomic consequences of capital loss deductions.

We introduce a corporate tax schedule (Summers, 1981; Abel, 1982) and show analytically that *after-tax investment frictions*—namely, the fixed adjustment cost relative to the after-tax frictionless profits and the price wedge relative to the after-tax frictionless profit-capital ratio—are the key objects affecting dynamic investment decisions. We then examine a regime shift from a high to a low corporate income tax rate. There are two opposing effects. On the one hand, a lower tax rate decreases after-tax fixed adjustment costs. On the other hand, a lower tax rate raises after-tax price wedge because deductions of capital losses fall. The calibrated model suggests that, other things equal, a corporate income tax cut from 42% to 25%—corresponding to the median decrease in OECD countries between 1980 and 2020—improves the allocation of capital, decreases capital valuation (q falls) and makes fluctuations more persistent (CIR rises).

Contributions to the literature. First, we contribute to the irreversibility literature (Bertola and Caballero, 1994; Dixit and Pindyck, 1994; Abel and Eberly, 1996; Ramey and Shapiro, 2001; Veracierto, 2002; Lanteri, 2018) by deriving sufficient statistics that capture the role of irreversibility for aggregate capital's allocation, valuation, and fluctuations. To do this, we extend the sufficient statistics approach in Álvarez, Le Bihan and Lippi (2016) and Baley and Blanco (2021) to environments with multi-reset points. While we focus on investment, our methodology is flexible enough to study the macroeconomic implications of irreversibility more generally in environments in which the marginal benefit (or cost) of taking an action is different from the marginal benefit (or cost) of reversing that action, such as durables, housing, and inventories.

Second, our work directly speaks to the quantitative investment literature. Following Cooper and Haltiwanger (2006), the literature has targeted a certain set of moments of the distribution of investment computed at the plant level, including investment spikes and the autocorrelation of investment rates, to assess the nature of investment frictions and derive conclusions about their role for capital allocation (Asker, Collard-Wexler and De Loecker, 2014) and capital fluctuations (Khan and Thomas, 2008, 2013; Bachmann, Caballero and Engel, 2013; Bachmann and Bayer, 2014; Winberry, 2021). Our analysis shows that, while partially informative, widely used targets do not fully reflect the effects of irreversibility because they do not consider the Markovian structure that arises. We provide researchers with the key moments of the joint distribution of investment and duration of inaction that fully encode the role of irreversibility by conditioning on the sign of the last adjustment. Our approach complements other direct methods designed to assess the magnitude of frictions, such as examining the stationary distribution of marginal products of capital (Hsieh and Klenow, 2009; Restuccia and Rogerson, 2013) or conditional transitions of marginal products (Caballero, Engel and Haltiwanger, 1997; Lanteri, Medina and Tan, 2020).

Lastly, we contribute to the literature by studying the interaction of corporate taxes with investment frictions. Early work focused on the user-cost channel of taxation in frictionless environments. Subsequent work incorporated firm heterogeneity and non-convex adjustment costs to investigate the frictional channel of taxation (Miao, 2019; Gourio and Miao, 2010; Miao and Wang, 2014). We show how to reduce the complex interactions between corporate taxes and investment frictions to a rescaling of the appropriate friction. This idea considerably simplifies the analysis and highlights the channels through which corporate tax reforms affect private investment.

2 Investment with a Fixed Cost and a Price Wedge

In this section, we develop a parsimonious investment model with the following features: idiosyncratic productivity shocks, fixed capital adjustment costs, a positive wedge between the purchase and resale prices of capital, and a constant interest rate.

2.1 A firm's problem

Time is continuous, extends forever, and it is denoted by s. The future is discounted at rate $\rho > 0$. For any stochastic process x_s, we use the notation $x_{s^-} \equiv \lim_{r \uparrow s} x_r$ to denote the limit from the left. We first present the problem of an individual firm and then consider a continuum of ex-ante identical firms to characterize the aggregate behavior of the economy.

Technology and shocks. The firm produces output y_s using capital k_s according to a production function with decreasing returns to scale

$$y_s \;=\; u_s^{1-\alpha} k_s^{\alpha}, \quad \alpha < 1. \tag{1}$$

Flow profits are equal to $\pi_s \equiv A y_s$, where $A > 0$ is a profitability parameter. Idiosyncratic productivity u_s follows a geometric Brownian motion with drift $\mu > 0$ and volatility $\sigma > 0$,

$$\log u_s \;=\; \log u_0 + \mu s + \sigma W_s, \qquad W_s \sim Wiener. \tag{2}$$

The capital stock, if uncontrolled, depreciates at a constant rate $\xi^k > 0$.

Investment frictions. The firm can control its capital stock through buying and selling investment goods. For every active adjustment, $i_s \equiv \Delta k_s = k_s - k_{s^-}$, the firm must pay a fixed cost proportional to its productivity

$$\theta_s = \theta u_s, \tag{3}$$

where $\theta > 0$ is measured in output units. Capital is bought at price $p^{\text{buy}} = p$ and sold at price $p^{\text{sell}} = p(1-\omega)$, where ω is the price wedge. Alternatively, $1-\omega$ is the recovery rate. To simplify notation, we define the price function

$$p\left(i_s\right) \;\equiv\; p^{\text{buy}} \mathbb{1}_{\{i_s > 0\}} + p^{\text{sell}} \mathbb{1}_{\{i_s < 0\}}. \tag{4}$$

Investment problem. Let $V(k,u)$ denote the value of a firm with capital stock k and productivity u. Given initial conditions (k_0, u_0), the firm chooses a sequence of adjustment dates $\{T_h\}_{h=1}^{\infty}$

and investments $\{i_{T_h}\}_{h=1}^{\infty}$, where h counts the number of adjustments, to maximize its expected discounted stream of profits. The sequential problem is

$$V(k_0, u_0) \; = \; \max_{\{T_h, i_{T_h}\}_{h=1}^{\infty}} \mathbb{E}\left[\int_0^{\infty} e^{-\rho s} \pi_s \, \mathrm{d}s - \sum_{h=1}^{\infty} e^{-\rho T_h} \left(\theta_{T_h} + p\left(i_{T_h}\right) i_{T_h}\right)\right], \tag{5}$$

subject to the production technology (1), the idiosyncratic productivity shocks (2), the fixed cost (3), the investment price function (4), and the law of motion for the capital stock

$$\log k_s \; = \; \log k_0 - \xi^k s + \sum_{h: T_h \leq s} \left(1 + i_{T_h} / k_{T_h^-}\right), \tag{6}$$

which describes a period's capital as a function of its initial value k_0, the physical depreciation rate ξ^k, and the sum of all adjustments made at prior adjustment dates.

2.2 Capital-productivity ratios $\hat{k}$

To characterize the investment decision, it is convenient to reduce the state-space and recast the firm problem using a new state variable, the log capital-productivity ratio:

$$\hat{k}_s \; \equiv \; \log\left(k_s / u_s\right). \tag{7}$$

The problem admits this reformulation because the production function is homothetic, the adjustment costs are proportional to productivity, and idiosyncratic shocks follow a Brownian motion with drift.[2] Note that in the absence of investment frictions, $\hat{k}_s$ is a constant. With investment frictions, between any two consecutive adjustment dates $[T_{h-1}, T_h]$, the capital-productivity ratio $\hat{k}$ follows a Brownian motion

$$\mathrm{d}\hat{k}_s \; = \; -\nu \, \mathrm{d}s + \sigma \, \mathrm{d}W_s, \tag{8}$$

where the drift $\nu \equiv \xi^k + \mu$ reflects the depreciation rate and productivity growth rate. At any adjustment date T_h, the log capital-productivity ratio changes by the amount

$$\Delta \hat{k}_{T_h} \; = \; \log\left(1 + i_{T_h} / k_{T_h^-}\right). \tag{9}$$

Using the Principle of Optimality, Lemma 1 rewrites the sequential problem in (5) as a recursive stopping-time problem. It also shows that the value of the firm equals a function of the log capital-productivity ratio $\hat{k}$ that scales with productivity, that is, $V(k, u) = uv(\hat{k})$. Since $\Delta \hat{k}_s$ and

[2] We can also reformulate the problem in terms of capital-productivity ratios assuming that adjustment costs scale with output or with the capital stock.

i_s have the same sign, we write the investment price as $p(\Delta\hat{k})$. All proofs appear in Appendix A.

Lemma 1. *Let $r \equiv \rho - \mu - \sigma^2/2$ be the adjusted discount factor and let $v(\hat{k}) : \mathbb{R} \to \mathbb{R}$ be a function of the log capital-productivity ratio equal to*

$$(10)\quad v(\hat{k}) = \max_{\tau,\Delta\hat{k}} \mathbb{E}\Big[\int_0^\tau A e^{-rs+\alpha\hat{k}_s}\, \mathrm{d}s + e^{-r\tau}\left(-\theta - p(\Delta\hat{k})(e^{\hat{k}_\tau+\Delta\hat{k}} - e^{\hat{k}_\tau}) + v(\hat{k}_\tau + \Delta\hat{k})\right)\Big|\hat{k}_0 = \hat{k}\Big].$$

Then the firm value equals $V(k,u) = uv(\hat{k})$.

2.3 Optimal investment policy

The optimal investment policy is characterized by four numbers, $\mathcal{K} \equiv \{\hat{k}^- \leq \hat{k}^{*-} \leq \hat{k}^{*+} \leq k^+\}$, which correspond to the lower and upper borders of the inaction region

$$(11)\qquad \mathcal{R} = \left\{\hat{k} : \hat{k}^- < \hat{k} < \hat{k}^+\right\},$$

and two reset points $\hat{k}^{*-} < \hat{k}^{*+}$. A firm adjusts if and only if its log capital-productivity ratio falls outside the inaction region, that is, $\hat{k}_s \notin \mathcal{R}$. Conditional on adjusting, the firm purchases capital to bring its state up to $\hat{k}^{*-}$ if it hits the lower border $\hat{k}^-$, and sells capital to bring its state down to $\hat{k}^{*+}$ if it hits the upper border $\hat{k}^+$. Given $\mathcal{R}$, the optimal adjustment dates are

$$(12)\qquad T_h = \inf\left\{s \geq T_{h-1} : \hat{k}_s \notin \mathcal{R}\right\} \quad \text{with} \quad T_0 = 0.$$

The duration of a complete inaction spell τ_h and the time elapsed since the last adjustment a_s (or the age of the capital-productivity ratio) are

$$(13)\qquad \tau_h = T_h - T_{h-1},$$

$$(14)\qquad a_s = s - \max\left\{T_h : T_h \leq s\right\}.$$

To save on notation, we write the reset points and the stopped capitals (an instant before adjustment) as functions of the sign of adjustment:

$$(15)\qquad \hat{k}^*(\Delta\hat{k}) = \begin{cases} \hat{k}^{*-} & \text{if} \quad \Delta\hat{k} > 0 \\ \hat{k}^{*+} & \text{if} \quad \Delta\hat{k} < 0, \end{cases}$$

$$(16)\qquad \hat{k}_\tau(\Delta\hat{k}) = \hat{k}^*(\Delta\hat{k}) - \Delta\hat{k}.$$

Lemma 2 characterizes the value function and the optimal investment policy through the standard sufficient optimality conditions. The firm value and the policy must satisfy: (i) the Hamilton-

Jacobi-Bellman equation in (17), which describes the evolution of the firm's value during periods of inaction, (ii) two value-matching conditions in (18) and (19), which set the value of adjusting equal to the value of not adjusting at the borders of the inaction region, and (iii) two smooth-pasting and optimality conditions in (20) and (21), which ensure the differentiability of the value function at the borders of inaction and the two reset points.

Lemma 2. *The value function $v(\hat{k})$ and the optimal policy $\mathcal{K} \equiv \{\hat{k}^-, \hat{k}^{*-}, \hat{k}^{*+}, \hat{k}^+\}$ satisfy:*

(i) In the inaction region $\mathcal{R}$, $v(\hat{k})$ solves the HJB equation:

$$rv(\hat{k}) = Ae^{\alpha\hat{k}} - \nu v'(\hat{k}) + \frac{\sigma^2}{2}v''(\hat{k}). \tag{17}$$

(ii) At the borders of the inaction region, $v(\hat{k})$ satisfies the value-matching conditions:

$$v(\hat{k}^-) = v(\hat{k}^{*-}) - \theta - p^{buy}(e^{\hat{k}^{*-}} - e^{\hat{k}^-}), \tag{18}$$
$$v(\hat{k}^+) = v(\hat{k}^{*+}) - \theta + p^{sell}(e^{\hat{k}^+} - e^{\hat{k}^{*+}}). \tag{19}$$

(iii) At the borders of the inaction region and the two reset states, $v(\hat{k})$ satisfies the smooth-pasting and the optimality conditions:

$$v'(\hat{k}) = p^{buy}e^{\hat{k}}, \quad \hat{k} \in \left\{\hat{k}^-, \hat{k}^{*-}\right\}, \tag{20}$$
$$v'(\hat{k}) = p^{sell}e^{\hat{k}}, \quad \hat{k} \in \left\{\hat{k}^{*+}, \hat{k}^+\right\}. \tag{21}$$

The optimal policy in terms of capital is recovered as $\{k^-, k^{*-}, k^{*+}, k^+\} = u \times \exp\{\hat{k}^-, \hat{k}^{*-}, k^{*+}, k^+\}$.

2.4 Tobin's marginal q

Next, we express the optimal investment decision using Tobin's marginal q, namely, the shadow value of installed capital. We identify a firm's marginal q as the marginal valuation of an extra unit of installed capital relative to its replacement cost of capital (the purchase price p):[3]

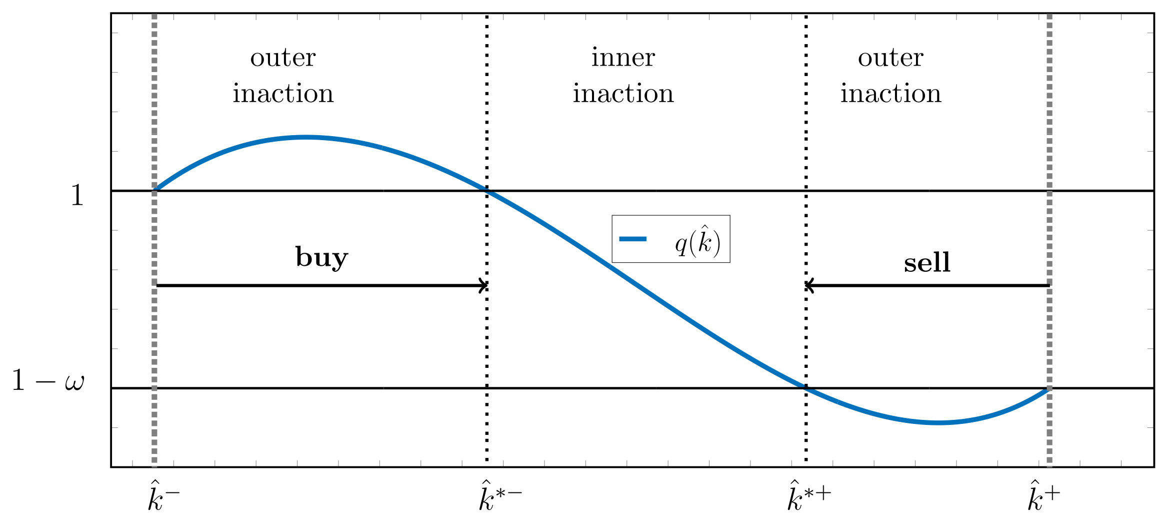
$$q(\hat{k}) \equiv \frac{1}{p}\frac{\partial V(k,u)}{\partial k} = \frac{v'(\hat{k})e^{-\hat{k}}}{p}. \tag{22}$$

Figure I describes the optimal investment policy using $q(\hat{k})$. We use this diagram to describe how each investment friction affects the firm's optimal policy. Let's consider first an environment

[3]Note that we define q relative to the purchase price $p^{buy} = p$. Alternatively, we could define q using the average investment price in the economy. These two formulations are proportional to each other.

with partial irreversibility and no fixed costs. Without the fixed adjustment cost ($\theta = 0$), a firm purchases capital if $q(\hat{k}) \geq 1$ (or $\hat{k} \leq \hat{k}^{*-}$) and sells capital if $q(\hat{k}) \leq 1 - \omega$ (or $\hat{k} \geq \hat{k}^{*+}$) without any delay. When $q(\hat{k})$ lies between the two prices (or the state between the two reset points), it is optimal to remain inactive. At that productivity level, it is too expensive to purchase capital and too cheap to sell it. This gives rise to an "inner" inaction region $[\hat{k}^{*-}, \hat{k}^{*+}]$ due exclusively to partial irreversibility. Next, let's consider an environment with fixed costs and no partial irreversibility. Without a price wedge ($\omega = 0$), the "inner" inaction region collapses to a unique reset point k^*. However, the fixed adjustment cost generates an "outer" inaction region $[\hat{k}^-, \hat{k}^+]$ that prevents firms from adjusting, even if $q(\hat{k})$ lies above or below the investment price. When both frictions are active, the policy features both "outer" and "inner" inaction regions and two reset points.

Figure I – Optimal Investment Policy



Notes: This figures illustrates a firm's marginal $q(\hat{k}) = v'(\hat{k})/pe^{\hat{k}}$ and its investment policy $\mathcal{K} = \{\hat{k}^-, \hat{k}^{*-}, \hat{k}^{*+}, \hat{k}^+\}$.

The interaction of the investment frictions generates two interesting features in the optimal investment behavior. First, as argued by Caballero and Leahy (1996), individual $q(\hat{k})$ is *not monotonic* in $\hat{k}$. Without fixed costs, $q(\hat{k})$ monotonically decreases with $\hat{k}$ due to decreasing returns to scale $\alpha < 1$. With fixed costs, however, firms anticipate large adjustments when approaching the inaction thresholds. As $\hat{k}$ approaches the lower threshold $\hat{k}^-$, firms anticipate that a future tiny change in the state $\mathrm{d}\hat{k} < 0$ will trigger a large positive adjustment $\Delta\hat{k} > 0$. The future positive investment lowers future $q(\hat{k})$ and feeds back into lower current $q(\hat{k})$, bending down the function. A reverse argument explains why $q(\hat{k})$ bends up as $\hat{k}$ approaches the upper threshold $\hat{k}^+$. As a result, individual $q(\hat{k})$ is *not a sufficient statistic for individual investment*, in contrast to the postulate in Tobin (1969).

Second, optimal investment features an *endogenous positive serial correlation in the sign of adjustments*. A firm is more likely to buy capital if it bought capital recently, and it is more likely to sell capital if it sold capital recently. This correlation arises because the inner inaction region generated by the price wedge widens the distance between the two borders of inaction but shortens the distance between each border of inaction and its corresponding reset point. Thus, it is more likely to reach $\hat{k}^-$ from the nearby $\hat{k}^{*-}$ than from the further $\hat{k}^{*+}$. The serial correlation in adjustment sign—which we label micro-history dependence—brings technical challenges for characterizing aggregate outcomes and for identifying the role of irreversibility with data. Later in the paper, we explain how to handle micro-history dependence conditioning behavior on the last reset point.

2.5 Frictionless and frictional investment

Next, Proposition 1 shows that the investment policy can be separated into a static frictionless component and a dynamic frictional component, where we characterize the latter introducing the notion of effective investment frictions. From a firm's perspective, what matters for investment decisions is the fixed adjustment cost relative to frictionless profits and the price wedge relative to the frictionless profits-capital ratio, respectively. To simplify the notation, we recall the definition of the discount r and define the user cost of capital $\mathcal{U}$:

$$r \equiv \rho - \mu - \frac{\sigma^2}{2}, \tag{23}$$

$$\mathcal{U} \equiv \rho + \xi^k - \sigma^2. \tag{24}$$

For the problem to be well-defined, we assume $r > 0$ and $\mathcal{U} > 0$.

Proposition 1. *Let $\mathcal{K} \equiv \{\hat{k}^-, \hat{k}^{*-}, \hat{k}^{*+}, \hat{k}^+\}$ denote the firms' optimal investment policy characterized in Lemma 2. The optimal policy can be decomposed as the sum of a static and a dynamic component $\mathcal{K} = \hat{k}^{ss} + \mathcal{X}$, where $\hat{k}^{ss}$ is the static log capital-productivity ratio $\hat{k}^{ss}$ that firms would set in the absence of frictions and equal to*

$$\hat{k}^{ss} = \frac{1}{1-\alpha} \log\left(\frac{\alpha A}{p\mathcal{U}}\right) \tag{25}$$

and $\mathcal{X} \equiv \{x^-, x^{*-}, x^{*+}, x^+\}$ solves the following stopping problem for the normalized capital-productivity ratio $x \equiv \hat{k} - \hat{k}^{ss}$:

$$\mathcal{V}(x) = \max_{\tau,\Delta x} \mathbb{E}\left[\int_0^\tau e^{-r\tau}\left(e^{\alpha x_s} - \alpha e^{x_s}\right) \mathrm{d}s + e^{\tilde{r}\tau}\left(-\tilde{\theta} - \tilde{\omega}\mathbb{1}_{\{\Delta x<0\}}(e^{x_\tau} - e^{x_\tau+\Delta x}) + \mathcal{V}(x_\tau + \Delta x)\right) \Big| x_0 = x\right], \tag{26}$$

$$\mathrm{d}x_t = -\nu\,\mathrm{d}t + \sigma\,\mathrm{d}W_t. \tag{27}$$

The effective fixed cost $\tilde{\theta}$ and the effective wedge $\tilde{\omega}$ are defined as:

$$\tilde{\theta} \equiv \frac{\theta}{Ae^{\alpha \hat{k}^{ss}}} = \left(\frac{1}{A}\right)^{\frac{1}{1-\alpha}} \left(\frac{p\mathcal{U}}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} \theta \tag{28}$$

$$\tilde{\omega} \equiv \frac{p\omega}{Ae^{(\alpha-1)\hat{k}^{ss}}} = \frac{\alpha}{\mathcal{U}}\omega. \tag{29}$$

Proposition 1 provides several insights. The static optimal policy $\hat{k}^{ss}$ in (25) sets the capital-productivity ratio to a constant, and its value reflects profitability αA, the average user cost of capital $\mathcal{U}$, and the investment price p. By definition, investment frictions do not affect the static choice $\hat{k}^{ss}$. In contrast, the dynamic policy $\mathcal{X}$ characterized by (26) and (27) takes into account the fixed cost and the price wedge, but these frictions enter scaled by static profits or by the profit-capital ratio. Moreover, the flow payoff in the dynamic problem $e^{\alpha x_s} - \alpha e^{x_s}$ only depends on the curvature of the profit function α and thus it is invariant to frictions. Finally, any price can be used to construct k^{ss}, because $\mathcal{X}$ moves accordingly so that $\mathcal{K}$ is invariant to the price.

The dynamic policy $\mathcal{X}$ that solves the stopping time problem above closely resembles the price-setting and investment problems with fixed costs, analyzed first by Barro (1972), Sheshinski and Weiss (1977), Dixit (1991), but with the addition of a price wedge. We will leverage on this literature and our Proposition 1 to characterize analytically the effect of frictions (and their interaction with corporate taxes) on aggregate outcomes in the next sections, extending previous results to include partial irreversibility.

A remark on the fixed adjustment cost. For simplicity, we specify the fixed cost parameter θ as deterministic, symmetric for positive and negative investments, and equal across firms. However, we prove all the results for the generalized hazard model proposed by Caballero and Engel (1999, 2007) and examined in contemporaneous work by Álvarez, Lippi and Oskolkov (2020). This generalized hazard model accommodates asymmetric fixed costs, random fixed costs, as well as time-dependent adjustments that can be motivated by information frictions (Verona, 2014) or search frictions (Kurmann and Petrosky-Nadeau, 2007; Ottonello, 2018). Appendix B presents the generalized hazard model. Also see Baley and Blanco (2021) for ex-ante heterogeneity in firms' production and adjustment technologies.

A remark on the price wedge. Our preferred interpretation of the wedge ω is a gap between the buying and selling prices, which may reflect asymmetric information about capital's quality (Akerlof, 1970; Kurlat, 2013; Bigio, 2015); imperfect substitutability (Lanteri, 2018); obsolescence (Caunedo and Keller, 2020); intermediary fees (Nosal and Rocheteau, 2011); tax credits (Altug, Demers and Demers, 2009); or VAT taxes (Chen *et al.*, 2019). However, to the extend that ω is a linear and asymmetric adjustment cost, it allows for alternative interpretations besides a price gap, such as installation or transaction costs that scale with investment (Cooper and Haltiwanger, 2006; Fang, 2021). Moreover, setting $\omega = 1$ delivers the extreme irreversibility case that eliminates the possibility to disinvest (Sargent, 1980; Veracierto, 2002).

3 Three macroeconomic outcomes

This section investigates how investment frictions shape aggregate capital's allocation, valuation, and fluctuations. We consider an economy populated by a continuum of ex ante identical firms that face the investment problem described before. Idiosyncratic shocks W_s are independent across firms, and as a result, the economy features a stationary cross-sectional joint distribution of capital-productivity ratios $g(\hat{k})$. It solves a KFE that describes the evolution of capital-productivity ratios inside the inaction region, excluding the two reset points (where it has kinks), together with continuity, border and resetting conditions. It is plotted in Panel A of Figure II. Next, we define and characterize the macroeconomic outcomes using steady-state moments of $g(\hat{k})$.[4]

3.1 Capital allocation

We identify capital allocation with the cross-sectional variance of the log marginal revenue product of capital. In our model, all firms produce the same good and the output price is normalized to one. Therefore, we measure instead the variance of marginal products. From the production function (1), the log of the marginal product of capital is collinear to the capital-productivity ratio $\hat{k}$, that is, $\log mpk_s = \log \alpha - (1-\alpha)\hat{k}_s$, implying that

$$\mathbb{V}ar[\log mpk] \;=\; (1-\alpha)^2 \mathbb{V}ar[\hat{k}]. \tag{30}$$

In a frictionless environment, $\hat{k}_s$ is constant and $\mathbb{V}ar[\log mpk] = 0$. With investment frictions, however, dispersion in the marginal product of capital arises. Given the collinear relationship in (30), we use both $\mathbb{V}ar[\hat{k}]$ and $\mathbb{V}ar[\log mpk]$ to refer to the allocation of capital. In what follows, we show that the allocation of capital is a common driver of capital valuation and capital fluctuations.

3.2 Capital valuation

We identify *capital's valuation* with the weighted average of individual marginal $q(\hat{k})$ in (22), with weights equal to $\phi(\hat{k}) \equiv e^{\hat{k}}/\hat{K}$:

$$q \;\equiv\; \int_{\hat{k}^-}^{\hat{k}^+} q(\hat{k})\phi(\hat{k})g(\hat{k})\,\mathrm{d}\hat{k} \;=\; \frac{\mathbb{E}[v'(\hat{k})]}{p\hat{K}}, \tag{31}$$

Without frictions, there is a unique investment price, and optimality implies that $q = 1$ at all times. Any changes in the costs or benefits of investing are immediately passed through to the

[4] According to Proposition 1, we can alternatively define aggregate outcomes in terms of normalized ratios x.

capital stock, eliminating any possibility for q to deviate from unity. With investment frictions, there is an incomplete passthrough from changes in the cost or benefits of investing to the capital stock, and deviations of q from unity reflect the shadow value of eliminating the frictions.

Proposition 2 characterizes aggregate q in terms of steady-state moments of $\hat{k}$. The proof combines the HJB equation for $v'(\hat{k})$, which specifies firms' optimal behavior, with the KFE satisfied by $g(\hat{k})$, which describes the evolution of firms through the cross-sectional distribution, into a single "master equation." Then we integrate to eliminate idiosyncratic noise and recover q.

Proposition 2. *Consider the weights $\phi(\hat{k}) \equiv e^{\hat{k}}/\hat{K}$. Then aggregate q equals:*

$$q = \frac{1}{r}\left(\underbrace{\frac{\alpha A\hat{Y}}{p\hat{K}} + \frac{\sigma^2}{2} - \nu}_{\textit{productivity}} + \underbrace{\mathbb{E}\left[\frac{1}{\mathrm{d}s}\mathbb{E}_s\left[\mathrm{d}(\mathcal{P}(\hat{k}_s)\phi(\hat{k}_s))\right]\right]}_{\textit{irreversibility} < 0}\right), \tag{32}$$

where $\hat{Y}/\hat{K}$ is equal, up to second order, to

$$\frac{\hat{Y}}{\hat{K}} = \frac{\mathbb{E}[e^{\alpha\hat{k}}]}{\mathbb{E}[e^{\hat{k}}]} = \exp\left\{-(1-\alpha)\left(\mathbb{E}[\hat{k}] + \frac{\alpha}{2}\mathbb{V}ar[\hat{k}]\right)\right\} + o(\hat{k}^3), \tag{33}$$

and $\mathcal{P}(\hat{k}) \in \mathbb{C}^2$ is a twice continuously differentiable function that extends capital losses per unit of capital across the entire domain $[\hat{k}^+, \hat{k}^-]$:

$$\mathcal{P}(\hat{k}) \equiv \begin{cases} 0 & \text{if } \hat{k} \in [\hat{k}^-, \hat{k}^{*-}], \\ -\omega & \text{if } \hat{k} \in [\hat{k}^{*+}, \hat{k}^+]. \end{cases} \tag{34}$$

Aggregate q in (32) equals the perpetuity value of three terms. The term $\hat{Y}/\hat{K}$ equals the average output-productivity ratio divided by the average capital-productivity ratio, and it is further characterized in (33) in terms of the average and variance of $\hat{k}$.[5] Because of decreasing returns to scale ($\alpha < 1$), this term decreases with both the average $\mathbb{E}[\hat{k}]$ and the dispersion $\mathbb{V}ar[\hat{k}]$ of capital-productivity ratios. Consequently, aggregate q also decreases with these moments. Both the fixed cost and the price wedge affect q indirectly through this channel.

The term $(\sigma^2/2-\nu)$ reflects the expected change in the average capital-productivity ratio. Since firms can upsize to exploit good outcomes and can downsize to insure against bad outcomes, they are effectively risk loving (Oi, 1961; Hartman, 1972; Abel, 1983). Thus, an increase in idiosyncratic risk σ^2 directly increases q. At the same time, an increase in idiosyncratic risk indirectly affects q by increasing $\mathbb{V}ar[\hat{k}]$ and lowering the aggregate output-capital ratio in (33). The overall effect of risk on q depends on the relative strength of these two opposing forces.

In addition to irreversibility's indirect effect on productivity through $\mathbb{E}[\hat{k}]$ and $\mathbb{V}ar[\hat{k}]$, it has a

[5] Aggregate productivity differs from the average output-capital ratio $\mathbb{E}[y/k] = \mathbb{E}[e^{(\alpha-1)\hat{k}}]$ due to heterogeneity.

direct negative effect on q. This irreversibility term equals the average local drift of the function $\mathcal{P}(\hat{k}_s)\phi(\hat{k}_s)$ in the cross-section of non-adjusters. Intuitively, this average drift reflects the capital losses "amortized" during periods of inaction, and by the optimal sampling theorem and the renewal principle, it is also equal to the capital losses of adjusting firms. Later in the paper, we show how to measure the irreversibility term with microdata.

Individual vs. aggregate q. In Section 2.4 we showed that individual $q(\hat{k})$ is a non-monotonic function of $\hat{k}$. This observation has led some economists to argue that the individual non-monotonicity translates into aggregate non-monotonicity, discarding q as a sufficient statistic for aggregate investment. Expression (32) shows that this argument is flawed. Fixed adjustment costs and partial irreversibility do not break the decreasing relation between aggregate q and aggregate $\hat{K}$. While this result is counterintuitive, it is a natural consequence of aggregating the behavior of individual firms. The anticipatory effects that bend individual $q(\hat{k})$ in the vicinity of the borders of the inaction region disappear when aggregating the cross-section, as positive and negative stances of expected changes in $q(\hat{k})$ cancel each other out in the aggregate. As a result, aggregate q *is a sufficient statistic for aggregate investment.*

3.3 Capital fluctuations

We identify capital fluctuations with the transitional dynamics of aggregate capital following an aggregate productivity shock. Starting from the steady state at date $s = 0$, we introduce a small, permanent, and unanticipated decrease in the (log) level of productivity of size $\delta > 0$ to all firms. All firms' productivity and capital-productivity ratios change to

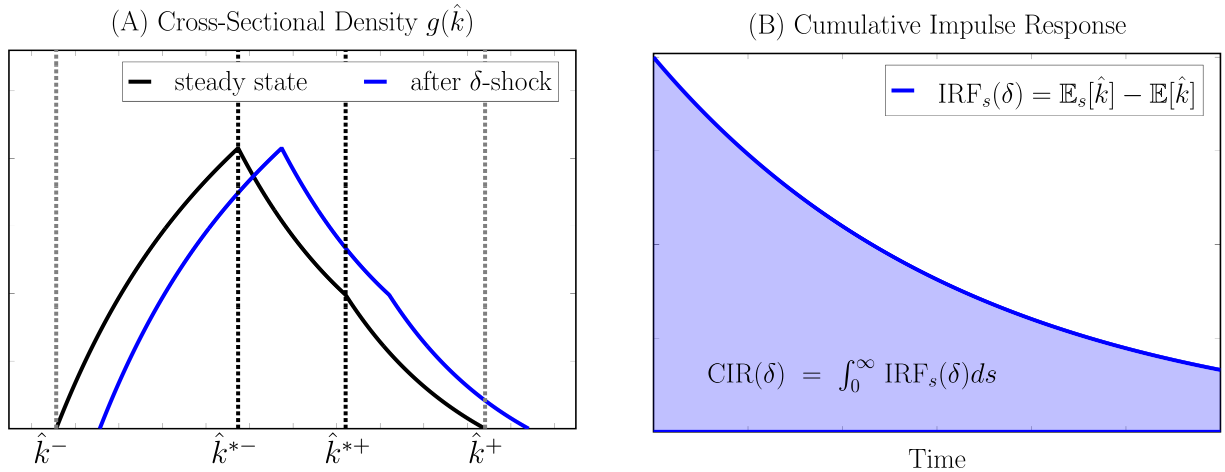
$$\log(u_0) \;=\; \log(u_{0^-}) \;-\; \delta; \qquad \log(\hat{k}_0) \;=\; \log(\hat{k}_{0^-}) \;+\; \delta. \tag{35}$$

Panel A of Figure II plots the initial density following the δ productivity shock (blue line) next to the steady-state density $g(\hat{k})$ (black line). The new distribution displaces horizontally to the right relative to the steady-state distribution. Our exercise consists in tracking the mean $\mathbb{E}_s[\hat{k}]$ as it makes its way back to its steady-state value $\mathbb{E}[\hat{k}]$. By assuming a constant interest rate, investment policies do not respond to changes in the distribution and remain fixed along the transition path. Thus, our analysis measures the strength of the partial equilibrium response to aggregate shock.[6]

We define the impulse-response function, denoted by $\mathrm{IRF}(\delta, s)$, measured s periods after an

[6] While assuming a constant interest rate (and investment policies) along the transition is an extreme assumption, Winberry (2021) shows that the interest rate response to aggregate productivity shocks is small and even countercyclical. Appendix C relaxes this assumption and presents a general equilibrium model that delivers constant prices as an equilibrium outcome.

Figure II – Distribution Dynamics and Cumulative Impulse Response



Notes: The figure illustrates the effects of an aggregate shock. Panel A shows the steady-state distribution $g(\hat{k})$ (blue line) and the initial distribution following a productivity shock (blue line). Panel B shows the $\text{IRF}(\delta, s)$ (solid blue line) and the CIR (area).

aggregate productivity shock of size δ as follows:

$$\text{IRF}(\delta, s) \equiv \mathbb{E}_s[\hat{k}] - \mathbb{E}[\hat{k}], \tag{36}$$

where $\mathbb{E}_s[\cdot]$ denotes expectations with the time-s distribution. We define the cumulative impulse response $\text{CIR}(\delta)$, as the area under the $\text{IRF}_s(\delta)$ function across all dates $s \in (0, \infty)$

$$\text{CIR}(\delta) \equiv \int_0^\infty \text{IRF}_s(\delta) \, \text{d}s. \tag{37}$$

Panel B in Figure II plots these two objects. The solid line is the impulse-response function $\text{IRF}(\delta, s)$, and the area underneath it is the cumulative impulse response function $\text{CIR}(\delta)$. The CIR is a useful metric. It summarizes both the impact and persistence of the response in one scalar and eases the comparison across different models.[7] Without frictions, firms respond instantly to the aggregate shock and the CIR is zero. With frictions, the larger the CIR the longer it takes firms to respond to the aggregate shock and the slower the transitional dynamics.

Proposition 3 characterizes the CIR as a function of cross-sectional moments of $\hat{k}$. To characterize the role of irreversibility, we use an analogous strategy to the one we employed above to characterize aggregate q through an average local drift.

[7] Álvarez, Le Bihan and Lippi (2016), Baley and Blanco (2019), Álvarez, Lippi and Oskolkov (2020), and Alexandrov (2021) use the CIR in the context of price-setting models to assess the effects of monetary shocks.

Proposition 3. *The CIR of the average log capital-productivity ratio $\mathbb{E}[\hat{k}]$ following a marginal aggregate productivity shock of size $\delta > 0$ is equal, up to first order, to*

$$\frac{CIR(\delta)}{\delta} = \underbrace{\frac{\mathbb{V}ar[\hat{k}]}{\sigma^2} + \frac{\nu \mathbb{C}ov[\hat{k}, a]}{\sigma^2}}_{responsiveness} + \underbrace{\mathbb{E}\left[\frac{1}{\mathrm{d}s}\mathbb{E}_s[\mathrm{d}(\mathcal{M}(\hat{k}_s)\hat{k}_s)]\right]}_{irreversibility} + o(\delta), \tag{38}$$

where $\mathcal{M}(\hat{k}) \in \mathbb{C}^2$ is a twice continuously differentiable function in the domain $[\hat{k}^+, \hat{k}^-]$ defined by

$$\mathcal{M}(\hat{k}) \equiv \begin{cases} \mathcal{M}^{buy} & \text{if } \hat{k} \in [\hat{k}^-, \hat{k}^{*-}] \\ \mathcal{M}^{sell} & \text{if } \hat{k} \in [\hat{k}^{*+}, \hat{k}^+]. \end{cases} \tag{39}$$

and $\mathcal{M}^{buy} < 0 < \mathcal{M}^{sell}$ are two numbers that measure the expected cumulative deviation of the capital-productivity ratio relative to the mean $\mathbb{E}[\hat{k}]$, conditional on the sign of the last adjustment.

According to (38), the CIR equals a linear combination of two steady-state moments and an irreversibility term. The moments are the cross-sectional variance of capital-productivity ratios $\mathbb{V}ar[\hat{k}]$ and the covariance of capital-productivity ratios $\hat{k}$ with the time elapsed since the last adjustment $\mathbb{C}ov[\hat{k}, a]$. These steady-state moments are informative about transitional dynamics because aggregate shocks δ and idiosyncratic shocks u enter symmetrically into $\hat{k}$, and as a consequence, how firms respond to idiosyncratic shocks inform how they respond to aggregate shocks. Specifically, the variance $\mathbb{V}ar[\hat{k}]$ reflects insensitivity to idiosyncratic shocks, while the covariance $\mathbb{C}ov[\hat{k}, a]$ reflects asymmetric costs of downsizing vs. upsizing. Baley and Blanco (2021) established the relationship between the CIR and these two steady-state moments in environments with drift, asymmetric fixed costs, and random opportunities of free adjustment, but without partial irreversibility. In all of those environments, the irreversibility term equals zero.

Irreversibility slows down the propagation of aggregate shocks through two channels. First, it has an indirect effect on the CIR by increasing the two cross-sectional moments $\mathbb{V}ar[\hat{k}]$ and $\mathbb{C}ov[\hat{k}, a]$. Second, it has a direct effect that reflects how the aggregate shock δ changes the mass of adjusters across the two reset points relative to the steady state. In principle, because of the correlated adjustments that arise with irreversibility, one should keep track of firms for a long time. Nevertheless, because the first adjustment after the aggregate shock eliminates all heterogeneity except for the sign of the adjustment, we only need to keep track of firms until their first adjustment. In other words, steady-state behavior is restored and two numbers are enough to characterize the CIR: $\mathcal{M}^{buy}$, which measures the expected cumulative deviations below to the steady-state mean conditional on a positive investment, and $\mathcal{M}^{sell}$, which measures the expected cumulative deviations above to the steady-state mean conditional on a negative investment. Given these two numbers, we characterize this irreversibility term with a similar strategy used in q, by "amortizing" the change in masses during periods of inaction (formally, the average local drift of the cumulative deviations from steady-state in the cross-section). Later, we provide expressions for these two numbers and show how to measure them in the microdata.

Identifying and characterizing the irreversibility term in the CIR is one of the key contributions of our analysis, as it opens to door to study transitional dynamics in environments with history dependence, that is, where the first stopping time does not fully absorb the effects of an aggregate shock. This type of problems are labeled as problems with reinjection by Álvarez and Lippi (2021).

4 On the Aggregate Consequences of Investment Frictions

In this section, we explore in depth how the nature of adjustment frictions matters for the macroeconomy. Specifically, we show that the size of the price wedge relative to the fixed cost matters and that the role of the price wedge hinges on the size of the drift (consisting of productivity growth and capital depreciation) relative to the volatility of the idiosyncratic shocks. We consider three cases which showcase how investment frictions map into aggregate outcomes and isolate the mechanisms at play.

In what follows, we exploit the decomposition between frictionless and frictional policies in Proposition 1, work in the space of normalized capital-productivity ratios $x \equiv \hat{k} - \hat{k}^{ss}$, and consider second-order approximations to the profit function.

4.1 Zero drift

First, we consider an environment with zero drift ($\nu = 0$). This case is relevant for economies or sectors with low productivity growth and/or low depreciation rate, in which idiosyncratic shocks are the main drivers of investment. In this driftless and symmetric environment the price wedge constitutes an important friction as firms expect to purchase and sell capital with equal probability. Proposition 4 characterizes two subcases: only fixed cost and only price wedge.[8]

Proposition 4. *Assume $\nu \to 0$. Construct $\hat{k}^{ss}$ using the price $(p^{buy} + p^{sell})/2 = p(1-\omega/2)$ such that the value function is symmetric. The effective price wedge equals $\tilde{\omega} = \omega\alpha/\mathcal{U}$ and the effective fixed cost equals $\tilde{\theta} = \left[p(1-\omega/2)\mathcal{U}/(\alpha A^{1/\alpha})\right]^{\frac{\alpha}{1-\alpha}}\theta$; the user cost of capital is $\mathcal{U} = \rho - \sigma^2$; and the discount factor is $r = \rho - \sigma^2/2$. In all symmetric cases: $\mathbb{E}[\hat{k}] = \hat{k}^{ss}$, $\mathbb{E}[x] = 0$, and $\mathbb{C}ov[x, a] = 0$.*

(i) ***Only fixed cost:*** *If $\omega = 0$, then the inaction thresholds are $\bar{x} = \pm\left(\frac{6\tilde{\theta}\sigma^2}{\alpha(1-\alpha)}\right)^{1/4}$, the reset*

[8] The driftless symmetric case with both frictions active is studied in the Appendix A.9. The main result is that a marginal increase in one friction, when the other is large, has tiny effects on the macro outcomes.

point is $x^* = 0$, and the macro outcomes are:

$$(40) \qquad \mathbb{V}ar[x] = \frac{\overline{x}^2}{6}; \qquad q = 1 - \frac{\mathcal{U}}{r}\frac{\alpha(1-\alpha)}{2}\mathbb{V}ar[x]; \qquad \frac{CIR(\delta)}{\delta} = \frac{\mathbb{V}ar[x]}{\sigma^2}.$$

(ii) ***Only price wedge:*** *If $\theta = 0$, then the inaction thresholds and the reset points coincide at $\overline{x}^* = \pm\left(\frac{3\tilde{\omega}\sigma^2}{4\alpha(1-\alpha)}\right)^{1/3}$, and the macro outcomes are:*

$$(41) \quad \mathbb{V}ar[x] = \frac{\overline{x}^{*2}}{3}; \quad q = 1 - \left(1 + \frac{2}{\alpha}\right)\frac{\mathcal{U}}{r}\frac{\alpha(1-\alpha)}{2}\mathbb{V}ar[x]; \quad \frac{CIR(\delta)}{\delta} = \left(1 + \frac{1}{\sigma^2}\right)\mathbb{V}ar[x].$$

In the two cases above, there is a positive relationship between the corresponding effective investment friction ($\tilde{\theta}$ or $\tilde{\omega}$) and the cross-sectional dispersion $\mathbb{V}ar[x]$. In turn, larger dispersion lowers aggregate productivity $\hat{Y}/\hat{K}$ because of decreasing returns, which reduces q; and larger dispersion reduces responsiveness and slows down the propagation of aggregate productivity shocks, increasing the CIR. Thus we learn that, with a small drift, the cross-sectional dispersion is the main driver behind capital valuation and fluctuations. We also learn that if effective frictions were of the same size, that is, $\tilde{\theta} = \tilde{\omega}$, a price wedge generates a higher $\mathbb{V}ar[x]$, a lower q, and a larger CIR compared to the case with only fixed costs.

4.2 Large drift

Second, we consider a very large drift ($\nu \to \infty$). This case is relevant for economies or sectors in which forces to upsize the capital stock dominate (productivity growth and depreciation) and idiosyncratic shocks play only a minor role for investment. The price wedge is irrelevant here, because firms prefer to downsize by allowing the drift to erode their capital instead of facing the penalty. Proposition 5 establishes this result. Note that instead of taking the drift to infinity, we take an equivalent limit towards zero volatility of idiosyncratic shocks.

Proposition 5. *Let $\nu > 0$ and $\sigma^2 \to 0$ such that $\nu/\sigma^2 \to \infty$. Construct $\hat{k}^{ss}$ using the purchase price p. The effective fixed cost equals $\tilde{\theta} = \left[p\mathcal{U}/(\alpha A^{1/\alpha})\right]^{\frac{\alpha}{1-\alpha}}\theta$ and the effective price wedge is irrelevant. In this case, the user cost is $\mathcal{U} = \rho + \xi^k$ and the discount is $r = \rho - \mu$. The policy is a one-sided inaction region with lower threshold x^- and one reset point x^*. The cross-sectional distribution is Uniform over $[x^-, x^*]$ with moments:*

$$(42) \qquad \mathbb{E}[x] = \frac{(x^* + \overline{x})}{12}; \qquad \mathbb{V}ar[x] = \frac{(x^* - \overline{x})^2}{12}.$$

The policy solves the non-linear system

$$(43) \qquad \mathbb{E}[x]\sqrt{\mathbb{V}ar[x]} \;=\; -\frac{r\tilde{\theta}}{\sqrt{12}\alpha(1-\alpha)}; \qquad \frac{\mathbb{E}[x]}{\mathbb{V}ar[x]+\mathbb{E}[x]^2} \;=\; -\left(\frac{r}{\nu}+\frac{\alpha+1}{2}\right),$$

and the macro outcomes are

$$(44) \qquad q \;=\; 1 \;-\frac{U}{r}(1-\alpha)\left(\mathbb{E}[x]+\frac{\alpha}{2}\mathbb{V}ar[x]\right); \qquad \frac{CIR(\delta)}{\delta} \;=\; 0.$$

The case with a large drift reveals new mechanisms absent in symmetric environments. As we have already mentioned, the price wedge has no effect. Comparing the expression for aggregate q and the CIR with large drift against the driftless cases in Proposition 4, we see that now the average $\mathbb{E}[x]$ matters. Moreover, it is the frictional average $\mathbb{E}[x]$ and not the frictionless average $\mathbb{E}[\hat{k}]$ the relevant statistic for the marginal value of capital. The non-linear system in (43) that pins down the investment policy implies that larger effective fixed costs $\tilde{\theta}$ increase both the average $\mathbb{E}[x]$ (in absolute value) and the variance $\mathbb{V}ar[x]$ of the normalized capital-productivity ratios x. In fact, the first equation in (43) is an indifference curve that mediates the trade-off between these two moments. The system also implies that the average $\mathbb{E}[x]$ is negative, thus $\mathbb{E}[\hat{k}] = \hat{k}^{ss} + \mathbb{E}[x] < \hat{k}^{ss}$. As the mean becomes more negative with higher fixed costs, q goes up; but as the variance increases, q goes down. The overall effect depends on the relative elasticities of these moments with respect to $\tilde{\theta}$. Lastly, the CIR equals zero: with an extreme drift, aggregate shocks are immediately absorbed and there are no deviations from steady-state (see Corollary 2 in Baley and Blanco, 2021).

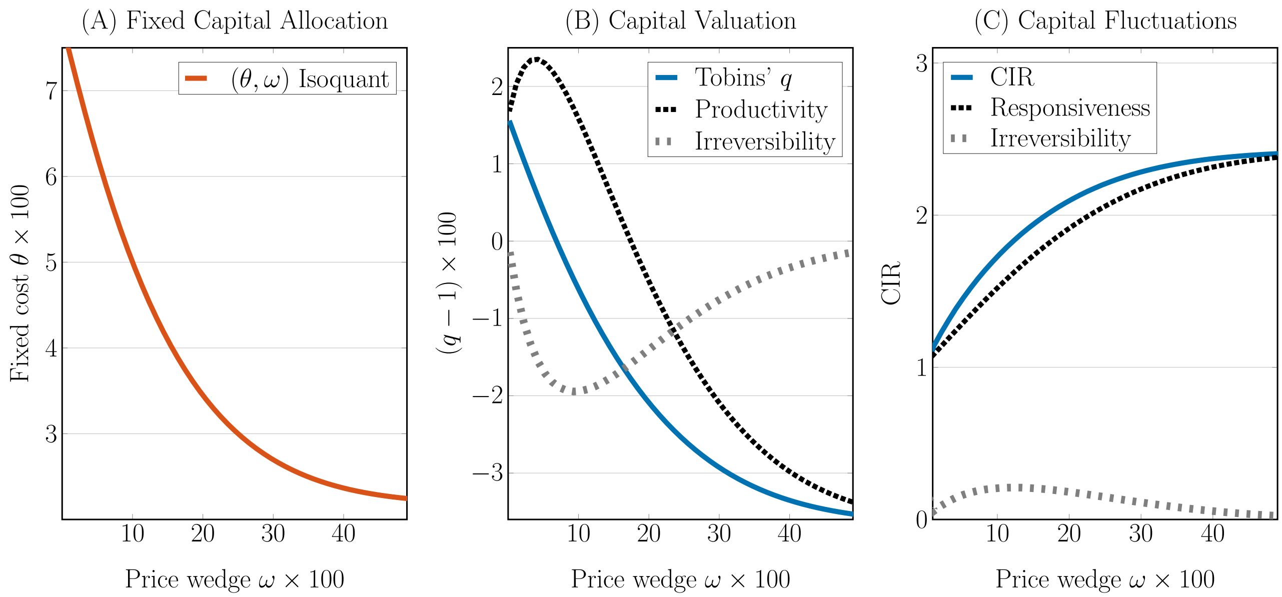
4.3 Intermediate drift

Finally, we consider the empirically-relevant case with an intermediate drift (about 11% in our calibration), in which the two investment frictions are relevant.[9] Clearly, varying the relative size of frictions changes many endogenous objects; thus, to deliver a consistent comparison across model configurations we must fix some target. In the spirit of Hsieh and Klenow (2009), we fix the cross-sectional dispersion of marginal products of capital, $\mathbb{V}ar[\hat{k}]$, as it is summarizes the pervasiveness of investment frictions. Fixing $\mathbb{V}ar[\hat{k}]$ implicitly fixes the frequency of adjustment and the dispersion of investment rates. Next, we use a calibrated version of the model (to be discussed in the following section) to illustrate the relevance of the price wedge in the aggregate.

For each price wedge $\omega \in [0.1, 0.5]$, we find the fixed cost θ that delivers a constant level of $\mathbb{V}ar[\hat{k}]$. Panel A in Figure III plots one of these isoquants (θ, ω), which happens to be convex. Panels B and C plot q and the CIR against the price wedge ω, computed along the isoquant. In all figures, going from left to right increases the relative importance of the price wedge vis-à-vis the

[9] We refer to Miao (2019) for a characterization of the case with full irreversibility ($\omega = 1$) for any drift $\nu \in \mathbb{R}$.

Figure III – Macroeconomic outcomes for fixed allocation



Notes: Panel A shows the (θ, ω) isoquant that matches the empirical value of $\mathbb{V}ar[\hat{k}] = 0.098$ (see Section 5.3). Panel B plots q and its components, normalizing them as $(q-1) \times 100$. Panel C plots the CIR and its components. Other parameters from Table I.

fixed cost while delivering the same level of cross-sectional dispersion $\mathbb{V}ar[\hat{k}]$. We observe that as price wedges become dominant, capital valuation decreases (q goes down) and capital fluctuations become more persistent (CIR goes up) relative to environments in which fixed costs dominate.

We further decompose q into its productivity and irreversibility components in (32). The irreversibility term (dashed line) is non-monotonic: It goes to zero for low ω (when there is no wedge) and for high ω (when no firm disinvests and the wedge becomes irrelevant). We find that irreversibility has the largest negative impact on q for a wedge of around 10%. To examine productivity term, note that since $\mathbb{V}ar[\hat{k}]$ remains fixed, this is exclusively determined by capital accumulation reflected in (minus) the average capital-productivity ratio $\mathbb{E}[\hat{k}]$. The productivity term (dotted line) is also non-monotonic: at low ω, an increase in the wedge decreases both prices (q) and quantities ($\mathbb{E}[\hat{k}]$) because firms are more cautious to invest; whereas at high ω an increase in the wedge decreases the price (q) but increases quantities ($\mathbb{E}[\hat{k}]$) because firms stop disinvesting.

We also decompose the CIR into its responsiveness and irreversibility components in (38). Because $\mathbb{V}ar[\hat{k}]$ remains fixed, the responsiveness term (dotted) is exclusively determined by the covariance of capital-productivity ratios with their age, $\mathbb{C}ov[\hat{k}, a]$. As shown in Baley and Blanco (2021), this covariance reflects asymmetry in the costs of downsizing vs. upsizing the capital stock. It is zero in driftless and symmetric environments. It is negative when there is drift and fixed costs and counteracts the effect of the variance for the CIR. With price wedges, the covariance turns positive because of the strong downsizing friction. Besides the covariance effect, price wedges have an additional (non-monotonic and quantitatively smaller) direct effect on the CIR (dashed line).

Summing up. The previous analysis teach us three lessons. First, the importance of the price wedge for macro outcomes crucially depends on the size of the drift. Second, the effect of frictions on q is ambiguous, as it depends on the mean $\mathbb{E}[x]$ and the variance $\mathbb{V}ar[x]$ of normalized capital-productivity ratios which may move in opposite directions as frictions increase. And third, keeping the capital allocation fixed, an economy or sector in which the price wedge is the dominant friction features lower capital valuation and more persistent capital fluctuations compared to an economy or sector in which fixed costs dominate.

In Section 6, we show that corporate income tax cuts correspond to movements along the (θ, ω) isoquant from left to right, increasing the relative importance of price wedges and changing long-run capital behavior.

5 Measuring macro outcomes with microdata

This section derives mappings from the microdata to parameters, policies and cross-sectional moments of the invariant distribution $g(\hat{k})$. Together with the relationship established between cross-sectional moments and macro outcomes in Section 3, these mappings connect macro outcomes with microdata.

Given an exogenously given price wedge ω and panel of observations $\Omega = \{\Delta\hat{k}, \tau\}$, which includes the adjustment size and the duration of inaction, we infer the behavior of non-adjusters in the cross-section and reverse-engineer the reset points $\{\hat{k}^{*-}, \hat{k}^{*+}\}$, the parameters of the productivity process $\{\nu, \sigma^2\}$, and the key cross-sectional moments that enter q and the CIR:

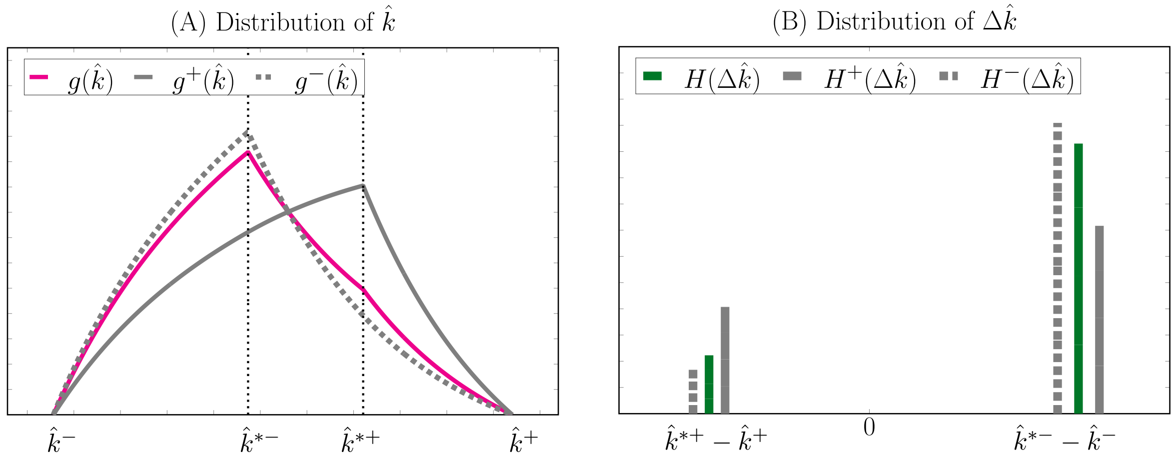
$$(45) \qquad \left\{\mathbb{E}[\hat{k}],\ \mathbb{V}ar[\hat{k}],\ \mathbb{C}ov[\hat{k}, a],\ \mathbb{E}\left[\frac{1}{\mathrm{d}s}\mathbb{E}_s\left[\mathrm{d}(\mathcal{P}(\hat{k}_s)\phi(\hat{k}_s))\right]\right],\ \mathbb{E}\left[\frac{1}{\mathrm{d}s}\mathbb{E}_s[\mathrm{d}(\mathcal{M}(\hat{k}_s)\hat{k}_s)]\right]\right\}.$$

To obtain these mappings, we condition adjusters' behavior on the sign of the last adjustment so that their actions remain Markovian. Then, we exploit properties of Markov processes and the fact that the two reset points are constant. Next, we define the conditional densities required for the handling the history dependence from irreversibility.

5.1 Conditional distributions

Let $g^-(\hat{k})$ and $g^+(\hat{k})$ denote the stationary density of $\hat{k}$ conditional on the last reset point being $\hat{k}^{*-}$ or $\hat{k}^{*+}$, respectively. They satisfy the same KFE as $g(\hat{k})$ except that they only have one kink at the corresponding reset point. Panel A in Figure IV plots the three densities g, g^- and g^+ (these are proper densities and integrate to 1). We denote expectations computed with these

Figure IV – Unconditional and Conditional Distributions of $\hat{k}$ and $\Delta\hat{k}$



Notes: The figures illustrates conditional and unconditional distributions. Panel A plots the unconditional density $g(\hat{k})$ and the densities conditional on the last reset $g^{\pm}(\hat{k})$. Panel B plots the unconditional distribution $H(\Delta\hat{k})$ and the distributions conditional on the last reset $H^{\pm}(\Delta\hat{k})$.

distributions as $\mathbb{E}$, $\mathbb{E}^{-}$, and $\mathbb{E}^{+}$.

Next, we consider the distribution over actions, denoted by $H(\Delta\hat{k},\tau)$, and the distributions of actions *conditional on the last reset point*: $H^{-}(\Delta\hat{k},\tau)$ and $H^{+}(\Delta\hat{k},\tau)$. Panel B of Figure IV plots the marginal distributions of adjustment size, $H(\Delta\hat{k})$, $H^{-}(\Delta\hat{k})$, $H^{+}(\Delta\hat{k})$, where we have integrated out the duration τ; these distributions correspond to probability masses at two points $\Delta\hat{k}=\hat{k}^{*+}-\hat{k}^{+}<0$ and $\Delta\hat{k}=\hat{k}^{*-}-\hat{k}^{-}>0$.

Note that the mass of upward adjustments $H(\hat{k}^{*-}-\hat{k}^{-})$ is larger than the mass of downward adjustments $H(\hat{k}^{*+}-\hat{k}^{+})$. This is because the drift shrinks capital-productivity ratios over time prompting upward adjustments and because partial irreversibility penalizes downward adjustments. This asymmetry is also observed in the firms' distribution, as g is closer to g^{-}. Second, the conditional masses reflect the autocorrelation in the investment sign; for instance, $H^{-}>H^{+}$ at $\Delta\hat{k}>0$ means that the probability of reseting to $\hat{k}^{*-}$ is larger whenever the last reset point was also $\hat{k}^{*-}$. In other words, positive investments beget future positive investments. We denote with bars the expectations computed with the distributions of adjusters: $\overline{\mathbb{E}}$, $\overline{\mathbb{E}}^{-}$ and $\overline{\mathbb{E}}^{+}$.

From conditional to unconditional distributions. Define the shares of upward $\mathcal{N}^{-}/\mathcal{N}$ and downward $\mathcal{N}^{+}/\mathcal{N}$ adjustments within the *population of adjusters*. By Bayes' law, the unconditional and conditional distribution of adjusters satisfy

$$H(\Delta\hat{k},\tau) \;=\; \frac{\mathcal{N}^{-}}{\mathcal{N}}H^{-}(\Delta\hat{k},\tau) \;+\; \frac{\mathcal{N}^{+}}{\mathcal{N}}H^{+}(\Delta\hat{k},\tau). \tag{46}$$

This relationship is useful to compute moments of adjusters. For example, the average duration of inaction equals the weighted sum of the average conditional durations:

$$\overline{\mathbb{E}}[\tau] \;=\; \overline{\mathbb{E}}\left[\mathbb{E}[\tau|\Delta k]\right] \;=\; \frac{\mathcal{N}^-}{\mathcal{N}}\overline{\mathbb{E}}^-[\tau] \;+\; \frac{\mathcal{N}^+}{\mathcal{N}}\overline{\mathbb{E}}^+[\tau]. \tag{47}$$

However, we need to leverage on another approach to recover the unconditional distribution of firms. In that case, the shares must be rescaled by the relative durations of inaction:

$$g(\hat{k}) \;=\; \frac{\mathcal{N}^-}{\mathcal{N}}\frac{\overline{\mathbb{E}}^-[\tau]}{\overline{\mathbb{E}}[\tau]}\, g^-(\hat{k}) \;+\; \frac{\mathcal{N}^+}{\mathcal{N}}\frac{\overline{\mathbb{E}}^+[\tau]}{\overline{\mathbb{E}}[\tau]}\, g^+(\hat{k}) \;=\; \mathcal{N}^-\, \overline{\mathbb{E}}^-[\tau]\, g^-(\hat{k}) \;+\; \mathcal{N}^+\, \overline{\mathbb{E}}^+[\tau]\, g^+(\hat{k}), \tag{48}$$

where we simplify the expression using $\overline{\mathbb{E}}[\tau] = \mathcal{N}^{-1}$, that is, the average duration of inaction equals the inverse of the total frequency of adjusters. This implies that the duration-adjusted frequencies also sum up to one, i.e., $\mathcal{N}^-\, \overline{\mathbb{E}}^-[\tau] + \mathcal{N}^+\, \overline{\mathbb{E}}^+[\tau] = 1$. Why do we need to rescale by duration? The answer is the *fundamental renewal property:* The average behavior in the economy is attributable to firms with longer periods of inaction (which are observed less frequently). Adjusting the shares with their relative duration corrects this observational bias. In environments with irreversibility, the slowly-adjusting firms are coincidentally those that make downward adjustments.[10]

To illustrate the power of our "conditional" approach, we use the law of total variance to decompose capital allocation $\mathbb{V}ar[\hat{k}]$ into two terms that condition on the sign last adjustment:

$$\underbrace{\mathbb{V}ar[\hat{k}]}_{\text{total}} \;=\; \underbrace{\mathbb{E}\left[\mathbb{V}ar[\hat{k}|\Delta\hat{k}]\right]}_{\text{within}} \;+\; \underbrace{\mathbb{V}ar\left[\mathbb{E}[\hat{k}|\Delta\hat{k}]\right]}_{\text{between}}. \tag{49}$$

The decomposition in (A.343) is useful to assess the relative importance of each investment friction in generating capital misallocation. The first term is the average of the variance *within* each conditional distribution g^+ and g^-, that is, the average of $\mathbb{V}ar^-[\hat{k}]$ and $\mathbb{V}ar^+[\hat{k}]$ (computed using (57) below conditioning on the sign of $\Delta\hat{k}$ and using the conditional renewal measure as in (47)). Both investment frictions add to this dispersion. The second term reflects the distance *between* the conditional means $\mathbb{E}^-[\hat{k}]$ and $\mathbb{E}^+[\hat{k}]$ (computed using (56) below conditioning on the sign of $\Delta\hat{k}$ and using the conditional renewal measure). This term arises exclusively from the price wedge that generates two different means. The larger the price wedge, the further apart are the conditional means and the larger the between variance. Note that this term is zero when only fixed costs are present as there is a unique reset point.

5.2 Data mappings

Next, we use the conditional distributions of adjusters to back out parameters, reset points and

[10] Appendix A.11 presents an illustrative example of how to use relative frequencies, Bayes' law, and renewal theory to back out unconditional distributions from conditional distributions.

cross-sectional moments. To facilitate the exposition, we present the mappings for these objects separately, but in fact they are all recovered simultaneously through a system of equations that can be solved iteratively and substituting the population moments with their sample counterpart (see Appendix D.4).

Parameters. We begin by recovering the parameters (ν, σ^2) of the stochastic process of capital-productivity ratios in Proposition 6.

Proposition 6. *The parameters of the stochastic process for productivity (ν, σ^2) are recovered from the microdata $\Omega \equiv (\Delta\hat{k}, \tau)$ as follows:*

$$\nu = \frac{\overline{\mathbb{E}}[\Delta\hat{k}]}{\overline{\mathbb{E}}[\tau]}, \tag{50}$$

$$\sigma^2 = \frac{\overline{\mathbb{E}}[(\hat{k}_\tau + \nu\tau)^2] - \overline{\mathbb{E}}[(\hat{k}^*)^2]}{\overline{\mathbb{E}}[\tau]}. \tag{51}$$

Expression (50) recovers the drift (which includes depreciation ξ^k and productivity growth μ) from the average adjustment size times the frequency of adjustment (the inverse of the expected duration of inaction $\mathcal{N} = \overline{\mathbb{E}}[\tau]^{-1}$), while expression (51) recovers the volatility of idiosyncratic shocks from the second moments of the adjustment size, also scaled by frequency.[11]

Reset points. We continue with the reset points. As a preliminary step, we note that the optimal stopping policy τ^* and the optimal reset points $\{\hat{k}^{*-}, \hat{k}^{*+}\}$ satisfy:

$$p^{buy} e^{\hat{k}^{*-}} = \mathbb{E}\left[\int_0^{\tau^*} \alpha e^{-rs+\alpha\hat{k}_s}\, ds \;+\; p(\Delta\hat{k})\; e^{-r\tau^*+\hat{k}_{\tau^*}} \middle| \hat{k}_0 = \hat{k}^{*-}\right], \tag{52}$$

$$p^{sell} e^{\hat{k}^{*+}} = \mathbb{E}\left[\int_0^{\tau^*} \alpha e^{-rs+\alpha\hat{k}_s}\, ds \;+\; p(\Delta\hat{k})\; e^{-r\tau^*+\hat{k}_{\tau^*}} \middle| \hat{k}_0 = \hat{k}^{*+}\right]. \tag{53}$$

We obtain this result by applying the envelope theorem for arbitrary choice sets (Milgrom and Segal, 2002) to expression (C.386) and then using the optimality condition at the reset points. The expressions say that, when a firm resets its capital-productivity ratio to either $\hat{k}^{*-}$ or $\hat{k}^{*+}$, it equalizes marginal costs to the marginal benefits. When purchasing capital, the marginal cost is the investment price $p^{buy}e^{\hat{k}^{*-}}$ and the marginal benefit includes the cumulative marginal profits obtained during the expected duration of its inaction period plus the expected value of its undepreciated capital at the next adjustment date. Since we work on the log scale, benefits and costs are expressed in percentage points (and thus multiplied by $e^{\hat{k}}$). Proposition 7 presents a

[11] We obtained similar mappings from the data to the parameters in Baley and Blanco (2021) for the case without irreversibility. Irreversibility does not change the mapping to the drift, but it changes the mapping to the volatility, as it affects the transition speed across the two reset points.

mapping from the microdata to the two reset points.

Proposition 7. *Let $\Phi(\nu,\sigma^2) \equiv \log\left(\alpha A/(r+\alpha\nu-\alpha^2\sigma^2/2)\right)$ be a function of structural parameters. For each $\Delta\hat{k}$, construct the stopped capital $\hat{k}_\tau(\Delta\hat{k})$ using (16). Then the reset points are recovered from the microdata $\Omega \equiv (\Delta\hat{k},\tau)$ as follows:*

$$\hat{k}^{*-} = \frac{1}{1-\alpha}\left[\Phi(\nu,\sigma^2) - \log(p^{buy}) + \log\left(\frac{1-\overline{\mathbb{E}}^{-}\left[e^{-\hat{r}\tau+\alpha\left(\hat{k}_\tau-\hat{k}^{*+}\right)}\right]}{1-\overline{\mathbb{E}}^{-}\left[\frac{p(\Delta\hat{k})}{p^{buy}}e^{-\hat{r}\tau+\hat{k}_\tau-\hat{k}^{*+}}\right]}\right)\right], \tag{54}$$

$$\hat{k}^{*+} = \frac{1}{1-\alpha}\left[\Phi(\nu,\sigma^2) - \log(p^{sell}) + \log\left(\frac{1-\overline{\mathbb{E}}^{+}\left[e^{-\hat{r}\tau+\alpha\left(\hat{k}_\tau-\hat{k}^{*-}\right)}\right]}{1-\overline{\mathbb{E}}^{+}\left[\frac{p(\Delta\hat{k})}{p^{sell}}e^{-\hat{r}\tau+\hat{k}_\tau-\hat{k}^{*-}}\right]}\right)\right]. \tag{55}$$

The first term $\Phi(\nu,\sigma^2)$ in expressions (54) and (55) reflects the ratio of marginal product to the user cost of capital. Through this ratio, both reset states increase with profitability A and idiosyncratic risk σ^2 and decrease with the discount r and the drift ν. The second term shows that reset points decrease with the corresponding investment price: firms invest more the lower is the purchasing price p^{buy} and disinvest less the lower is the selling price p^{sell}. Lastly, the third term shows how investment frictions shape the reset points through the marginal profits accrued during periods of inaction (in the numerator) and the resale value (in the denominator).

As a direct measure of irreversibility, consider the difference between the two reset points $(\hat{k}^{*+}-\hat{k}^{*-})$. The term $\Phi(\nu,\sigma^2)$ cancels out in the difference. The second term equals $-(\log(1-\omega))/(1-\alpha) > 0$ and reflects the exogenous price wedge, which is further amplified by the elasticity of output to capital α. The third term reflects history dependence. As long as the optimal policy $(\Delta\hat{k},\tau)$ depends on the last reset point, endogenous irreversibility arises beyond the exogenous price wedge.

Cross-sectional moments. With the parameters and reset points at hand, Proposition 8 recovers steady-state moments of capital-productivity ratios $\hat{k}$.

Proposition 8. *For each inaction spell find the departing point $\hat{k}^*$ and the ending point $\hat{k}_\tau$ using (15) and (16). Then the unconditional mean and variance of $\hat{k}$, and the covariance between $\hat{k}$ and age a are recovered from the microdata $\Omega \equiv (\Delta\hat{k},\tau)$ as follows:*

$$\mathbb{E}[\hat{k}] = \overline{\mathbb{E}}\left[\overline{\mathbb{E}}\left[\left(\frac{\hat{k}^*+\hat{k}_\tau}{2}\right)\left(\frac{\hat{k}^*-\hat{k}_\tau}{\overline{\mathbb{E}}[\Delta\hat{k}]}\right)\middle|\Delta\hat{k}\right]\right] + \frac{\sigma^2}{2\nu}, \tag{56}$$

$$\mathbb{V}ar[\hat{k}] = \overline{\mathbb{E}}\left[\overline{\mathbb{E}}\left[\left((\hat{k}^*-\mathbb{E}[\hat{k}])(\hat{k}_\tau-\mathbb{E}[\hat{k}]) + \frac{(\hat{k}^*-\hat{k}_\tau)^2}{3}\right)\left(\frac{\hat{k}^*-\hat{k}_\tau}{\overline{\mathbb{E}}[\Delta\hat{k}]}\right)\middle|\Delta\hat{k}\right]\right], \tag{57}$$

$$\mathbb{C}ov[\hat{k},a] = \frac{1}{2\nu}\left(\mathbb{V}ar[\hat{k}] - \frac{\overline{\mathbb{E}}[(\hat{k}_\tau-\mathbb{E}[\hat{k}])^2\tau]}{\overline{\mathbb{E}}[\tau]} + \frac{\sigma^2}{2}\frac{\overline{\mathbb{E}}[\tau]}{2}(1+\overline{\mathbb{CV}}^2[\tau])\right). \tag{58}$$

The mapping in (56) recovers the population mean $\mathbb{E}[\hat{k}]$ from the average midpoint between the departing and the ending points of an inaction spell $(\hat{k}^* + \hat{k}_\tau)/2$, where the average is computed under a change of measure induced by the renewal weights $(\hat{k}^* - \hat{k}_\tau)/\overline{\mathbb{E}}[\Delta\hat{k}]$. To recover the population mean, the renewal measure overweighs the midpoints of adjusters with longer periods of inaction, which are more representative in the population.[12] The term $\sigma^2/2\nu$ corrects for the accumulated drift between adjustments. Similarly, the mapping in (57) recovers the population variance $\mathbb{V}ar[\hat{k}]$ from the average distance between the departing point and the mean $(\hat{k}^* - \mathbb{E}[\hat{k}])$, the ending point and the mean $(\hat{k}_\tau - \mathbb{E}[\hat{k}])$, and the between departing and ending points $(\hat{k}^* - \hat{k}_\tau)^2$, again computed using the renewal distribution. In these expressions, we compute the inner expectation with H^- or H^+ depending on the sign of the last adjustment and compute the outer expectation with shares of upward $\mathcal{N}^-/\mathcal{N}$ and downward $\mathcal{N}^+/\mathcal{N}$ adjustment in the population. Lastly, expression (58) recovers the covariance $\mathbb{C}ov[\hat{k}, a]$ from the difference in the second moments of the distribution of adjusters relative to non-adjusters.

Irreversibility component of q. Finally, we derive data mappings to back out the irreversibility terms. Proposition 9 recovers the irreversibility term for q in (32), which reflects the way in which expected capital losses affect capital valuation.

Proposition 9. *The irreversibility component of q is negative and we recover it from the microdata as:*

$$\mathbb{E}\left[\frac{1}{\mathrm{d}s}\mathbb{E}_s\left[\mathrm{d}(\mathcal{P}(\hat{k}_s)\phi(\hat{k}_s))\right]\right] = -\frac{\overline{\mathbb{C}ov}\left[\Delta\hat{k}, \mathcal{P}(\hat{k}^*(\Delta\hat{k}))\right]}{\overline{\mathbb{E}}[\tau]} < 0. \tag{59}$$

According to (59), the irreversibility term of q maps to minus the covariance of investment $\Delta\hat{k}$ and capital losses $\mathcal{P}(\Delta\hat{k})$, scaled by average duration. This covariance is positive since firms purchase capital at the high price and sell capital at the low price. Since the covariance is positive, irreversibility reduces q. Intuitively, firms seek to avoid histories in which, after upsizing, negative productivity shocks will force them to downsize and face the penalty of selling their capital at a discount. Firms also seek to avoid histories in which, after downsizing, positive productivity shocks will force them to upsize and face the penalty of purchasing back capital at a higher price. To minimize the likelihood of these "switching" situations, firms under-invest and under-disinvest, effectively reducing capital valuation. The strength of this mechanism is measured in the data through this covariance.

Irreversibility component for CIR. Proposition 10 below recovers the irreversibility term for the CIR in (38), which reflects the expected cumulative deviations from the steady-state mean of capital-productivity ratios that arise following the aggregate productivity shock that

[12] Without the price wedge, the renewal weights are equal to the relative size of adjustment $\Delta k/\overline{\mathbb{E}}[\Delta\hat{k}]$.

changes the masses of adjusters across reset points. Before we proceed to the Proposition, we provide expressions for $\mathcal{M}^{buy}$ and $\mathcal{M}^{sell}$ that we used to define the function $\mathcal{M}(\hat{k})$ in (39). These two numbers equal to the cumulative deviations below and above the steady-state mean $\mathbb{E}[\hat{k}]$ conditional on the sign of the last reset. Formally, they are equal to:

$$\mathcal{M}^{buy} \equiv (\mathbb{E}^{-}[\hat{k}] - \mathbb{E}[\hat{k}])\overline{\mathbb{E}}^{-}[\tau]\,\frac{\mathbb{E}[\mathbb{P}^{+}]}{\mathbb{P}^{-+}} < 0, \tag{60}$$

$$\mathcal{M}^{sell} \equiv (\mathbb{E}^{+}[\hat{k}] - \mathbb{E}[\hat{k}])\overline{\mathbb{E}}^{+}[\tau]\,\frac{\mathbb{E}[\mathbb{P}^{-}]}{\mathbb{P}^{+-}} > 0. \tag{61}$$

The number $\mathcal{M}^{buy}$ reflects firms' upsizing behavior in steady state (analogously, $\mathcal{M}^{sell}$ reflects firms' downsizing behavior in steady state, mutatis mutandis). Upsizing firms reset their capital-productivity ratio below the unconditional mean and, on average, remain below the mean for the duration of their inaction spell. The average deviation accumulated during one inaction spell is then $(\mathbb{E}^{-}[\hat{k}] - \mathbb{E}[\hat{k}])\overline{\mathbb{E}}^{-}[\tau]$. Since investment sign is serially correlated, upsizing firms remain in an upsizing phase contributing to negative deviations for several periods; they would only leave this phase after a series of negative shocks makes them downsize. The ratio $\mathbb{E}[\mathbb{P}^{+}]/\mathbb{P}^{-+}$ exactly reflects the average time spent in the transient upsizing phase, where $\mathbb{E}[\mathbb{P}^{+}] \equiv \Pr[\Delta\hat{k}' < 0]$ is the unconditional probability of downsizing and $\mathbb{P}^{-+} \equiv \Pr[\Delta\hat{k}' < 0|\Delta\hat{k} > 0]$ is the probability of downsizing conditional on being currently in an upsizing phase.

Proposition 10. *Then irreversibility component in the CIR is positive and is recovered from the microdata as:*

$$\mathbb{E}\left[\frac{1}{\mathrm{d}s}\mathbb{E}_s\left[\mathrm{d}(\hat{k}_s\mathcal{M}(\hat{k}_s))\right]\right] = -\frac{\overline{\mathbb{C}ov}[\Delta\hat{k}, \mathcal{M}(\hat{k}^{*}(\Delta\hat{k}))]}{\overline{\mathbb{E}}[\tau]} > 0. \tag{62}$$

To compute $\mathcal{M}^{buy}$ and $\mathcal{M}^{sell}$, we recover means conditional on the sign of past adjustment as

$$\mathbb{E}^{-}[\hat{k}] = \frac{(\hat{k}^{*-})^2 - \overline{\mathbb{E}}^{-}[\hat{k}_\tau^2]}{2\overline{\mathbb{E}}^{-}[\Delta\hat{k}]} + \frac{\sigma^2}{2}\frac{\overline{\mathbb{E}}^{-}[\tau]}{\overline{\mathbb{E}}^{-}[\Delta\hat{k}]}; \qquad \mathbb{E}^{+}[\hat{k}] = \frac{(\hat{k}^{*+})^2 - \overline{\mathbb{E}}^{+}[\hat{k}_\tau^2]}{2\overline{\mathbb{E}}^{+}[\Delta\hat{k}]} + \frac{\sigma^2}{2}\frac{\overline{\mathbb{E}}^{+}[\tau]}{\overline{\mathbb{E}}^{+}[\Delta\hat{k}]}, \tag{63}$$

we recover unconditional probabilities as:

$$\mathbb{E}[\mathbb{P}^{+}] = \frac{\overline{\mathbb{E}}[\tau\mathbb{I}(\Delta\hat{k} < 0)]}{\overline{\mathbb{E}}[\tau]}, \qquad \mathbb{E}[\mathbb{P}^{-}] = \frac{\overline{\mathbb{E}}[\tau\mathbb{I}(\Delta\hat{k} > 0)]}{\overline{\mathbb{E}}[\tau]}; \tag{64}$$

and we recover transition probabilities between reset points as:

$$\mathbb{P}^{+-} = \Pr[\Delta\hat{k}' > 0|\Delta\hat{k} < 0], \qquad \mathbb{P}^{-+} = \Pr[\Delta\hat{k}' < 0|\Delta\hat{k} > 0]. \tag{65}$$

According to (62), the irreversibility term in the CIR equals minus the covariance of investment $\Delta\hat{k}$ with the capital deviations $\mathcal{M}(\Delta\hat{k})$. The covariance is negative: due to micro-history dependence, firms that do negative investments are expect to remain above the steady-state mean and contribute with positive deviations (similarly, firms that do negative investment are expect to contribute with positive deviations). Because a negative aggregate shock increases the mass of selling firms relative to steady state, and these firms contribute with positive deviations, the CIR goes up.

In summary, expressions (50) to (62) provide inverse mappings from the microdata $\Omega = \{\Delta\hat{k}, \tau\}$ to parameters, reset points, capital allocation $\mathbb{V}ar[\hat{k}]$, capital valuation q, and capital fluctuations CIR. In the next section, we apply these mappings to estimate these aggregate outcomes from the microdata and assess the role of investment frictions in shaping their empirical values.

5.3 Putting the theory to work

We apply the mappings using yearly investment data on manufacturing plants in Chile from the Annual National Manufacturing Survey (*Encuesta Nacional Industrial Anual*) for the period 1980 to 2011. To construct the capital series, we use information on depreciation rates and price deflators from national accounts and Penn World Tables. The sample considers plants that appear in the sample for at least 10 years (more than 60% of the sample) and have more than 10 workers. Appendix D presents all the details of the data.

Capital stock and investment rates. We construct the capital stock series using the perpetual inventory method. We include structures, machinery, equipment, and vehicles. Following the theory, a plant's capital stock in year s, k_s, evolves as

$$k_s \;=\; (1-\xi^k)k_{s-1} \;+\; I_s/(p(I_s)D_s), \tag{66}$$

where ξ^k is the physical depreciation rate; I_s is the nominal value of investment; $p(I_s)$ is the investment pricing function, which considers different prices for capital purchases and sales; D_s is the gross fixed capital formation deflator, and k_0 is a plant's self-reported nominal capital stock at current prices for the first year in which it is nonnegative. Note that the ratio $I_s/(p(I_s)D_s)$ is the real investment in capital units (the data counterpart to $i_s = \Delta k_s$ in the model).

With irreversibility, we need information on the price wedge in order to compute real investment.[13] We pick a price wedge of 25% and set $\omega = 0.25$. This is a conservative number, relative to the estimates in Ramey and Shapiro (2001) and Kermani and Ma (2020). We pick a smaller number to reflect heterogeneity across sectors and types of capital, the possibility of internal transfers of capital through mergers and acquisitions, as well as the fact that empirical estimates are obtained using information from liquidating firms and thus are likely to suffer from selection.

[13]Note that only the price wedge matters for computing investment, not the price level.

We construct gross nominal investment i_s with information on purchases, reforms, improvements, and sales of fixed assets, and define the investment rate ι_s as the ratio of real gross investment to the capital stock:[14]

$$\iota_s \equiv \frac{I_s/(p(I_s)D_s)}{k_{s-1}}. \tag{67}$$

For each plant and each inaction spell h, we record the change in the capital-productivity ratio upon action $\Delta\hat{k}_h$ and the spell's duration τ_h. We construct $\Delta\hat{k}_h$ with investment rates from (67):

$$\Delta\hat{k}_h = \begin{cases} \log(1+\iota_h) & \text{if} \quad |\iota_h| > \underline{\iota}, \\ 0 & \text{if} \quad |\iota_h| < \underline{\iota}. \end{cases} \tag{68}$$

The threshold $\underline{\iota} > 0$ reflects the idea that small maintenance investments should be excluded. Following Cooper and Haltiwanger (2006), we set $\underline{\iota} = 0.01$, such that all investment rates below 1% in absolute value are considered to be part of an inaction spell. Then we define an adjustment date T_h from $\Delta\hat{k}_{T_h} \neq 0$ and compute a spell's duration as the difference between two adjacent adjustment dates: $\tau_h = T_h - T_{h-1}$. Finally, we truncate the investment distribution at the 2nd and 98th percentiles to eliminate outliers.[15]

Figure V plots the resulting cross-sectional distribution of non-zero changes of the capital-productivity ratios $\Delta\hat{k}$ and completed inaction spells τ, conditional on a past positive or negative investment. The data shows investment patterns that are consistent with partial irreversibility. In particular, the distribution of investment conditional on a last negative investment $H^+(\Delta k)$ is skewed toward the left of the distribution conditional on a last positive investment H^-, meaning that the probability of a negative investment is larger after a negative investment and vice versa.

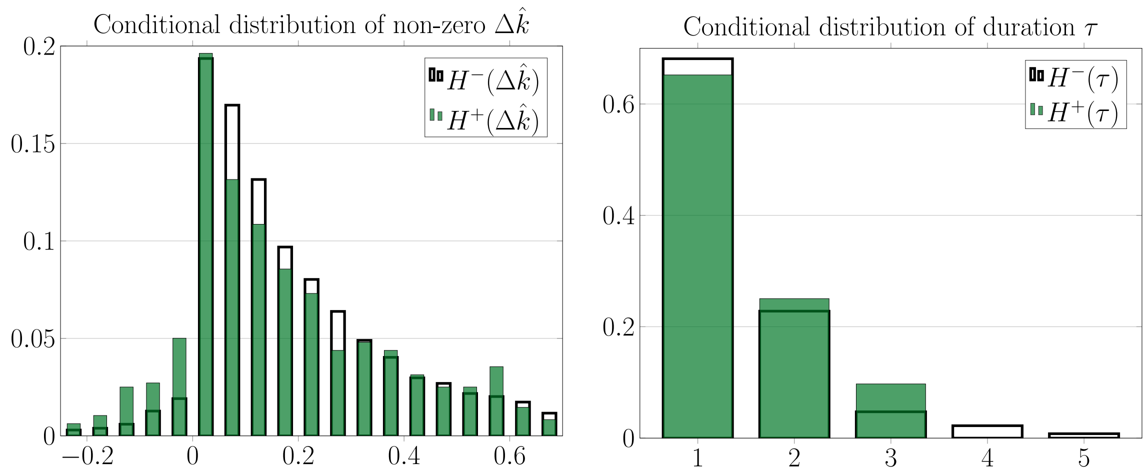
Externally-calibrated parameters. We calibrate externally several parameters to match average statistics from the Chilean economy between 1980 and 2011. One period equals a year. We set the real interest rate to 6.6% ($\rho = 0.066$) to match the average real interest rate computed by the IMF. The productivity growth rate is 3.3% ($\mu = 0.033$) to match the average GDP growth rate. The elasticity of output to capital is set to $\alpha = 0.5$. We set the price level $p = 6$ to match an aggregate output-capital ratio of $\hat{Y}/p\hat{K} = 0.23$. Finally, we set $A = 1$ as a normalization.

Estimated parameters. Using the mappings from the microdata to the parameters of the productivity process in (50) and (51), we recover a drift of $\nu = 0.118$ and a volatility of $\sigma^2 = 0.054$.

[14] Note that the investment rate equals $\iota_{T_h} \equiv i_{T_h}/k_{T_h^-} = (k_{T_h} - k_{T_h^-})/k_{T_h^-}$, where $k_{T_h^-} = \lim_{s\uparrow T_h} k_s$. In contrast to the continuous-time model, in which investment is computed as the difference in the capital stock between two consecutive instants, in the data we compute it as the difference between two consecutive years.

[15] Table I in Appendix D presents descriptive statistics on investment rates. In particular, the inaction rate ($|\iota| < 0.01$) equals 40.1%.

Figure V – Empirical Distribution of Observable Actions



Notes: Own calculations using establishment data from Chile. Panel A plots the distribution of *non-zero* changes in capital-productivity ratios and Panel B plots the duration of inaction spells. Solid bars = conditional on departing from $\hat{k}^{*+}$ (last negative investment); white bars = conditional on departing from $\hat{k}^{*-}$ (last positive investment). Sample: Firms with at least 10 years of data, truncation at 2nd and 98th percentiles of investment rate distribution, and inaction threshold of $\underline{\iota} = 0.01$.

Table I – Parametrization

Technology				**Productivity**		**Normalization**	
μ	α	ρ	ω	ν	σ^2	A	p
0.033	0.500	0.066	0.250	0.118	0.054	1.000	6.000

Notes: Baseline parameterization.

Together with the productivity growth rate, the value for the drift ν implies a physical depreciation rate of $\xi^k = \nu - \mu = 0.085$. Given these values, the adjusted discount is $r = \rho - \mu - \sigma^2/2 = 0.006$ and the user cost is $\mathcal{U} = \rho + \xi^k - \sigma^2 = 0.097$. The completely parameterization is summarized in Table I.

5.4 Aggregate capital behavior: 1980-2011

Given parameters and data, we apply all the data mappings. Table II reports the investment policy and average macro outcomes in Chile for the period between 1980 and 2011.

We begin by examining the investment policy. From (54) and (55), we recover the gap between the two reset points, $\hat{k}^{*+} - \hat{k}^{*-} = 0.914$, is a tell-tale sign of partial irreversibility. This gap is almost equally explained by the exogenous price wedge, $\log(p^{\text{buy}}/p^{\text{sell}}) = 0.575$, and the endogenous response (computed as a residual) equals 0.339. Using (56) and (57), we estimate a dispersion in marginal products of $\mathbb{V}ar[\hat{k}] = 0.098$.

We use (32) to recover an average capital valuation of $q = 1.05$. While q is not far from its frictionless value (unity), it would be erroneous to conclude that dynamic frictions are not

Table II – Aggregate Capital Behavior

Investment Policy		**Capital Allocation**	
Difference in reset capitals ($\hat{k}^{*+} - \hat{k}^{*-}$)	0.914	Variance	0.098
Exogenous price wedge	0.575		
Endogenous response	0.339		
Capital Valuation		**Capital Fluctuations**	
Tobins q	1.05	CIR	2.619
Productivity	1.10	Responsiveness	1.933
Irreversibility	-0.05	Irreversibility	0.686

Notes: Objects recovered from theory mappings applied to establishment-level data from Chile. Parameters described in Table I.

present; in fact, they are both important but affect mainly the stock of capital. The productivity component in (33) is 1.10 and the irreversibility component in (59) is -0.05. As predicted by the theory, irreversibility decreases q.

Lastly, using (38), we recover an average CIR of 2.619, meaning that a 1% decrease in aggregate productivity generates a total deviation of aggregate capital above its steady-state value of 2.6%. We further decompose the CIR into its components: responsiveness 1.933 from (57) and irreversibility 0.686 from (62). As predicted by the theory, the CIR's irreversibility term is positive. Indirectly, irreversibility reduces the sensitivity to idiosyncratic shocks (increasing $\mathbb{V}ar[\hat{k}]$) and increases the relative cost of downsizing the capital stock (increasing $\mathbb{C}ov[\hat{k}, a]$).

6 The Macroeconomic Effects of Corporate Taxes

This section applies our methodology to study the effects of corporate tax reforms. We introduce a comprehensive tax schedule and characterize the role of corporate taxation in shaping macroeconomic outcomes. We do this in three steps. First, we show that corporate taxes change four parameters: profitability A, the discount factor ρ, the fixed cost θ, and the investment prices $p(\Delta\hat{k})$. Once we redefine these parameters, the investment problem is identical to the one described in Section 2. Second, we analyze the effect of taxation using the notion of *after-tax* effective frictions, which summarize the complex interaction of taxes with investment frictions with an appropriate rescaling of each friction. Third, we assess the effects of corporate income tax cuts using the mappings between macroeconomic outcomes, cross-sectional moments, and microdata.

6.1 A comprehensive tax schedule

Following Summers (1981) and Abel (1982), we introduce a corporate tax system to the firm problem. It includes a corporate income tax, deduction allowances, a personal income tax, and capital gains tax.[16] The firm pays the corporate income tax rate t^c on its cash flow Ay_s net of deductions, where ξ^d denotes the deduction rate. Since the physical and the legal depreciation rates differ, we distinguish deductions from the capital stock and denote these with d_s. The state space now includes deductions $V(k, u, d)$.

The after–tax profits per unit of time π_s is given by

$$\pi_s = (1 - t^c)Ay_s + \underbrace{t^c \xi^d d_s}_{\text{deductions}} - \underbrace{t^c p\omega i_s \mathbb{1}_{\{i_s<0\}}}_{\text{capital losses}}, \tag{69}$$

where deductions evolve as:

$$\log d_s = \log d_0 - \xi^d s + \sum_{h:T_h \le s} \left(1 + \frac{\theta_{T_h} + p i_{T_h}}{d_{T_h^-}}\right). \tag{70}$$

Let us explain how we model the interaction of taxes with investment frictions. First we assume that the payments of fixed adjustment cost are capitalized and enter the law of motion of capital deductions in (70). This assumption is sensible to the extent that the fixed costs associated with investment or disinvestment are not fully deducted when they are paid. In contrast, when a firm sells its capital, it loses the future deductions valued at the buying price. For example, if $\omega = 1$ and the firm buys and sells capital immediately, then the capital depreciation allowance does not change. Second, we assume that capital losses are fully deducted from ordinary income in (69) on the date in which they are accrued.[17]

The personal income tax t^p and the capital gain tax t^g alter the firms' discount factor. We assume that equity is purchased by a representative investor with access to a riskless bond with return ρ per unit of time. Let D_s be the dividend per share, P_s the equity price per share, and $E_s = 1$ the number of shares, which we normalize to unity without loss of generality. From the investor's perspective, dividends and bond returns are taxed at the rate t^p, while capital gains arising from changes in equity prices are taxed at the rate t^g. For any dividend process, no-arbitrage implies equal after-tax returns:

$$(1 - t^p)\rho \, \mathrm{d}s = (1 - t^g)\frac{\mathbb{E}[\mathrm{d}P_s]}{P_s} + (1 - t^p)\frac{D_s}{P_s}\,\mathrm{d}s. \tag{71}$$

[16] This taxation schedule is also used in the investment models by Poterba and Summers (1983); King and Fullerton (1984); Auerbach (1986); Auerbach and Hines (1986) and Hassett and Hubbard (2002).

[17] In the U.S., the capital gains tax in the corporate sector is computed on observed transactions, exactly as for households. Differently from household taxation of capital gains, however, firms do not receive a preferential tax rate, that is, firms are not allowed to use capital losses to offset ordinary income taxation. Instead, as with the corporate income tax, losses are carried backward or forward to compensate gains. We abstract from these dimension and assume that the corporate income tax is linear and applies to the sum of ordinary and capital income. See Desai and Gentry (2004) a detailed discussion of capital gains taxes for the corporate sector. We thank Jim Hines for helpful advice on these assumptions.

Condition (71) pins down the time-0 value of the firm, which equals the equity price:

$$V(k_0, u_0, d_0) \; = \; P_0 \; = \; \frac{1-t^p}{1-t^g}\, \mathbb{E}_0 \left[\int_0^\infty e^{-\rho \frac{1-t^p}{1-t^g} s} D_s \, \mathrm{d}s \right]. \tag{72}$$

This expression says that the firm maximizes the cum-dividends market value of equity P_0 using the investor's after-tax discount factor $\rho(1-t^p)/(1-t^g)$, as in Auerbach (1979). We follow the "tax capitalization" view of the dividend decision and consider dividends as residuals, equal to the cash flow π_s net of investment and capital adjustment costs[18]

$$D_s \, \mathrm{d}s \; = \; \pi_s \, \mathrm{d}s - (\theta_s + (p(i_s) - t^c p \omega \mathbb{1}_{\{i_s<0\}}) i_s) \mathcal{D}(s = T_h), \qquad \mathcal{D}(\cdot) \sim \mathit{Dirac}. \tag{73}$$

Given the tax schedule, Lemma 3 characterizes the firm's problem with corporate taxation. The strategy consists of defining the discounted value of deductions per unit of investment z and using it to rewrite the 3-state problem (k, u, d) as the 1-state problem in terms of the capital-productivity ratio $\hat{k} = \log(k/u)$ already studied in Section 2.3, with four parametric changes and an additive term that reflects deductions d.

Lemma 3. *Define the discounted value of deductions:*

$$z \; \equiv \; \frac{\xi^d}{\rho \frac{1-t^p}{1-t^g} + \xi^d} < 1. \tag{74}$$

The firm value with taxes can be decomposed as:

$$V(k, u, d) \; = \; \frac{1-t^p}{1-t^g} \left[u v(\hat{k}) \; + \; t^c z d \right], \tag{75}$$

where $v(\hat{k})$ solves the investment problem in Lemma 2 with the following four parametric changes:

$$A \; \rightarrow \; (1-t^c)A, \tag{76}$$

$$\rho \; \rightarrow \; \frac{(1-t^p)}{(1-t^g)} \rho, \tag{77}$$

$$\theta \; \rightarrow \; (1-t^c z)\theta, \tag{78}$$

$$p(\Delta\hat{k}) \; \rightarrow \; \left(1 - t^c z - \omega(1-t^c) \mathbb{1}_{\{\Delta\hat{k}<0\}}\right) p. \tag{79}$$

The parametric changes established in Lemma 3 highlight the different channels through which

[18] In the model without corporate taxes, the Modigliani-Miller theorem holds, that is, the firms' values and investment policies—and the implicit dividend policy—were independent of the capital structure. Introducing taxes, in principle, could break this independence (for example, under the trade-off theory of the capital structure, see Hines and Park, 2017). Nevertheless, following the arguments in Miller (1977), and more recently in Abel (2018), we continue to work under the Modigliani-Miller paradigm.

taxes affect the firm value and optimal policy. The corporate income tax t^c directly affects after-tax profitability A in (76). The personal income tax t^p and the capital gains tax t^g scale the discount factor ρ in (77).[19] The discounted value of deductions z affects the firm value through an income effect, as deductions increase additively the firm value in (75), and a substitution effect, as deductions promote investment by reducing the after-tax adjustment costs and after-tax prices in (78) and (79). Additionally, t^p and t^g operate indirectly through z and t^c affects directly the effective level of partial irreversibility.

Next, we formalize the channels through which taxes affect investment through their interaction with frictions. To simplify the notation, we define the *after-tax* discount $\tilde{r}$ and the *after-tax* user cost of capital $\tilde{\mathcal{U}}$:

$$\tilde{r} \equiv \frac{1-t^p}{1-t^g}\rho - \mu - \frac{\sigma^2}{2}, \tag{80}$$

$$\tilde{\mathcal{U}} \equiv \frac{1-t^p}{1-t^g}\rho + \xi^k - \sigma^2. \tag{81}$$

In particular, the after-tax user cost $\tilde{\mathcal{U}}$ decreases with the personal income tax and increases with the capital gains tax. For the problem to be well-defined, we assume $\tilde{r} > 0$ and $\tilde{\mathcal{U}} > 0$.

6.2 After-tax investment frictions

Proposition 11 introduces the notion of after-tax investment frictions, which are analogous to the effective frictions defined in (28) and (29) with the addition of taxes. Then, it signs the relationships with investment frictions.

Proposition 11. *Let $z \equiv \xi^d/(\rho\frac{1-t^p}{1-t^g} + \xi^d) < 1$. With corporate taxes, the effective fixed cost $\tilde{\theta}$ and effective price wedge $\tilde{\omega}$ are given by*

$$\tilde{\theta} = \left(\frac{1-t^c z}{1-t^c}\frac{1}{A}\right)^{\frac{1}{1-\alpha}} \left(\frac{p\tilde{\mathcal{U}}}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} \theta, \tag{82}$$

$$\tilde{\omega} = \frac{(1-t^c)\alpha}{\tilde{\mathcal{U}}}\omega. \tag{83}$$

Corporate taxes have the following effects on the after-tax investment frictions:

1. *The effective fixed costs $\tilde{\theta}$ increases with t^c and the after-tax user cost $\tilde{\mathcal{U}}$, and decreases with the PDV of deductions z.*

[19] The factor $(1-t^p)/(1-t^g)$ also scales A, θ, and $p(\hat{k})$. However, these parameters divide each other in all the expressions that follow, so we can safely ignore this factor.

2. *The effective price wedge $\tilde{\omega}$ <u>decreases</u> with the corporate income tax t^c and after-tax user cost $\tilde{\mathcal{U}}$, and does not change with the PDV of deductions z.*

Let us focus on the effects of the corporate income tax. It has opposing effects on after-tax investment frictions. On the one hand, a lower corporate income tax t^c increases profits and therefore *reduces the after-tax fixed cost*. This effect is mediated by the depreciation allowance rate, being lowest when $z = 1$ (in this case t^c is a pure profit tax) and highest when $\xi^d = z = 0$. On the other hand, a lower corporate income tax t^c *increases the after-tax price wedge* because it provides a smaller subsidy to capital losses. As anticipated, a corporate income tax cut generates an inward *displacement* together with a movement *along* the isoquant of after-tax frictions $(\tilde{\theta}, \tilde{\omega})$ (recall Figure III). Next, we apply these insights to study the aggregate effects of corporate taxes.

6.3 A regime shift from high to low taxes

This section explores the macroeconomic effects of a regime change from high to low taxes, focusing on a reduction in the corporate income tax rate.[20] We motivate this exercise with the observation that the top marginal corporate income tax rate experienced a median drop of 17 percentage points across OECD countries, from 42% in 1980 to 25% in 2020.[21] According to the theory, a decline in the corporate income tax rate t^c is equivalent to a reduction in the effective fixed cost $\tilde{\theta}$ and an increase in the effective price wedge $\tilde{\omega}$. Given our calibration, this reform unambiguously reduced misallocation, other things equal, but the consequences for capital valuation and capital fluctuations depend on the magnitude of the various forces that we characterized before.

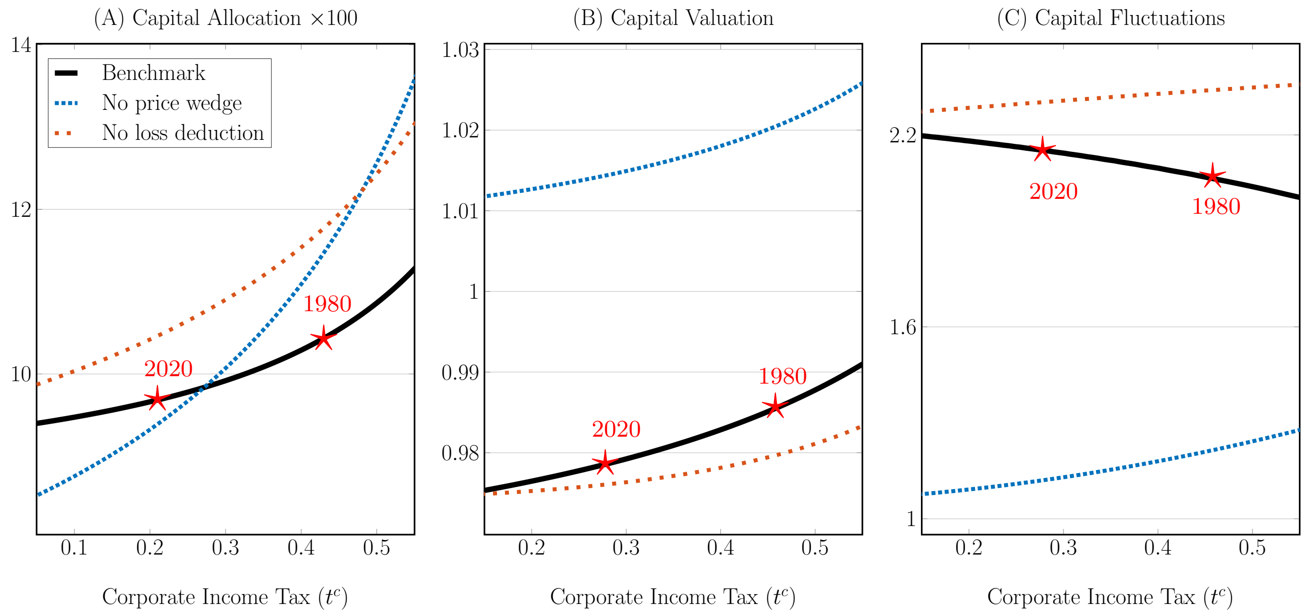
To discipline these forces, we use the parameterization that matches the average Chilean experience summarized in Table I. Additionally, we must take a stand on the size of the fixed cost θ in order to map the changes in the corporate tax into changes in the after-tax fixed cost $\tilde{\theta}$. Note that taking a stand on the size of the fixed cost was not necessary for applying the mappings from microdata to macro outcomes in the previous section. Here, we search for a fixed-cost parameter using the method of moments to minimize the relative distance between two moments in the model and the data: the variance of capital-productivity ratios $\mathbb{V}ar[\hat{k}]$ and the covariance of capital-productivity ratios with the time elapsed since their last adjustment $\mathbb{C}ov[\hat{k}, a]$ (below we discuss these choices and the implied model fit). We obtain a fixed cost of $\theta = 0.2$.

Figure VI illustrates the macroeconomic consequences of a drop in the corporate tax rate. In each panel, we consider three versions of the model. The first version is the benchmark model (solid black) that consider a price wedge of $\omega = 0.25$ and allows for capital loss deductions. The second version eliminates the price wedge by setting $\omega = 0$ (by construction, there are no capital

[20] We leave the analysis of the other three tax instruments for future work.

[21] In Chile, the evolution of the corporate tax rate is U-shaped. It was 40% in 1980, dropped to 10% in 1984, and then consistently (but infrequently) moved upward until reaching 20% in 2020 (see Appendix D). In this exercise, we identify the high-tax regime with 1980 and the low-tax regime with 2020.

Figure VI – Macro Outcomes and Corporate Income Tax



Notes: Panels A, B, and C plot capital allocation, capital valuation, and capital fluctuations for various levels of the corporate income tax rate t^c and three specifications: Benchmark = baseline model calibrated as in Table I ; No price wedge = baseline model with $\omega = 0$; No loss deduction = Stars correspond to the median values for t^c in the OECD: 1980 = 42%, 2020 = 25%.

losses). The third version maintains the price wedge at the benchmark value but does not allow for capital loss deductions.

Panel A shows the capital allocation $\mathbb{V}ar[\hat{k}]$. We observe that a decline in the tax rate improves the allocation of capital. As predicted by equation (82) and Propositions 4 and 5, lower t^c reduces the effective fixed cost $\tilde{\theta}$ and shrinks inaction regions, lowering firms' tolerance for mismatch between their capital and their productivity and decreasing the dispersion of capital-productivity ratios. At the same time, it raises the price wedge which in principle can increase dispersion. In the calibration, the first effect dominates and cross-sectional dispersion falls with the lower tax.

Panel B shows capital valuation q (in black). We discover that q moves in the same direction as the tax rate. When t^c decreases, firms invest more and the average capital-productivity ratio $\mathbb{E}[\hat{k}]$ goes up. Abundant capital is less valuable and q goes down. At the same time, the allocation of capital improves, $\mathbb{V}ar[\hat{k}]$ falls (see Panel A) and q goes up. However, the price wedge becomes more important and the implied subsidy for capital losses falls. This force also pushes q down.

Finally, Panel C shows capital fluctuations measured by the CIR (in black). We find that the CIR increases with the tax cut, meaning that aggregate productivity shocks propagate more slowly when taxes are low. While there is lower dispersion, the larger after-tax price wedge generates more history dependence and persistent deviations from steady-state.

Comparing the results across the various model configurations—with and without price wedge and with and without capital loss deductions—highlights the importance of price wedges and their tax treatment for the macroeconomy. We observe important differences in the size and in the sign of the elasticities of aggregate outcomes with respect to corporate taxes. For instance, ignoring the price wedge reverts the sign of the relationship between the CIR and the corporate tax rate.

In summary, a drop in the corporate income tax rate reduces capital misallocation, reduces capital valuation, and slows down the propagation of aggregate productivity shocks. While we focus here on the aggregate outcomes, our results can be also applied to think about cross-sectional responses to corporate tax changes. In particular, cross-sectional differences in the relative importance of fixed costs θ and price wedges ω—across firms, industries, sectors, or types of capital—may bring heterogenous responses to an identical change in t^c across the board. Alternatively, cross-sectional differences in depreciation allowances z, as documented in House and Shapiro (2008) and Zwick and Mahon (2017), should bring heterogenous responses to t^c, controlling for fixed costs and price wedges. These observations offer a complementary identification strategy that exploits ex-ante heterogeneity instead of heterogeneity in the treatment. We leave for future work testing this prediction using cross-sectoral data.

A remark on the calibration of fixed costs. Let us compare the values for the macro outcomes reported in Table II—recovered directly from the microdata mappings—with the corresponding values in Figure VI—obtained by simulating the calibratd model. We see that all values in the data are consistently larger than those produced by the model. The reason for this discrepancy is that our model with a symmetric fixed cost is extremely parsimonious and cannot reproduce the large variance of capital-productivity ratios $\mathbb{V}ar[\hat{k}]$ and the large covariance of capital-productivity ratios and their age $\mathbb{C}ov[\hat{k}, a]$ recovered from the data. The simulated method of moments strikes a balance between these two moments in the data, but falls below their empirical values.

In previous work (Baley and Blanco, 2021), we demonstrated that the symmetric fixed cost model is unable to replicate the empirical values of these two moments and showed how it should be augmented in order to match them. Introducing a time-dependent component in adjustment, such as random opportunities for free adjustment, increases the variance $\mathbb{V}ar[\hat{k}]$. Introducing asymmetric fixed costs that depend on the adjustment sign increases the covariance $\mathbb{C}ov[\hat{k}, a]$ (the price wedge already pushes the covariance up, but it is not quantitatively enough). Augmenting the model in these two directions is straightforward and necessary to conduct a fully-fledged quantitative analysis. Nevertheless, we have opted to keep the model as simple as possible to facilitate its exposition and to highlight the key mechanisms at work in the cleanest way.

7 Final thoughts

We propose a new laboratory to study the macroeconomic implications of partial irreversibility, its interaction with fixed adjustment costs, and corporate taxes. Our results, and particularly the tax application, highlight the need of disentangling the role of price wedges vis-à-vis fixed adjustment to correctly assess the aggregate effects of tax policy. Moreover, the analysis puts forward a new channel for policy intervention: Corporate tax policy can change the effective size of fixed costs and price wedges—technological constraints or market prices typically outside the control of a policymaker—and structurally change long-run behavior of aggregate capital and the macroeconomy more broadly.

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