Returns to Labor Mobility

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Online Appendix

A Equilibrium computation

Here we outline the structure of the algorithm that we used to compute equilibria.[1]

A.1 General algorithm structure

The algorithm centers around approximating the joint continuation values $g_i(z)$ by using linear projections on a productivity grid. It employs the following steps:

1. Fix a parameterization and construct productivity distributions over a grid of size N_z.

2. Guess initial values for:

   - ζ_i^k: coefficients for linear approximations $\hat{g}_i(z) = \zeta_i^0 + \zeta_i^1 z$ to $g_i(z)$
   - b_j: unemployment benefits
   - ω_{ij}^w: workers' outside values, not including current payment of benefit
   - $\omega^f = 0$: firms' outside value
   - τ: tax rate
   - u_{ij}, e_{ij}: masses of unemployed and employed workers

3. Given linear approximations $\hat{g}_i(z)$, use (2)–(5) to compute reservation productivities $\underline{z}_{ij}^o, \underline{z}_{ij}$.

4. Given cutoffs $\underline{z}_{ij}^o, \underline{z}_{ij}$, compute rejection probabilities ν_{ij}^o, ν_{ij} using (6) and compute E_{ij} using (7).

5. Compute the expected match surplus of a vacancy that encounters an unemployed worker:

$$\bar{s} \equiv \sum_{(i,j)} \frac{u_{ij}}{u} \int_{\underline{z}_{ij}^o}^{\infty} s_{ij}^o(y)\ dv_i^o(y).$$

6. Compute joint continuation values $g_i(z)$ using (8) and (9). Then update coefficients ζ_i^0, ζ_i^1 described in step 2 by regressing $g_i(z)$ on $[1 \;\; z]$.

7. Update the value of posting a vacancy, market tightness, and matching probabilities:

$$w^f = 0, \qquad \theta = \left(\frac{\beta A(1-\pi)\bar{s}}{\mu}\right)^{1/\alpha}, \qquad \lambda^w(\theta) = A\theta^{1-\alpha}, \qquad \lambda_{ij}^f(\theta) = A\theta^{-\alpha}\frac{u_{ij}}{u}.$$

[1] We are grateful to Wouter den Haan, Christian Haefke, and Garey Ramey for generously sharing their computer code. That code was augmented and modified by LS and further by us.

8. Update values ω_{ij}^w of being unemployed using (10) and (11).

9. Compute net changes in worker flows (all must be zero in a steady state)

$$\begin{aligned}\Delta u_{ll} &= \rho^r + (1-\rho^r)\{\rho^x + (1-\rho^x)(1-\gamma^u)\gamma^s \nu_{ll}\}\, e_{ll} \\ &- \rho^r u_{ll} - (1-\rho^r)\lambda^w(\theta)(1-\nu_{ll}^o)u_{ll} \end{aligned} \tag{A.1}$$

$$\begin{aligned}\Delta u_{lh} &= (1-\rho^r)\left\{\rho^x \gamma^{d,x} e_{hh} + (1-\rho^x)\nu_{hh}\gamma^d(\gamma^s e_{hh} + \gamma^u e_{ll})\right\} \\ &- \rho^r u_{lh} - (1-\rho^r)\lambda^w(\theta)(1-\nu_{lh}^o)u_{lh} \end{aligned} \tag{A.2}$$

$$\begin{aligned}\Delta u_{hh} &= (1-\rho^r)\left\{\rho^x(1-\gamma^{d,x})e_{hh} + (1-\rho^x)\nu_{hh}(1-\gamma^d)(\gamma^s e_{hh} + \gamma^u e_{ll})\right\} \\ &- \rho^r u_{hh} - (1-\rho^r)\lambda^w(\theta)(1-\nu_{hh}^o)u_{hh} \end{aligned} \tag{A.3}$$

$$\begin{aligned}\Delta e_{ll} &= (1-\rho^r)\lambda^w(\theta)\{(1-\nu_{ll}^o)u_{ll} + (1-\nu_{lh}^o)u_{lh}\} \\ &- \rho^r e_{ll} - (1-\rho^r)[\rho^x + (1-\rho^x)(\gamma^u + (1-\gamma^u)\gamma^s\nu_{ll}]e_{ll} \end{aligned} \tag{A.4}$$

$$\begin{aligned}\Delta e_{hh} &= (1-\rho^r)\{\lambda^w(\theta)(1-\nu_{hh}^o)u_{hh} + (1-\rho^x)\gamma^u(1-\nu_{hh})e_{ll}\} \\ &- \rho^r e_{hh} - (1-\rho^r)[\rho^x + (1-\rho^x)\gamma^s\nu_{hh}]e_{hh} \end{aligned} \tag{A.5}$$

10. Compute average wages $\bar{p}_i$ and average productivities $\bar{z}_i$ as described in Appendix A.2, to determine government expenditures for unemployment benefits and government tax revenues using the left side and right side of (23), respectively.

11. Adjust tax rate τ in (23) to balance the government budget.

12. Check convergence of a set of moments. If convergence has been achieved, stop. If convergence has not been achieved, go to step 2 and use the last values computed as guesses.

A.2 Average wages and productivities

Our computation of the equilibrium measures of workers in equations (A.1)–(A.5) involve only two groups of employed workers, e_{ll} and e_{hh}, but each of these groups needs to be subdivided when we compute average wages and productivities. For employed low-skilled workers, we need to single out those who gained employment after first having belonged to group u_{lh}, i.e., low-skilled unemployed workers who received high benefits b_h. In the first period of employment, those workers will earn a higher wage $p_{lh}^o(z) > p_{ll}^o(z) \geq p_{ll}(z)$. And even afterwards, namely until their first on-the-job productivity draw, those workers will on average continue to differ from other employed low-skilled workers because of their higher reservation productivity at the time they regained employment, $\underline{z}^o_{lh} > \underline{z}^o_{ll} \geq \underline{z}_{ll}$.

Let e'_{ll} denote the measure of unemployed low-skilled workers with high benefits who gain employment in each period (they are in their first period of employment):

$$e'_{ll} = (1-\rho^r)\lambda^w(\theta)(1-\nu^o_{lh})u_{lh}.$$

Let e''_{ll} be the measure of such low-skilled workers who remain employed with job tenures greater than one period and who have not yet experienced any on-the-job productivity draw:

$$\begin{aligned} e''_{ll} &= (1-\rho^r)(1-\rho^x)(1-\gamma^u)(1-\gamma^s)\left[e'_{ll}+e''_{ll}\right] \\ &= \frac{(1-\rho^r)(1-\rho^x)(1-\gamma^u)(1-\gamma^s)}{1-(1-\rho^r)(1-\rho^x)(1-\gamma^u)(1-\gamma^s)}\, e'_{ll}. \end{aligned}$$

Given these measures of workers, we can compute the average wage of all employed low-skilled workers and also their average productivity

$$\begin{aligned} \bar{p}_l &= \int_{\underline{z}^o_{lh}}^{\infty}\left[\frac{e'_{ll}}{e_{ll}}p^o_{lh}(y)+\frac{e''_{ll}}{e_{ll}}p_{ll}(y)\right]\frac{dv^o_l(y)}{1-v^o_l(\underline{z}^o_{lh})} + \frac{e_{ll}-e'_{ll}-e''_{ll}}{e_{ll}}\int_{\underline{z}_{ll}}^{\infty}p_{ll}(y)\,\frac{dv_l(y)}{1-v_l(\underline{z}_{ll})} \\ \bar{z}_l &= \frac{e'_{ll}+e''_{ll}}{e_{ll}}\int_{\underline{z}^o_{lh}}^{\infty}y\,\frac{dv^o_l(y)}{1-v^o_l(\underline{z}^o_{lh})} + \frac{e_{ll}-e'_{ll}-e''_{ll}}{e_{ll}}\int_{\underline{z}_{ll}}^{\infty}y\,\frac{dv_l(y)}{1-v_l(\underline{z}_{ll})}. \end{aligned}$$

For employed high-skilled workers, we need to single out those just hired from the group of unemployed high-skilled workers u_{hh} who earn a higher wage in their first period of employment, $p^o_{hh}(z) > p_{hh}(z)$. This is because they do not face the risk of quit turbulence if no wage agreement is reached and hence, no employment relationship is formed.

For the same reason discussed above, we also need to keep track of such workers until their first on-the-job productivity draw (or layoff or retirement, whatever comes first). Reasoning as we did earlier, let e'_{hh} and e''_{hh} denote these respective groups of employed high-skilled workers;

$$\begin{aligned} e'_{hh} &= (1-\rho^r)\lambda^w(\theta)(1-\nu^o_{hh})u_{hh} \\ e''_{hh} &= \frac{(1-\rho^r)(1-\rho^x)(1-\gamma^s)}{1-(1-\rho^r)(1-\rho^x)(1-\gamma^s)}\, e'_{hh}. \end{aligned}$$

Given these measures of workers, we can compute the average wage of all employed high-skilled workers and also their average productivity

$$\bar{p}_h = \int_{\underline{z}_{hh}^o}^{\infty} \left[\frac{e'_{hh}}{e_{hh}} p_{hh}^o(y) + \frac{e''_{hh}}{e_{hh}} p_{hh}(y) \right] \frac{dv_h^o(y)}{1 - v_h^o(\underline{z}_{hh}^o)} + \frac{e_{hh} - e'_{hh} - e''_{hh}}{e_{hh}} \int_{\underline{z}_{hh}}^{\infty} p_{hh}(y) \frac{dv_h(y)}{1 - v_h(\underline{z}_{hh})}$$

$$\bar{z}_h = \frac{e'_{hh} + e''_{hh}}{e_{hh}} \int_{\underline{z}_{hh}^o}^{\infty} y \frac{dv_h^o(y)}{1 - v_h^o(\underline{z}_{hh}^o)} + \frac{e_{hh} - e'_{hh} - e''_{hh}}{e_{hh}} \int_{\underline{z}_{hh}}^{\infty} y \frac{dv_h(y)}{1 - v_h(\underline{z}_{hh})}.$$