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Cross-phenomenon restrictions: Unemployment effects of layoff costs and quit turbulence ☆

Isaac Baley [a,b,∗], Lars Ljungqvist [c,d,b], Thomas J. Sargent [d,e]

[a] *Universitat Pompeu Fabra, Barcelona School of Economics, CREI, Spain*
[b] *CEPR, United Kingdom*
[c] *Stockholm School of Economics, Sweden*
[d] *New York University, United States of America*
[e] *Hoover Institution, United States of America*

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ABSTRACT

Cross-phenomenon restrictions associated with returns to labor mobility can inform calibrations of productivity processes in macro-labor models. We exploit how returns to labor mobility influence effects on equilibrium unemployment of changes in (a) layoff costs, and (b) distributions of skill losses coincident with quits ("quit turbulence"). Returns to labor mobility intermediate both effects. Ample labor reallocations observed across market economies that have different layoff costs imply that a turbulence explanation of trans-Atlantic unemployment experiences is robust to adding plausible quit turbulence.

> *"... often the most important constraint on a new theory is ... that it should agree with the whole body of past observations, as crystallized in former theories. ... New theories of course do not agree entirely with any previous theory – otherwise they would not be new – but they must not throw out all the success of former theories. This sort of thing makes the work of the theorist far more conservative than is often thought.*
>
> *"The wonderful thing is that the need to preserve the successes of the past is not only a constraint, but also a guide."* Steven Weinberg (2018, p. 197)

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∗ Corresponding author.
E-mail addresses: isaac.baley@upf.edu (I. Baley), lars.ljungqvist@hhs.se (L. Ljungqvist), thomas.sargent@nyu.edu (T.J. Sargent).

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1. Introduction

This paper illustrates benefits brought by the calibration approach, codified for macroeconomics by Cooley and Prescott (1995), that Tom Cooley used so well. We aspire to live up to high standards that Tom Cooley set when he practiced what he liked to call "economic science." We present a macro-labor economics application of principles stated by particle physicist Steven Weinberg in the epigraph above. Our research strategy combines essential ingredients of Tom Cooley's approach to research. We study (1) a substantive economic puzzle presented by a structural break in post World War II trans-Atlantic unemployment experiences and (2) an apparently successful turbulence explanation of it that (3) had been challenged for not being robust to a model perturbation that activated a force that the turbulence explanation had neglected. The challengers provided no direct evidence about that new channel that could help calibrate critical parameters. At this point, Steven Weinberg's rules help: we bring in another phenomenon that earlier macro-labor models had explained well and that is also tied to the challengers' new force. By using an associated "cross-phenomenon restriction" to calibrate critical parameters in the perturbed model, we can resolve the challengers' robustness challenge in favor of the original turbulence explanation of post-World War II differences in trans-Atlantic unemployment outcomes.

After Ljungqvist and Sargent (1998) had used an extended McCall (1970) search model to quantify adverse macroeconomic consequences coming from interactions between microeconomic turbulence and generous unemployment compensation in European welfare states,[1] two complementary studies added forces and phenomena that Ljungqvist and Sargent had excluded. Ljungqvist and Sargent (1998) modeled turbulence in terms of risks of human capital losses coincident with *involuntary* job losses ("layoff turbulence"). While their model explained the persistently higher unemployment rates observed in Europe since the late 1970s, it excluded losses of human capital coincident with *voluntary* separations from jobs. Neglect of such "quit turbulence" risk is the starting point of our story because in 1998 an astute observer, Alan Greenspan (1998, p. 743), suggested that a more hazardous job market had suppressed mobility among employed workers and had decreased upward pressures on wages:

> "... the sense of increasing skill obsolescence has also led to an apparent willingness on the part of employees to forgo wage and benefit increases for increased job security. Thus, despite the incredible tightness of labor markets, increases in compensation per hour have continued to be relatively modest."

Greenspan's words inspired den Haan et al. (2005, henceforth DHHR) to construct a Diamond-Mortensen-Pissarides matching model that they used to represent Greenspan's idea by including possible "quit turbulence" in the form of an immediate stochastic depreciation of a worker's human capital that in turbulent times would be triggered by a worker's decision to quit a job. DHHR's calibrated model implied that even small amounts of quit turbulence made workers reluctant to quit and consequently suppressed both quits and overall job reallocations, thereby reversing the unemployment-increasing interactions between turbulence and welfare state generosity that Ljungqvist and Sargent (1998) had used to explain trans-Atlantic differences in unemployment rates. DHHR's representation and calibration thus cast doubt on Ljungqvist and Sargent's inference that a rise in turbulence explains the outbreak of high European unemployment in the late 1970s.

What parts of DHHR's structure are responsible for reversing Ljungqvist and Sargent's inference about the interaction of heightened turbulence and trans-Atlantic differences in unemployment outcomes? Was it DHHR's adding quit turbulence in the form of skill deterioration risks brought by quitting? Or was it DHHR's decision to replace Ljungqvist and Sargent's extended McCall framework with their version of a Diamond-Mortensen-Pissarides matching model? Or was it something else, such as different calibrations of the processes governing productivity distributions and dynamics that are exogenous to both the DHHR model and the Ljungqvist and Sargent model?

Hornstein et al. (2005, section 8.3) suggested some answers. They accepted that DHHR's finding showed a lack of robustness of Ljungqvist and Sargent's explanation of those trans-Atlantic unemployment rate differences:

> "... once the Ljungqvist and Sargent mechanism is embedded into a model with endogenous job destruction, the comparative statics for increased turbulence are reversed, i.e., unemployment falls. The reason is that as the speed of skill obsolescence rises, workers become more reluctant to separate, and job destruction falls."

Hornstein et al. thus concluded that what had allowed DHHR to overturn the Ljungqvist and Sargent inference about how higher turbulence had affected Europe and America differently was Ljungqvist and Sargent's reliance on a model that had mostly excluded endogenous job separations.[2] To address that concern, in this paper we too adopt a Diamond-Mortensen-Pissarides matching model by using a version of Ljungqvist and Sargent (2007, henceforth LS) as our benchmark model. We

[1] Unlike the situations in particle physics and cosmology, there is no "standard model" of forces that shape an equilibrium unemployment rate. Each of three workable classes of models of frictional unemployment has persuasive advocates and skillful users: (1) matching models in the Diamond-Mortensen-Pissarides tradition; (2) equilibrium versions of McCall (1970) search models; and (3) search-island models in the tradition of Lucas and Prescott (1974). Calibrated versions of all three types of models have succeeded in fitting data on labor market flows and generating plausible responses of unemployment rates to government policies like generous unemployment insurance and layoff taxes.

[2] Learning-by-doing human capital accumulation induces endogenous job separations in the model of Ljungqvist and Sargent (1998). But besides exogenous layoffs, there are no on-the-job shocks to productivity per unit of human capital.

include quit turbulence. In contrast to DHHR, we find that plausible amounts of skill loss at times of voluntary quits have only small effects on outcomes: for quit turbulence to suppress unemployment, it has to be raised to become about 50% of layoff turbulence, not DDHR's 5%, and both kinds of turbulence must also be high.

The big disagreement between the matching model analysis of DHHR and an LS model augmented to incorporate quit turbulence comes from differences in returns to labor mobility that are implied by different widths of the productivity distributions calibrated by DHHR and LS.[3] The spread of the productivity distribution matters because of how it affects returns to labor mobility. Equilibrium returns to labor mobility must be suppressed markedly for the introduction of quit turbulence to be able to reverse the unemployment-increasing interactions between layoff turbulence and welfare state generosity featured by LS. It follows that evidence about returns to labor mobility sheds light on the potential impact of quit turbulence. Where might we find pertinent evidence?

Informative sources include the establishment data on firm and worker turnover assembled by Davis and Haltiwanger (1990), as well as similar data sets from other countries. Taken together they provide compelling evidence that extensive reallocations occur within different market economies that operate under a variety of government policies directed at influencing job separations, some heavy-handed, others light-handed. Central to the present paper is our insistence that calibrated labor productivity processes in macro-labor models have to imply high enough returns to labor mobility if they are to be consistent with the high reallocation rates across economies that have very different public policies toward restraining or promoting resource reallocations. Earlier models that have provided sufficiently high returns to labor mobility to do that despite large cross-economy differences in layoff costs include Alvarez and Veracierto (2001), Mortensen and Pissarides (1999), and Ljungqvist and Sargent (2008).

Taking our cue from the patterns studied in those papers, to infer quantitatively plausible returns to labor mobility, we exploit how they also shape effects on unemployment from the introduction of quit turbulence. Thus, we proceed by first inferring reasonable parameter values for productivity processes from a consensus view about quantitative effects on unemployment from imposing layoff costs. Then we study the associated potential impact of quit turbulence on the relationship between turbulence and unemployment.

Section 2 sets forth a matching model augmented to include DHHR's quit turbulence. The productivity process of LS brings high returns to labor mobility while DHHR's productivity process brings low returns. Section 3 studies effects on unemployment of layoff costs and quit turbulence. Their common dependence on returns to labor mobility ties together the magnitudes of effects of layoff costs and quit turbulence on unemployment. To highlight that link, Section 4 constructs mappings from the parameters of the productivity process to distinct outcome criteria for layoff costs and quit turbulence, respectively. The layoff tax criterion is a minimum layoff tax that serves to shut down all voluntary job separations when turbulence is absent. The quit-turbulence criterion is a minimum level of quit turbulence that suffices to turn the relationship between unemployment and turbulence from positive, as it is according to LS, to negative, as it is according to DHHR. As we vary the width of the productivity distribution and the on-the-job arrival rate of new productivity draws, the two criteria move together. The criteria reveal that in a parameter vicinity where substantial voluntary separations continue to occur under plausible layoff costs that can be inferred from observed cross-country outcomes, plausible amounts of quit turbulence do not reverse a positive relationship between unemployment and turbulence. Section 5 offers concluding remarks. Auxiliary materials appear in online Appendices.

2. A matching model with quit turbulence

Our benchmark is a standard matching model to which we add human capital dynamics that incorporate turbulence. It is a version of the LS (2007) matching model that represents layoff turbulence as more adverse skill transition probabilities for workers who suffer involuntary layoffs. We include DHHR quit turbulence in the form of adverse skill transition probabilities for workers who voluntarily quit.[4]

2.1. Environment

Workers There is a unit mass of workers who are either employed or unemployed. Workers are risk neutral and rank consumption streams according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_t, \tag{1}$$

where $\beta \equiv \hat{\beta}(1-\rho^r)$, $\hat{\beta} \in (0,1)$ is a subjective time discount factor, and $\rho^r \in (0,1)$ is a constant probability of retirement. Retired workers exit the economy and are replaced by equal measures of newborn workers.

[3] This finding illustrates an assertion of Baley et al. (2023) that "returns to labor mobility have too often escaped the attention they deserve as conduits of important forces in macro-labor models." In this paper, we shall use a cross-phenomenon restriction to calibrate those returns.

[4] LS thanked Wouter den Haan, Christian Haefke, and Garey Ramey for generously sharing computer code that LS then modified. Much of our notation and mathematics follow DHHR closely. For an account of differences between the models of LS and DHHR, see Appendix B.

Worker heterogeneity Besides employment status, workers differ along two dimensions: a current skill level i that can be either low (l) or high (h) and an administrative skill level j that determines a worker's entitlement to unemployment benefits. An employed worker has $j = i$; but for an unemployed worker, j is her skill level during her last employment spell. Workers gain or lose skills with transition probabilities that depend on their employment status and instances of layoffs and quits. We assume that all newborn workers enter the labor force with low skills and a low benefit entitlement. Thus, each worker carries along two indices (i, j), the first denoting current skill and the second denoting benefit entitlement.

Firms and matching technology There is free entry of firms who can post vacancies at a cost μ per period. Aggregate numbers of unemployed u and vacancies v are inputs into an increasing, concave and linearly homogeneous matching function $M(v, u)$. Let $\theta \equiv v/u$ be the vacancy-unemployment ratio, also called market tightness. The probability $\lambda^w(\theta) = M(v, u)/u = M(\theta, 1) \equiv m(\theta)$ that an unemployed worker encounters a vacancy is increasing in market tightness. The probability $M(v, u)/v = m(\theta)/\theta$ that a vacancy encounters an unemployed worker is decreasing in market tightness.

Worker-firm relationships and productivity processes A job opportunity is a productivity draw z from a cumulative distribution function $v_i^o(z)$ that is indexed by a worker's skill level i. We assume that the high-skill distribution first-order stochastically dominates the low-skill distribution: $v_h^o(z) \leq v_l^o(z)$. Wages are set through Nash bargaining, with π and $1 - \pi$ as the bargaining weights of a worker and a firm, respectively.

Idiosyncratic shocks within a worker-firm match determine an employed worker's productivities. Productivity in an ongoing job is governed by a first-order Markov process with a transition probability matrix Q_i, also indexed by the worker's skill level i, where $Q_i(z, z')$ is the probability that next period's productivity becomes z', given current productivity z. Specifically, an employed worker retains her last period productivity with probability $1 - \gamma^s$, but with probability γ^s draws a new productivity from the distribution $v_i(z)$. As in the case of the productivity distributions for new matches, the high-skill distribution in ongoing jobs first-order stochastically dominates the low-skill distribution: $v_h(z) \leq v_l(z)$. Furthermore, an employed worker's skills may get upgraded from low to high with probability γ^u. A skill upgrade is accompanied by a new productivity drawn from the high-skill distribution $v_h(z)$. A skill upgrade is realized immediately, regardless of whether the worker remains with her present employer or quits.

We can now define our notions of layoffs and quits.

(i) **Layoffs:** At the beginning of each period, a job is exogenously terminated with probability ρ^x. We call this event a layoff. An alternative interpretation of the job-termination probability ρ^x is that productivity z becomes zero and stays zero forever. A layoff is involuntary.
(ii) **Quits:** After any new on-the-job productivity draw and any skill upgrade, a relationship can continue or be endogenously terminated. We call a separation after such events a voluntary quit because a firm and a worker agree to separate after Nash bargaining.

Turbulence We define layoff and quit varieties of turbulence in terms of risks of losing skills at times of job separations. When a high-skilled worker is laid off, she becomes a low-skilled worker with probability γ^ℓ. We call this risk *layoff turbulence*. When a high-skilled worker *quits*, she becomes a low skilled worker with probability γ^q. We call this risk *quit turbulence*.

At the beginning of a period, exogenous job terminations occur and displaced high-skilled workers face layoff-turbulence risk. Continuing employed workers might receive new on-the-job productivity draws and they might also receive skill upgrades. High-skilled workers face quit turbulence risk whenever they quit. All separated workers join other unemployed workers and wait in the matching function before they have chances to encounter vacancies next period.

Government policy The government provides unemployment compensation. An unemployed worker who was low (high) skilled in her last employment receives a benefit b_l (b_h).[5] Unemployment benefit b_i is calculated as a fraction ϕ of the average wage of employed workers with skill level i. The government imposes a layoff tax Ω on every job termination except for retirements.

The government levies a flat-rate tax τ on production and runs a balanced budget. If layoff tax revenues fully cover payments of unemployment benefits, the government sets $\tau = 0$ and returns the surplus as lump-sum transfers to workers. Since surpluses typically don't arise in our analyses, we choose to omit such lump-sum transfers in various equations below.[6]

[5] As mentioned above, newborn workers are entitled to b_l. Also, for simplicity, we assume that a worker who receives a skill upgrade and chooses to quit, is entitled to high benefits.

[6] The exceptional case in which a government surplus has to be returned to workers as lump-sum transfers is described in our reference in Section 3.2 to a layoff tax analysis in a version of the LS economy without unemployment benefits.

2.2. Match surpluses

A match between a firm and a worker with skill level i and benefit entitlement j that has drawn productivity z will form an employment relationship, or continue an existing one, if a match surplus is positive. The match surplus for a new job $s^o_{ij}(z)$ or a continuing job $s_{ij}(z)$ is given by after-tax productivity $(1-\tau)z$ plus the future joint continuation value $g_i(z)$ minus the outside values of the match that consist of the worker's receiving unemployment benefit b_j and a future value ω^w_{ij} associated with entering the unemployment pool in the current period; and the firm's value ω^f from entering the vacancy pool in the current period, net of paying the vacancy cost μ. Define $\omega_{ij} \equiv \omega^w_{ij} + \omega^f$.

The match surplus $s^o_{lj}(z)$ for a new job or $s_{lj}(z)$ for a continuing job for a low-skilled worker with benefit entitlement j equal to

$$s^o_{lj}(z) \;=\; s_{lj}(z) \;=\; (1-\tau)z + g_l(z) - [b_j + \omega_{lj}], \qquad j = l, h. \tag{2}$$

To compute the match surplus for jobs with high-skilled workers, we distinguish between new and continuing jobs. The match surplus s^o_{hh} for forming a new job with an unemployed high-skilled worker involves outside values without risk of skill loss if the match does not result in employment is

$$s^o_{hh}(z) \;=\; (1-\tau)z + g_h(z) - [b_h + \omega_{hh}]. \tag{3}$$

In contrast, the match surplus for a continuing job with a high-skilled worker or for a job with an earlier low-skilled worker who gets a skill upgrade that is immediately realized involves quit turbulence:

$$s_{hh}(z) \;=\; (1-\tau)z + g_h(z) - [b_h + \underbrace{(1-\gamma^q)\omega_{hh} + \gamma^q \omega_{lh}}_{\text{quit turbulence}}]. \tag{4}$$

Reservation productivities and rejection rates A worker and a firm split the match surplus through Nash bargaining with outside values as threat points. The splitting of match surpluses ensures mutual agreement whether to start (continue) a job. For a new (continuing) match, the reservation productivity $\underline{z}^o_{ij}$ $(\underline{z}_{ij})$ is the lowest productivity that makes a match profitable and satisfies

$$s^o_{ij}(\underline{z}^o_{ij}) \;=\; 0 \qquad \Big(s_{ij}(\underline{z}_{ij}) \;=\; -\Omega\Big). \tag{5}$$

Note that in a continuing match, the surplus must fall to the negative of the layoff tax before a job is terminated.

Given the reservation productivity $\underline{z}^o_{ij}$ $(\underline{z}_{ij})$, let ν^o_{ij} (ν_{ij}) denote the rejection probability, which is given by the probability mass assigned to all draws from productivity distribution $v^o_i(y)$ $(v_i(y))$ that fall below a threshold:

$$\nu^o_{ij} \;=\; \int_{-\infty}^{\underline{z}^o_{ij}} dv^o_i(y) \qquad \left(\nu_{ij} \;=\; \int_{-\infty}^{\underline{z}_{ij}} dv_i(y)\right). \tag{6}$$

Define

$$E_{ij} \equiv \int_{\underline{z}_{ij}}^{\infty} [(1-\tau)y + g_i(y)]\, dv_i(y). \tag{7}$$

2.3. Joint continuation values

Consider a match between a firm and a worker with skill level i. Given a current productivity z, $g_i(z)$ is the joint continuation value of the associated match. We now describe value functions for low- and high-skilled workers.

High-skilled worker The joint continuation value of a match of a firm with a high-skilled worker with current productivity z, denoted $g_h(z)$, is affected by prospects of future layoff turbulence if by chance the worker is laid off and of by chance experiencing future quit turbulence should the worker decide to quit after receiving an unacceptable on-the-job new productivity draw:

$$\begin{aligned}
&\text{Exogenous separation:} \qquad && g_h(z) = \beta\Big[\rho^x(b_h + \underbrace{(1-\gamma^\ell)\omega_{hh} + \gamma^\ell \omega_{lh}}_{\text{layoff turbulence}})\\
&\text{Productivity switch:} && \quad + (1-\rho^x)\gamma^s(E_{hh} + \nu_{hh}(b_h + \underbrace{(1-\gamma^q)\omega_{hh} + \gamma^q \omega_{lh}}_{\text{quit turbulence}}))
\end{aligned}$$

$$\text{Status quo:} \qquad + (1-\rho^x)(1-\gamma^s)((1-\tau)z + g_h(z))\Big]. \tag{8}$$

Low-skilled worker The joint continuation value of a firm match with a low-skilled worker accounts for these contingencies: no changes in productivity or skills, an exogenous separation, a productivity switch, and a skill upgrade. When a skill upgrade occurs, even if the worker chooses to quit, the worker immediately becomes entitled to high unemployment benefits. Furthermore, a skill upgrade coincides with a new draw from the high-skill productivity distribution ν_h. Thus, the joint continuation value of a match between a firm and a low-skilled worker with current productivity z, denoted by $g_l(z)$, is

$$\begin{aligned}
&\text{Exogenous separation:} && g_l(z) = \beta\Big[\rho^x(b_l + \omega_{ll}) \\
&\text{Immediate skill upgrade:} && \quad + (1-\rho^x)\gamma^u(E_{hh} + \nu_{hh}(b_h + \underbrace{(1-\gamma^q)\omega_{hh} + \gamma^q\omega_{lh}}_{\text{quit turbulence}})) \\
&\text{Productivity switch:} && \quad + (1-\rho^x)(1-\gamma^u)\gamma^s(E_{ll} + \nu_{ll}(b_l + \omega_{ll})) \\
&\text{Status quo:} && \quad + (1-\rho^x)(1-\gamma^u)(1-\gamma^s)((1-\tau)z + g_l(z))\Big].
\end{aligned} \tag{9}$$

2.4. Outside values

Value of unemployment An unemployed worker with current skill level i and benefit entitlement j receives benefits b_j and has a future value ω_{ij}^w. Recall that the probability that an unemployed worker becomes matched next period is $\lambda^w(\theta)$.

A low-skilled unemployed worker with benefit entitlement j obtains $b_j + \omega_{lj}^w$, where

$$\omega_{lj}^w = \beta\Big[\underbrace{\lambda^w(\theta)\int_{\underline{z}_{lj}^o}^{\infty} \pi s_{lj}^o(y)\, d\nu_l^o(y)}_{\text{match + accept}} + \underbrace{b_j + \omega_{lj}^w}_{\text{outside value}}\Big] \qquad j = l, h. \tag{10}$$

A high-skilled unemployed worker with benefit entitlement h obtains $b_h + \omega_{hh}^w$, where

$$\omega_{hh}^w = \beta\Big[\underbrace{\lambda^w(\theta)\int_{\underline{z}_{hh}^o}^{\infty} \pi s_{hh}^o(y)\, d\nu_h^o(y)}_{\text{match + accept}} + \underbrace{b_h + \omega_{hh}^w}_{\text{outside value}}\Big]. \tag{11}$$

Value of a vacancy A firm that searches for a worker pays an upfront cost μ to enter the vacancy pool and thereby obtains a fraction $(1-\pi)$ of the match surplus if an employment relationship is formed next period. Let $\lambda_{ij}^f(\theta)$ be the probability of filling the vacancy with an unemployed worker of type (i, j). Then a firm's value ω^f of entering the vacancy pool is:

$$\omega^f = -\mu + \beta\Big[\underbrace{\sum_{(i,j)} \lambda_{ij}^f(\theta)\int_{\underline{z}_{ij}^o}^{\infty} (1-\pi)s_{ij}^o(y)\, d\nu_i^o(y)}_{\text{match + accept}} + \underbrace{\omega^f}_{\text{outside value}}\Big]. \tag{12}$$

2.5. Market tightness and matching probabilities

Let u_{ij} be the number of unemployed workers with current skill i and benefit entitlement j. The total number of unemployed workers is $u = \sum_{i,j} u_{ij}$. The probability $\lambda^w(\theta)$ that an unemployed worker encounters a vacancy depends only on market tightness θ; the probability $\lambda_{ij}^f(\theta)$ that a vacancy encounters an unemployed worker with skill level i and benefit entitlement j also depends on the mix of workers in the unemployment pool. Free entry of firms implies that a firm's expected value of posting a vacancy is zero. Equilibrium market tightness can be inferred from equation (12) with $w^f = 0$. In summary, labor market outcomes are:

$$\omega^f = 0 \tag{13}$$

$$\mu = \beta(1-\pi)\sum_{(i,j)} \lambda_{ij}^f(\theta)\int_{\underline{z}_{ij}^o}^{\infty} s_{ij}^o(y)\, d\nu_i^o(y) \tag{14}$$

$$\lambda^w(\theta) = m(\theta) \tag{15}$$
$$\lambda_{ij}^f(\theta) = \frac{m(\theta)}{\theta}\frac{u_{ij}}{u}. \tag{16}$$

2.6. Wages

We assume Nash bargaining between a worker and a firm, each getting a share of the match surplus every period.[7] Given a productivity draw z in a new match with a positive match surplus, wage $p_{lj}^o(z)$ of a low-skilled worker with benefit entitlement $j = l, h$ and wage $p_{hh}^o(z)$ of a high-skilled worker, respectively, solve

$$\max_{p_{lj}^o(z)} \left[(1-\tau)z - p_{lj}^o(z) + g_l^f(z) - \omega^f\right]^{1-\pi} \left[p_{lj}^o(z) + g_l^w(z) - b_j - \omega_{lj}^w\right]^{\pi} \tag{17}$$
$$\max_{p_{hh}^o(z)} \left[(1-\tau)z - p_{hh}^o(z) + g_h^f(z) - \omega^f\right]^{1-\pi} \left[p_{hh}^o(z) + g_h^w(z) - b_h - \omega_{hh}^w\right]^{\pi},$$

where $g_i^w(z)$ and $g_i^f(z)$ are future values obtained by the worker and the firm, respectively, from continuing an employment relationship[8]; and ω^f and $b_j + \omega_{ij}^w$ are outside values defined in (10), (11), and (12). Solutions to these wage determination problems set the sum of the worker's wage and continuation value equal to the worker's share π of the match surplus plus her outside value:

$$p_{lj}^o(z) + g_l^w(z) = \pi s_{lj}^o(z) + b_j + \omega_{lj}^w \qquad j = l, h \tag{18}$$
$$p_{hh}^o(z) + g_h^w(z) = \pi s_{hh}^o(z) + b_h + \omega_{hh}^w,$$

where worker continuation values are

$$g_l^w(z) = \beta(1-\rho^x)\pi\left\{(1-\gamma^u)\left[(1-\gamma^s)s_{ll}(z) + \gamma^s\int_{\underline{z}_{ll}}^{\infty} s_{ll}(y)\,dv_l(y)\right] + \gamma^u\int_{\underline{z}_{hh}}^{\infty} s_{hh}(y)\,dv_h(y)\right\}$$
$$+\beta(\rho^x + (1-\rho^x)(1-\gamma^u))\left(b_l + \omega_{ll}^w\right) + \beta(1-\rho^x)\gamma^u\left(b_h + (1-\gamma^q)\omega_{hh}^w + \gamma^q\omega_{lh}^w\right) \tag{19}$$
$$g_h^w(z) = \beta(1-\rho^x)\pi\left[(1-\gamma^s)s_{hh}(z) + \gamma^s\int_{\underline{z}_{hh}}^{\infty} s_{hh}(y)\,dv_h(y)\right]$$
$$+\beta\rho^x\left(b_h + (1-\gamma^\ell)\omega_{hh}^w + \gamma^\ell\omega_{lh}^w\right) + \beta(1-\rho^x)\left(b_h + (1-\gamma^q)\omega_{hh}^w + \gamma^q\omega_{lh}^w\right).$$

For ongoing employments, wages $p_{ll}(z)$ and $p_{hh}(z)$ satisfy counterparts of the above equations that use appropriate match surpluses $s_{ll}(z)$ and $s_{hh}(z)$:

$$p_{ll}(z) + g_l^w(z) = \pi s_{ll}(z) + b_l + \omega_{ll}^w \tag{20}$$
$$p_{hh}(z) + g_h^w(z) = \pi s_{hh}(z) + b_h + \underbrace{(1-\gamma^q)\omega_{hh}^w + \gamma^q\omega_{lh}^w}_{\text{quit turbulence}},$$

where the latter expression for the high-skilled wage now involves quit turbulence on the right side.

2.7. Government budget constraint

Unemployment benefits Benefit entitlement j awards an unemployed worker benefit b_j equal to a fraction ϕ of the average wage $\bar{p}_j$ of employed workers with skill level j. Therefore, total government expenditure on unemployment benefits is

$$b_l u_{ll} + b_h(u_{lh} + u_{hh}) = \phi(\bar{p}_l u_{ll} + \bar{p}_h(u_{lh} + u_{hh})). \tag{21}$$

[7] Nash bargaining implies that workers pay part of the layoff tax upon a job separation. An alternative assumption is that once a worker is hired, firms are the only ones liable for the layoff tax. That generates a two-tier wage system à la Mortensen and Pissarides (1999). Risk-neutral firms and workers would be indifferent between adhering to period-by-period Nash bargaining or a two-tier wage system. Ljungqvist (2002) showed that the wage profile, but not the allocation, is affected by the two-tier wage system. Match surpluses, reservation productivities, and market tightness remain the same. Under the two-tier wage system, an initial wage concession by a newly hired worker is equivalent to her posting a bond that equals her share of a future layoff tax.

[8] *Joint* continuation values defined in (8) and (9) equal sums of the individual continuation values: $g_i(z) = g_i^w(z) + g_i^f(z)$, $i = l, h$.

Layoff taxes The measure Ξ of total separations excluding retirements equals

$$\Xi = (1-\rho^r)\Big[\rho^x(e_{ll}+e_{hh}) + (1-\rho^x)[(1-\gamma^u)\gamma^s \nu_{ll} + \gamma^u \nu_{hh}]e_{ll} + (1-\rho^x)\gamma^s \nu_{hh} e_{hh}\Big]. \tag{22}$$

Then government revenue from layoff taxation equals $\Omega\,\Xi$.

Income taxes Output is taxed at a constant rate τ. Where $\bar{z}_i$ is average productivity of employed workers with skill level i, total tax revenue equals $\tau(\bar{z}_l e_{ll} + \bar{z}_h e_{hh})$, where e_{ll} (e_{hh}) is the number of employed workers with low skills and low benefit entitlement (high skills and high benefit entitlement).

Balanced budget The government runs a balanced budget. The tax rate τ on output is set to cover the expenditures on unemployment benefits described in (21) net of layoff tax revenues $\Omega\,\Xi$:

$$\phi(\bar{p}_l u_{ll} + \bar{p}_h(u_{lh}+u_{hh})) - \Omega\,\Xi = \tau(\bar{z}_l e_{ll} + \bar{z}_h e_{hh}). \tag{23}$$

Calculations of average wages $\bar{p}_i$ and average productivities $\bar{z}_i$ appear in Appendix A.2.

2.8. Worker flows

Workers move across employment and unemployment states, skill levels, and benefit entitlements. Here we focus on low-skilled unemployed with high benefits, workers at the center of our analysis. (Appendix A.1 describes flows for other groups of workers.)

Inflows to the pool of low-skilled unemployed with high benefits u_{lh} come from the following sources. Layoff turbulence affects high-skilled workers e_{hh} who get laid off; with probability γ^ℓ, they become low-skilled unemployed workers entitled to high unemployment benefits. Quit turbulence affects high-skilled workers e_{hh} who reject productivity switches, as well as low-skilled workers e_{ll} who get skill upgrades and then reject their new productivity draws. All of those quitters face probability γ^q of entering the pool of low-skilled unemployed workers entitled to high unemployment benefits. Outflows from unemployment coincide with successful matches and retirements. Thus, the net change of low-skilled unemployed with high benefits (equaling zero in a steady state) is

$$\Delta u_{lh} = (1-\rho^r)\Bigg\{\underbrace{\rho^x\gamma^\ell e_{hh}}_{\text{1. layoff turbulence}} + \underbrace{(1-\rho^x)\gamma^q \nu_{hh}[\gamma^s e_{hh} + \gamma^u e_{ll}]}_{\text{2. quit turbulence}} \\ \underbrace{-\lambda^w(\theta)(1-\nu^o_{lh})u_{lh}}_{\text{3. successful matches}}\Bigg\} - \rho^r u_{lh}. \tag{24}$$

Terms numbered 1 and 3 in expression (24) identify sources of a positive layoff-turbulence, unemployment relationship in a welfare state in the LS model. Although more layoff turbulence in term 1 – a higher probability γ^ℓ of losing skills after layoffs – has a small effect on equilibrium unemployment in a “laissez-faire” environment in which $\phi = 0, \Omega = 0$, it provokes a strong turbulence-unemployment relationship in a welfare state that offers a generous unemployment benefit replacement rate for a worker’s earnings in her last job. After a layoff with skill loss, those benefits are high relative to a worker’s earnings prospects at her now diminished skill level. As a consequence, the acceptance rate $(1-\nu^o_{lh})$ in term 3 is low; the relatively high outside value of a low-skilled unemployed with high benefits implies that fewer matches have positive match surpluses, as reflected in a high reservation productivity z^o_{lh}. Moreover, given those suppressed match surpluses, equilibrium market tightness θ falls to restore firm profitability enough to make vacancy creation break even. Lower market tightness, in turn, reduces the probability $\lambda^w(\theta)$ that a worker encounters a vacancy, which further decreases the fraction of successful matches and thereby contributes to a positive layoff-turbulence, unemployment relationship.

Presence of quit turbulence adds the term numbered 2 in expression (24). On the one hand, an additional source of turbulence $\gamma^q > 0$ can further increase the equilibrium unemployment rate since there is one more channel for high-skilled workers to lose skills and become low-skilled unemployed with high benefits. On the other hand, quit turbulence also exerts a countervailing force that could attenuate or even reverse a positive turbulence-unemployment relationship. When voluntary quits are also subject to risks of skill loss, there will be fewer voluntary quits in turbulent times; exposing themselves to a risk of skill loss makes high-skilled workers more reluctant to quit, lowering the rejection rate ν_{hh} in term 2. That lower rejection rate causes lower inflows u_{lh} into the pool of low-skilled unemployed who are entitled to high benefits as well as inflows u_{hh} into the pool of high-skilled unemployed who are entitled to high unemployment benefits. This is the force activated by DHHR to reverse a positive turbulence-unemployment relationship.

Table 1
Parameterization of benchmark model.

Parameter	Definition	Value
Preferences		
$\hat{\beta}$	discount factor	0.99425
ρ^r	retirement probability	0.0031
$\beta = \hat{\beta}(1-\rho^r)$	adjusted discount	0.991
Sources of risk		
ρ^x	exogenous breakup probability	0.005
γ^u	skill upgrade probability	0.0125
γ^s	productivity switch probability	0.05
γ^ℓ	layoff turbulence	[0, 1]
$\gamma^q = \epsilon\gamma^\ell$	quit turbulence	$\epsilon \in [0, 1]$
Labor market institutions		
π	worker bargaining power	0.5
ϕ	replacement rate	0.7
Ω	layoff tax	0
Matching function		
A	matching efficiency	0.45
α	elasticity of matches w.r.t. u	0.5
μ	cost of posting a vacancy	0.5

2.9. Steady state equilibria

A steady state equilibrium consists of measures u_{ij} of unemployed workers and e_{ij} employed workers; labor market tightness θ, probabilities $\lambda^w(\theta)$ that workers encounter vacancies and $\lambda^f_{ij}(\theta)$ that vacancies encounter workers; reservation productivities $\underline{z}^o_{ij}, \underline{z}_{ij}$, match surpluses $s^o_{ij}(z), s_{ij}(z)$, future values of an unemployed worker ω^w_{ij} and of a firm posting a vacancy ω^f; wages $p^o_{ij}(z), p_{ij}(z)$; unemployment benefits b_i and a tax rate τ; such that

a) Match surplus conditions (5) determine reservation productivities.
b) Free entry of firms implies zero-profit condition (14) in vacancy creation that pins down market tightness.
c) Nash bargaining outcomes (18) and (20) set wages.
d) The tax rate balances the government's budget (23).
e) Net worker flows, such as expression (24), are all equal to zero: $\Delta u_{ij} = \Delta e_{ij} = 0, \quad \forall i, j$.

2.10. Parameterization

Apart from considering alternative assumptions about the productivity process and different values of the layoff tax, the benchmark model shares the remaining parameterization with LS, in conjunction with DHHR's codification of quit turbulence, as reported in Table 1.[9] The model period is half a quarter.

Preference parameters In light of our semi-quarterly model with its eight periods per year, we specify a discount factor $\hat{\beta} = 0.99425$ and a retirement probability $\rho^r = 0.0031$, which together imply an adjusted discount of $\beta = \hat{\beta}(1-\rho^r) = 0.991$. The retirement probability implies an average time of 40 years in the labor force.

Stochastic processes for productivity Exogenous layoffs occur with probability $\rho^x = 0.005$, on average a layoff every 25 years. We set a probability of upgrading skills $\gamma^u = 0.0125$ so that it takes on average 10 years to move from low to high skill, conditional on no job loss. The probability of a productivity switch on the job equals $\gamma^s = 0.05$, so a worker expects to retain her productivity for 2.5 years.

Layoff and quit turbulence Following DHHR, we parameterize quit turbulence as a fraction ϵ of layoff turbulence so that $\gamma^q = \epsilon\gamma^\ell$. We vary ϵ from zero – denoting complete absence of quit turbulence – to one – in which case layoff and quit turbulence risks are equal.

Labor market institutions We set a worker's bargaining power to be $\pi = 0.5$. We set the replacement rate in unemployment compensation at $\phi = 0.7$ and initially set the layoff tax $\Omega = 0$. When we study the effects of layoff taxes on unemployment in section 4, we'll set $\Omega > 0$.

[9] Subject to the caveat of DHHR assuming a fixed population of firms of the same measure as that of workers and hence, an exogenous market tightness equal to 1, the remaining parameterization in Table 1 is identical or similar to that of DHHR. For a detailed account, see Appendix B.

Table 2
Productivity distributions of LS and DHHR.

Properties	Original model		Benchmark model version	
	LS	DHHR	LS	DHHR
Functional form, $v_i(z)$	Normal	Uniform	Uniform	Uniform
Mean, low-skilled	1	1	1	1
high-skilled	2	2	2	2
Width of support	4	1	2.25	0.6
Standard deviation	1	0.289	0.650	0.173

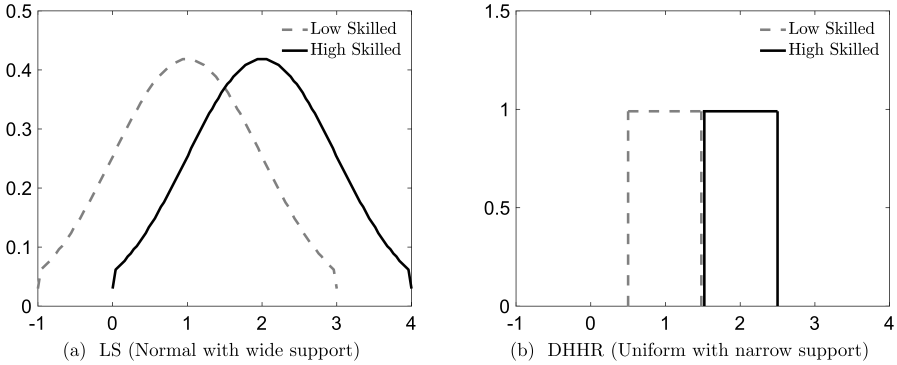


Fig. 1. Productivity distributions of LS and DHHR.

Matching We assume a Cobb-Douglas matching function $M(v,u) = Au^{\alpha} v^{1-\alpha}$, which implies that the probability that a worker encounters a vacancy and that the probability that a vacancy encounters a worker of a particular type, respectively, are:

$$\lambda^w(\theta) = A\theta^{1-\alpha}, \qquad \lambda^f_{ij}(\theta) = A\theta^{-\alpha}\frac{u_{ij}}{u}. \tag{25}$$

The elasticity of matches with respect to unemployment is $\alpha = 0.5$, in line with a consensus that plausible values fall in the mid range of the unit interval (e.g., see Petrongolo and Pissarides (2001)). We adopt LS's parameterization of the matching efficiency $A = 0.45$ and the cost of posting a vacancy $\mu = 0.5$.

3. High (LS) and low (DHHR) returns to labor mobility

This section describes implications for returns to labor mobility of disparate productivity distributions calibrated by LS and DHHR. We start with two models, one representing LS's specification, the other representing DHHR's. In subsection 3.4 we proceed to project each of these models into a common benchmark model and verify that projected versions of the two models do good jobs of representing outcomes in the original models. We then use two calibrations of the productivity distribution in the benchmark model to isolate their effects on outcomes.

Parameterizations of LS and DHHR reported in the first two columns of Table 2 and depicted in Fig. 1 provide examples of different productivity distributions that imply different returns to labor mobility. LS and DHHR both assume that productivity distributions are the same for new and ongoing matches, so that $v^o_i(z) = v_i(z)$. LS parameterize truncated normal distributions in Fig. 1a whereas DHHR in Fig. 1b assume uniform distributions with narrow ranges.[10] These probability distributions imply different returns to labor mobility that in turn affect how much equilibrium unemployment respond to changes in either layoff taxes or in quit turbulence. As indicated in the previous paragraph, we start by studying these effects in the original models of LS and DHHR. Then we map each of their productivity processes into uniform distributions within our benchmark model, a model-projection exercise that allows us in Section 4 to characterize LS and DHHR versions of our benchmark model that differ only in the widths of their uniform productivity distributions. This machinery lets us bring out implications of the disparate LS and DHHR productivity distributions for (1) effects of layoff taxes on unemployment, and (2) effects of quit turbulence on unemployment.

[10] LS incorrectly implemented the quadrature method at the truncation points of the normal distributions; nevertheless, the constructed distributions are still proper. Therefore, instead of recalibrating the LS model under a correct implementation of the quadrature method, we have chosen for reasons of comparability to retain the distributions presented in the published LS analysis.

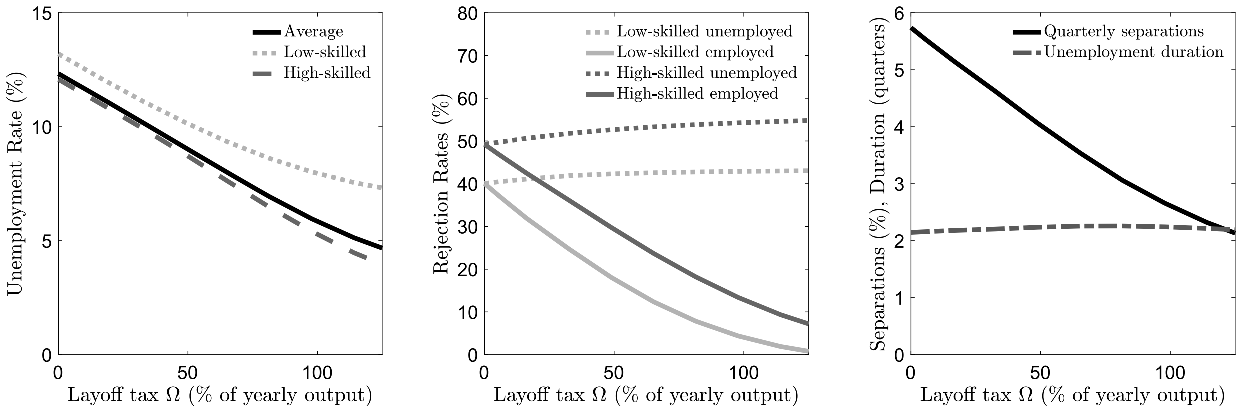


Fig. 2. Layoff taxes in LS when $\phi = 0.7$ and $\gamma^{\ell} = 0$.

3.1. The LS and DHHR models

We can obtain the LS original model by simply importing the LS productivity distributions into our benchmark model. What we refer to as the DHHR model is their original framework, except for two modifications that, although they facilitate our way of mapping DHHR into our benchmark model, do not alter outcomes substantially.[11] We verify and extend DHHR's finding that with their narrow distribution of productivities, small amounts of quit turbulence reverse the Ljungqvist-Sargent unemployment-increasing interactions between turbulence and welfare state generosity, but that this does not happen with LS's wider productivity distribution. We'll eventually see that this difference in outcomes is a tell tale sign of differences in the returns to labor mobility that come from different widths of productivity distributions.

Next, we map the LS and DHHR models into our benchmark model under the assumption of uniform distributions. For the LS. model, this is just a matter of converting LS's truncated normal distributions into uniform distributions. For DHHR, things are more complicated because their matching framework differs from our benchmark model in two ways that, for our purposes, are inconsequential.[12] In the end, mapping DHHR into our benchmark model only requires transforming DHHR's productivity distributions. So we calibrate the widths of the uniform distributions in our benchmark model to generate unemployment effects of quit turbulence like those in our analyses of the LS and DHHR models. It turns out that effects of layoff taxes on unemployment for each such calibration of the benchmark model aligns with outcomes in the corresponding analyses of the LS and DHHR model, an alignment that reflects the cross-phenomenon restriction featured in Section 4.

3.2. Layoff taxes

Layoff taxes in LS In the tranquil zero-turbulence times $\gamma^{\ell} = 0$ LS model, Fig. 2 shows unemployment and rejection rates of various types of workers, as well as aggregate labor flows, as functions of the layoff tax Ω expressed as a fraction of the average yearly output per worker in a $\phi = 0, \Omega = 0$ "laissez-faire" economy.[13] As the layoff tax increases, the unemployment rate falls (left panel) due primarily to a decline in endogenous separations (right panel). The rejection rates plotted in the middle panel refer to the arrival rate of new on-the-job draws of z that prompt employed workers to quit (solid lines) and the draws of z in new job offers rejected by unemployed workers (dotted lines), for both skill levels. Raising the layoff tax causes rejection rates of both high-skilled and low-skilled employed workers to fall markedly. But even at pretty substantial layoff taxes, these workers still remain mobile. Thus, if the layoff tax reaches the average annual output of a worker $\Omega = 100\%$, employed high-skilled workers reject about 12% of offers.

Incidentally, Fig. 2 expresses forces that LS used to explain why, before the arrival of layoff turbulence, a welfare state with generous unemployment can actually have *lower* unemployment than a $\phi = 0, \Omega = 0$ laissez-faire economy (also see Mortensen and Pissarides (1999)). Thus, despite the generous Table 1 unemployment benefit replacement rate of $\phi = 0.7$,

[11] Our first modification is that instead of the zero benefits that they receive in the original DHHR setup, we assume that newborn workers are eligible for the same unemployment benefits as low-skilled workers. The second modification concerns the risk of losing skills following unsuccessful job market encounters. As a "simplifying assumption," DHHR assume that after an encounter between a firm and an unemployed worker that does not result in an employment relationship, the worker faces the same risk of losing skills as she would after quitting a job; an added risk that we omit. For an assessment of these alternative assumptions, see Appendix D.

[12] As described in Appendix B, these structural differences pertain to i) how vacancies are created, and ii) how the capital gain from a skill upgrade is split between firm and worker. To show that among these two differences and the parameterization of productivity distributions it is the latter one that is the sole important source for how unemployment responds to quit turbulence, we proceed as follows. Appendix C starts with the benchmark model with LS productivity distributions and outcomes as depicted in Fig. 4a, and then successively perturbs the three potential sources one by one, to see which one brings us closest to outcomes in the DHHR model in Fig. 4b. In Appendix D, we start from the DHHR model in Fig. 4b and work through the perturbations in reverse. Both procedures detect productivity distributions as being the critical source for differences in outcomes.

[13] In the LS laissez-faire economy with $\phi = 0, \Omega = 0$, a worker's average semi-quarterly output is 2.3 goods in tranquil zero-turbulence times.

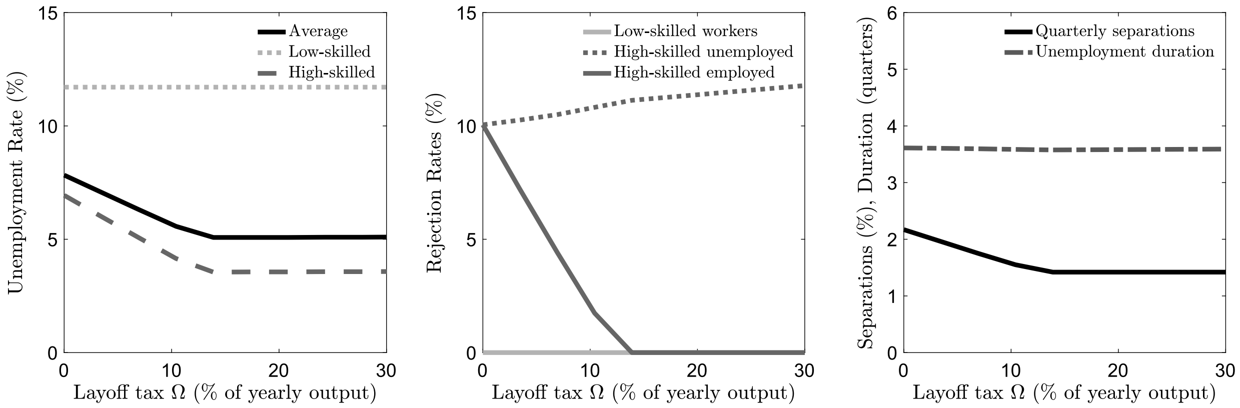


Fig. 3. Layoff taxes in DHHR when $\phi = 0.7$ and $\gamma^\ell = 0$.

the left panel of Fig. 2 shows that sufficiently high layoff taxes cause unemployment to fall below its 5% rate in the laissez-faire $\phi = 0, \Omega = 0$ economy.

For later use, we note that in the LS model, layoff taxes above 184% of the average yearly output per worker completely suppress endogenous separations. This can be discovered by extrapolating the dark solid curve in the middle panel of Fig. 2; evidently, high-skilled workers are more resilient in their mobility before eventually no longer quitting. The corresponding minimum layoff tax required to close down all endogenous separations in the laissez-faire economy with no unemployment insurance is 163%. When $\phi = 0$, gains from quitting and searching for another job are smaller, requiring a smaller layoff tax to suppress endogenous separations.

Layoff taxes in DHHR Fig. 3 shows how a higher layoff tax affects equilibrium outcomes in tranquil zero-turbulence times $\gamma^\ell = 0$ DHHR model.[14] A layoff tax equivalent to 14% of the average annual output per worker in the $\phi = 0, \Omega = 0$ DHHR laissez-faire economy completely suppresses the mobility of high-skilled employed workers.[15] Above this low level of layoff taxes, the rate of which high-skilled workers reject on-the-job new draws of z becomes zero, so that job-separation rates become constant at exogenous job-termination rates. Imposing a small layoff tax devalues labor mobility. Note that at all levels of the layoff tax the rejection rate is zero for both employed and unemployed low-skilled workers with the DHHR parameterization.

Endogenous separations occur in our DHHR model only because they are encouraged by a generous replacement rate of $\phi = 0.7$. None occurs in a $\phi = 0, \Omega = 0$ laissez-faire version. This situation is symptomatic of the low returns to labor mobility in the DHHR model, a topic that we take up in Section 4.

3.3. *Quit turbulence*

How should a model represent that different job separators can find themselves in different situations? For example, workers with valuable skills who separate to find better-paying jobs differ from laid-off workers whose skills are no longer in demand, e.g., due to changing technologies or their types of work moving abroad to low-wage countries.

To capture such differences, the benchmark model treats involuntary separations as earlier theories did by assuming that they worsen circumstances for job separators by presenting the highest possible risks of skill losses. The benchmark model also introduces quit turbulence in the form of risk of human capital loss for workers who voluntarily separate from jobs after draws of poor job-specific productivities at their current employment. We specify that voluntary quitters are more fortunately situated than workers who have just been laid off, both in terms of their having the option to continue working at their current jobs after receiving shocks to productivity, as well as, conditional on separating, facing a lower risk of skill loss, than are workers who suffer involuntary separations.

Like DHHR, we can study the robustness to quit turbulence of LS's attribution of high and persistent European unemployment to interactions between microeconomic turbulence and Europe's more generous welfare states. We can accomplish this by measuring how much the risk of skill loss at times of voluntary separations must be relative to the risk at times of involuntary separations to generate a negative rather than a positive turbulence-unemployment relationship. Because contending forces push for and against the LS outcome, this is a quantitative issue.

Fig. 4a depicts unemployment outcomes as a function of turbulence when productivity distributions of the benchmark model are those of LS. The x-axis shows layoff turbulence γ^ℓ and the y-axis the unemployment rate in percent.

[14] In addition to the two simplifying modifications of the original DHHR framework described in footnote 11, here we assume that skill upgrades are realized immediately in the DHHR model as in the LS framework. Appendix D.2 documents a small impact on equilibrium outcomes in the DHHR model of this change in assumptions.

[15] In the DHHR laissez-faire economy with $\phi = 0, \Omega = 0$, a worker's average quarterly output is 1.8 goods in tranquil zero-turbulence times.

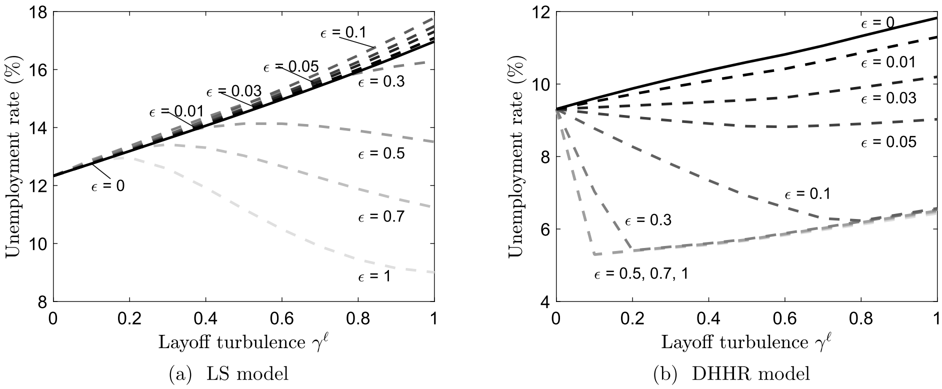


Fig. 4. Quit turbulence in LS and DHHR. Layoff turbulence γ^ℓ on the *x*-axis. Each line represents a different quit turbulence γ^q as a fraction ϵ of layoff turbulence, i.e., $\gamma^q = \epsilon\gamma^\ell$. Panel a shows the benchmark model with LS productivity distributions, i.e., the LS model with no layoff tax. Panel b is the DHHR model with our two simplifying modifications in footnote 11.

Each line has its own quit turbulence γ^q represented as a fraction ϵ of layoff turbulence γ^ℓ, i.e., $\gamma^q = \epsilon\gamma^\ell$ where $\epsilon \in \{0, 0.01, 0.03, 0.05, 0.1, 0.3, 0.5, 0.7, 1\}$. In Fig. 4a, we observe that the quit turbulence fraction ϵ must be *large*, about 50% of layoff turbulence, before the aggregate unemployment rate varies inversely with layoff turbulence, and even then only for relatively high levels of layoff turbulence.

However, these consequences of adding quit turbulence to the LS model differ markedly from those in DHHR's paper. DHHR find that the turbulence-unemployment relationship already becomes negative at very *small* skill loss probabilities for voluntary separators relative to those for involuntary separators:

> "… allowing for a skill loss probability following [voluntary] separation that is only 3% of the probability following [involuntary] separation eliminates the positive turbulence-unemployment relationship. Increasing this proportion to 5% gives rise to a strong *negative* relationship between turbulence and unemployment." (DHHR, p. 1362)

Fig. 4b reproduces DHHR's findings in our version of their model with its two modifications described in footnote 11, inconsequential though they are for the questions we are now addressing. Evidently, DHHR's assertion remains essentially intact; under our two modifications of their model, it just requires a bit more quit turbulence to recover DHHR's critical findings of a negative turbulence-unemployment relationship. Thus, as cited above for the original DHHR model, the relationship becomes markedly negative at 5% of quit turbulence ($\epsilon = 0.05$), while subject to our modifications, quit turbulence needs to be 7% ($\epsilon = 0.07$).

What accounts for these different outcomes emerging after adding just small amounts of quit turbulence to the LS model and the DHHR model? These forces are at work. Productivity draws on the job bring incentives for workers to change employers in search of higher productivities. The small dispersion of productivities under DHHR's uniform distributions with narrow support in Fig. 1b reduce returns to labor mobility. Fig. 4b shows that returns to labor mobility are so low that they fail to compensate for even small amounts of quit turbulence. Consequently, a positive turbulence-unemployment relationship at zero quit turbulence ($\epsilon = 0$) turns negative with even small amounts of quit turbulence. Notice that high-skilled workers choose to remain on the job and accept productivities at the lower end of the productivity distribution rather than quit and have to face even small probabilities of skill loss.

Fig. 4b also shows that DHHR's negative turbulence-unemployment relationship can eventually turn positive, as starkly illustrated by a quit turbulence of $\epsilon = 0.3$ and higher. Those high levels of quit turbulence are initially characterized by a steep negative relationship that ends abruptly at a kink that precedes a gentler upward-sloping turbulence-unemployment relationship. At such kinks, all endogenous separations shut down. The source of unemployment suppression – reductions in quits – has vanished. What leads to a positive turbulence-unemployment relationship is that higher turbulence generates more low-skilled unemployed who are entitled to high benefits. These workers must draw relatively high productivities in order to want to join employment relationships for two reasons. First, relative to low-skilled workers who are entitled to low benefits, such workers are reluctant to give up their high benefits: a high benefit entitlement brings a stronger bargaining position. Second, a bargained wage not only must be *high* enough to induce workers to surrender their high benefits; it also must be *low* enough to induce firms to fill vacancies. As described in footnote 9, DHHR assume a fixed measure of firms, with each idle firm being endowed with a vacancy. The opportunity cost for a firm is the option value of waiting to fill the vacancy as it anticipates prospects of meeting either a high-skilled unemployed worker or a low-skilled unemployed worker who is entitled only to low benefits and who therefore has less bargaining power. Consequently, productivities drawn by low-skilled unemployed workers with high benefits have to be relatively high in order for there to exist a wage acceptable to a worker, firm pair. The resulting low hazard rate for low-skilled workers with high benefits to escape unemployment means that unemployment has to increase with layoff turbulence after all endogenous separations have shut down.

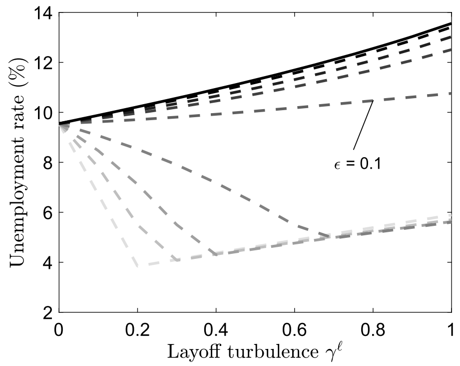


Fig. 5. Benchmark model with DHHR productivity distributions.

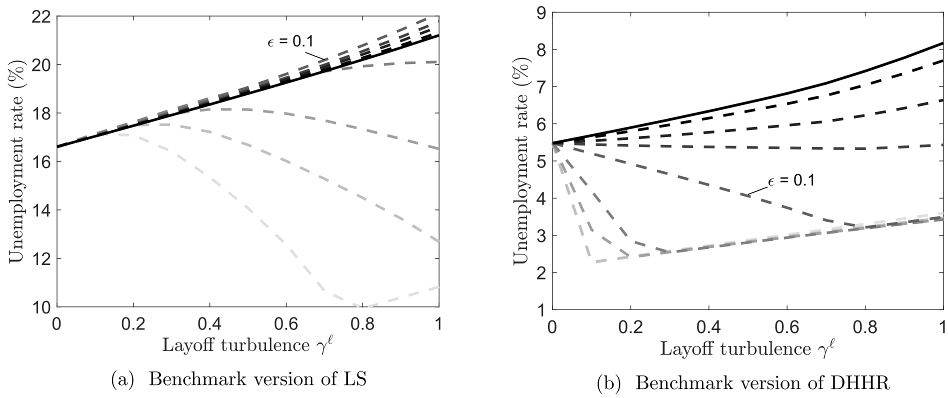


Fig. 6. Benchmark model versions of LS and DHHR.

3.4. Benchmark model versions of LS and DHHR

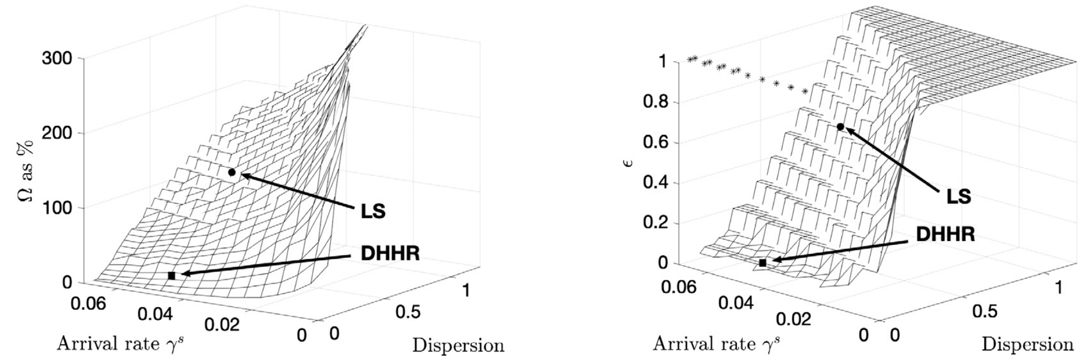
Differences in the spreads of their assumed productivity distributions explain the markedly different implications of quit turbulence in the two models analyzed in Fig. 4.[16] Indeed, by simply switching from the LS to DHHR productivity distributions in the benchmark model, outcomes in Fig. 4a transform into those of Fig. 5: the positive turbulence-unemployment relationship is weakened so much that we get DHHR-like outcomes. We can arrive at what we call the benchmark model version of DHHR by shrinking the width of the uniform productivity distributions from DHHR's original value of 1 to 0.6. This result in Fig. 6b where the responses of unemployment to layoff and quit turbulence closely approximate those of the DHHR model in Fig. 4b. The good approximation prevails while also preserving the two structural differences between the models in Figs. 6b and 4b, as described in footnote 12.

We can also construct a benchmark model version of LS with a uniform productivity distributions. The LS model's high returns to labor mobility require a fairly big width of 2.25 for the uniform distributions. The resulting Fig. 6a generates unemployment responses to turbulence that resemble those of the LS model presented in Fig. 4a. Although to calibrate the benchmark model versions of LS and DHHR we target only the effects of turbulence on unemployment, a cross-phenomenon restriction should ensure that associated effects layoff taxes for unemployment survive our mappings into our benchmark model. We confirm that in the next section.

4. Cross-phenomenon restriction

We present a cross-phenomenon restriction that emerged from our investigation of LS and DHHR by describing interrelated effects of layoff costs on unemployment and of quit turbulence on unemployment that are swept out across environments as we vary the width of the productivity processes. How much layoff costs can suppress unemployment is linked to the potency of quit turbulence risk for reversing a positive turbulence-unemployment relationship. The strengths of both forces on unemployment are intermediated by rates of returns to labor mobility, outcomes that are vitally influenced by the widths and dynamics of the productivity process. We convey these links between the consequences of layoff costs

[16] Please see footnote 12.



(a) Layoff cost criterion: minimum layoff cost at which all voluntary separations shut down when $\gamma^\ell = 0$

(b) Quit turbulence criterion: minimum amount of quit turbulence that makes the turbulence-unemployment relationship negative when $\gamma^\ell = 0.3$

Fig. 7. Cross-phenomenon restriction.

and quit turbulence by computing two outcome criteria as functions of parameters that describe the width and dynamics of the productivity process.

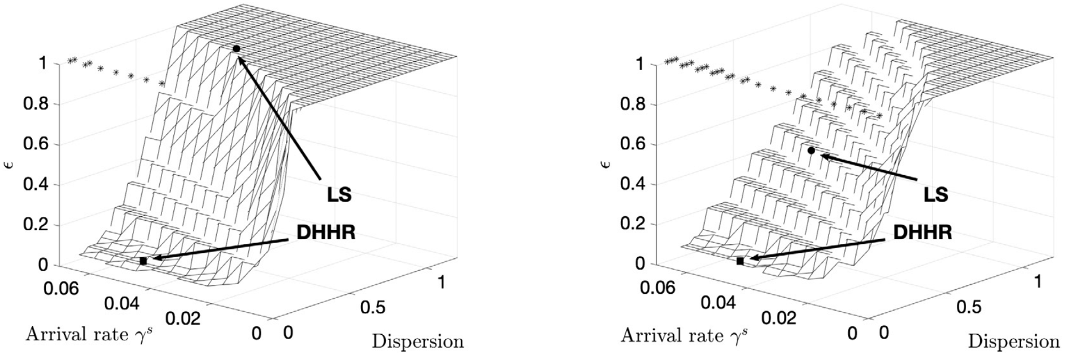
We ferret out these associations by watching two outcome criteria vary as we sweep through a set of uniform productivity processes parameterized by both their widths and their arrival rates γ^s of productivity shocks in continuing matches. We take the minimum layoff cost for which all voluntary separations shut down in $\gamma^\ell = 0$ tranquil times as our outcome criterion for the influence of layoff costs on unemployment. We express the layoff cost as a proportion of the annual output per worker in a corresponding laissez-faire $\phi = 0, \Omega = 0$ economy. We take a minimum amount of quit turbulence that makes the turbulence-unemployment relationship negative, conditional on a magnitude of layoff turbulence γ^ℓ as our outcome criterion for the effect of quit turbulence on unemployment. We measure quit turbulence relative to the magnitude of layoff turbulence, i.e., as a fraction $\epsilon \in [0, 1]$. So conditional on a value of γ^ℓ, our quit turbulence criterion is the minimum value of ϵ that yields an inverse turbulence-unemployment relationship, i.e., that makes the unemployment rate fall with an incremental increase in layoff turbulence at the conditioned value of γ^ℓ. (When the turbulence criterion equals a maximum value of 1, indicates either a knife-edged case at an interior solution when the minimum value of ϵ that yields a negative turbulence-unemployment relationship occurs at 1 or, more often, a corner solution in which there exists no $\epsilon \in [0, 1]$ that can overturn the positive turbulence-unemployment relationship.)

Fig. 7 presents the two outcome criteria as functions of the arrival rate γ^s of new on-the-job productivity draws and the standard deviation of the uniform productivity distribution in our benchmark model, here denoted as "dispersion."[17] The layoff cost criteria in Fig. 7a indicate that the minimum layoff tax required to shut down voluntary separations increases with dispersion and decreases with the arrival rate of new on-the-job productivity draws. Because a higher dispersion brings higher returns to labor mobility, a higher layoff cost is required to shut down voluntary separations. A higher arrival rate of productivity shocks in continuing matches implies a lower expected duration of a productivity draw and thereby suppresses returns to labor mobility via two forces. First, a relatively low productivity draw becomes less costly to bear when it is expected to persist for a shorter period of time. Second, the prospective gain from quitting and finding a higher productivity match becomes less attractive when the new productivity draw can be anticipated to last for less time. These considerations make the minimum layoff tax required to shut down voluntary separations decrease in the arrival rate. At the far right corner of Fig. 7a that indicates high dispersion and very small arrival rates, the layoff cost criterion explodes when the graph is extended. Here the supports of the Fig. 1b uniform productivity distributions extend ever further into negative territory; combined with a low arrival rate, a poor productivity draw is expected to last for a long time. Consequently, firms are willing to incur very high layoff costs to terminate exceptionally poor productivity draws.[18]

The Fig. 7b presents the quit turbulence criterion when layoff turbulence $\gamma^\ell = 0.3$. It reveals how outcomes are linked to those revealed by the layoff cost criterion in Fig. 7a. Both outcome criteria are driven by the returns to labor mobility implied by the productivity process. The interrelatedness of the effects of layoff costs on unemployment and of quit turbulence on unemployment reflects a cross-phenomenon restriction.

[17] All outcome criteria figures are drawn for dispersion greater than 0.0722 (a support of 0.25). By omitting zero dispersion, we stay clear of economies that trivially have no endogenous separations. In such degenerate economies, the layoff cost criterion is zero and all turbulence criteria equal 1 since, in the absence of quits, no force could reverse the positive turbulence-unemployment relationship.

[18] As a point of reference, the axis for dispersion ends at 1.2 in the outcome criterion figures, which implies a width of just above 4 for the support of the uniform distributions. Thus, at a dispersion of 1.2, the combined productivity distributions for low- and high-skilled workers cover the entire range of the x-axis in Fig. 1b.



(a) Minimum amount of quit turbulence that makes the turbulence-unemployment relationship negative when $\gamma^\ell = 0.1$

(b) Minimum amount of quit turbulence that makes the turbulence-unemployment relationship negative when $\gamma^\ell = 0.5$

Fig. 8. Quit turbulence criterion, $\gamma^\ell = 0.1$ and $\gamma^\ell = 0.5$.

A notable difference between the two panels in Fig. 7 is that the quit turbulence criterion plateaus at a maximum value of 1 when rates of return to labor mobility are so high that there exists no amount of quit turbulence that can reverse a positive turbulence-unemployment relationship. The stars at the front end of Fig. 7b occur at very low values of dispersion and also indicate a quit turbulence criterion equal to 1. In this vicinity, for a given arrival rate, very small dispersions imply rates of return to labor mobility so low that, even without quit turbulence, no voluntary separations occur. Without voluntary separations, there is nothing to be shut down by introducing quit turbulence and hence there is no force coming from quit turbulence to reverse a positive turbulence-unemployment relationship.

Fig. 8 portrays the dependence of the quit turbulence criterion on the amount of layoff turbulence γ^ℓ. A lower layoff turbulence $\gamma^\ell = 0.1$ in Fig. 8a implies a steeper slope that quickens an ascent to a plateau where no amount of quit turbulence can reverse a positive turbulence-unemployment relationship. A higher layoff turbulence $\gamma^\ell = 0.5$ in Fig. 8b slows down the ascent. At very low dispersions, the two panels show corresponding decreases and increases in the numbers of stars.

Figs. 7 and 8 include two points denoted LS and DHHR that are our benchmark model versions of those frameworks with turbulence-unemployment outcomes as shown in Fig. 6. For each framework, the arrival rate is $\gamma^s = 0.05$ as reported in Table 1, while the dispersion was chosen to target turbulence-unemployment outcomes in the appropriate framework. Recall that the width of support for the uniform distributions in the benchmark model version of DHHR is 0.6 and so that dispersion (measured as a standard deviation) equals $\sqrt{0.6^2/12} = 0.173$; corresponding numbers for the benchmark model version of LS are a width of support of 2.25 and hence a dispersion equal to 0.650. In line with Fig. 6b, the quit turbulence criterion for DHHR is very low, about 0.05 for all three values of γ^ℓ in Figs. 7b, 8a and 8b, respectively. Likewise, outcomes for LS are ones that can be inferred from Fig. 6a; specifically, the quit turbulence criterion equals 0.58 at layoff turbulence $\gamma^\ell = 0.3$, 1 at the lower turbulence $\gamma^\ell = 0.1$, and 0.45 at higher turbulence $\gamma^\ell = 0.5$. These do good jobs of representing the quit turbulence outcomes in Fig. 4 that we set out to explain.[19]

The cross-phenomenon restriction portrayed in Figs. 7 and 8 helps assess the potential scope that quit turbulence brings for undermining LS's turbulence explanation of trans-Atlantic unemployment experiences. Starting with the DHHR analysis, its location in the space of productivity processes confirms our Section 3 conclusion that DHHR's reversal of LS relies on assuming a very compressed productivity distribution. DHHR's compressed productivity process renders their model incapable of explaining observed relationships between layoff costs and unemployment across countries. Furthermore, DHHR's productivity process rests perilously downstream on the border of a parameter region with no voluntary separations (marked by stars). Hence, a small parameter perturbation could ironically turn DHHR's feeble positive turbulence-unemployment relationship into a strong one, as discussed above. Moving upstream to the other side of DHHR's productivity process would quickly raise the quit turbulence criterion before it reaches a parameter region consistent with observations on layoff costs and unemployment. Assuming higher values of layoff turbulence γ^ℓ provides little help to DHHR's point of view. In contrast, the LS analysis falls within a parameter region with quantitatively plausible implied returns to labor mobility, in terms of its implications for the effects of layoff costs on unemployment.

[19] For the record, the layoff cost criteria in Fig. 7a for the benchmark model versions of DHHR and LS are 23% and 129%, respectively, while the corresponding numbers are 14% and 186% in our layoff cost analyses in Section 3.2. The different numbers for the DHHR framework are due to the structural differences between the benchmark model version and the DHHR model described in footnote 12. In the case of LS, the difference is solely driven by the uniform productivity distributions in the benchmark model version of LS versus LS's own assumption of truncated normal distributions. Not surprisingly, it takes a higher layoff cost to shut down voluntary separations under the latter distributions with longer tails that include worse productivities than the narrower support of the uniform distributions. For our present argument, these differences are immaterial.

Another application We gather further insights from our parameter perturbation exercises by revisiting two celebrated macro-labor studies of layoff taxes. The first is a Mortensen and Pissarides (1999) matching model that calibrates productivity processes to unemployment statistics and outcomes in an unemployment insurance system. The second is a search-island model of Alvarez and Veracierto (2001) that enlists establishment data on firm and worker turnover to calibrate firm size dynamics. Baley et al. (2023) show in both frameworks how high returns to labor mobility are required to accompany empirically plausible unemployment responses to variations in layoff costs. Furthermore, they show how those high returns to labor mobility also sustain a positive turbulence-unemployment relationship even when quit turbulence is present.[20] Thus, the cross-phenomenon restriction that prevails within our Section 2 benchmark model extends more broadly.

5. Concluding remarks

That the magnitude of returns to labor mobility contributes to several aggregate outcomes brings informative cross-phenomenon restrictions that can guide calibrations of productivity processes. Exploiting such restrictions adheres to the advice offered by Lucas (1980, pp. 696-697):

> "... we are interested in models because we believe they may help us to understand matters about which we are currently ignorant, we need to test them as useful imitations of reality by subjecting them to shocks for which we are fairly certain how actual economies, or parts of economies, would react. The more dimensions on which the model mimics the answers actual economies give to simple questions, the more we trust its answers to harder questions."

For us, Lucas's relatively "simple question" is about how differences in layoff costs have affected labor reallocations, while the "harder question" concerns effects of quit turbulence on unemployment, about which much less is known. We recommend further studies of the role that returns to labor mobility play in macro-labor models.

Having recalibrated DHHR's model of quit turbulence to align it with a "Weinberg constraint," we rejoin the conversation with Alan Greenspan, with which DHHR began their paper. In the passage that DHHR cited, reproduced in Section 1 above, Greenspan does indeed seem to be concerned with the DHHR's quit turbulence force as well as its role in reducing job mobility that comes with DHHR's calibration. But Greenspan refrained from emphasizing such possible effects of increased turbulence more broadly. Earlier in that same paragraph, Greenspan (1998, p. 743) said that it was higher, not lower, labor mobility (i.e., "churning") that concerned him:

> "... the perception of increased churning of our workforce in the 1990s has understandably increased the sense of accelerated job-skill obsolescence among a significant segment of our workforce, especially among those most closely wedded to older technologies. The pressures are reflected in a major increase in on-the-job training and a dramatic expansion of college enrollment, especially at community colleges. As a result, the average age of full-time college students has risen dramatically in recent years as large numbers of experienced workers return to school for skill upgrading."

We read Greenspan as writing about US workers who had suffered the type of adverse human capital destruction shock that Ljungqvist and Sargent (1998, 2007, 2008) used to capture increased turbulence. Greenspan pointed out that such workers have ways of rebuilding their human capital in addition to the ways that are open to them in the Ljungqvist and Sargent models, thereby opening other ramifications of increased turbulence for outcomes studied by neither DHHR nor Ljungqvist and Sargent. It would be worthwhile to add such activities to models of trans-Atlantic unemployment experiences, while adhering to Weinberg's rules.

Data availability

No data was used for the research described in the article.

Appendix. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.red.2023.07.004.

[20] Baley et al. (2023) also demonstrate that for parameterizations calibrated to fit firm size dynamics, even when parameters are perturbed, high returns to labor mobility prevail in models like Alvarez and Veracierto's (2001) in which shocks to productivity are intermediated through neo-classical production functions. But other macro-labor models that rely solely on unemployment statistics to calibrate per-worker productivity processes can have returns to labor mobility that are fragile with respect to perturbations of parameters that still fit targeted unemployment statistics. Baley et al. (2023) show that this is the case for Mortensen and Pissarides's (1999) calibration. Baley et al. conjecture that, because they focused on employment effects of layoff taxes, equilibrium outcomes probably would have prompted Mortensen and Pissarides to explore more of their parameter space if their calibration had wandered into the region with extremely low returns to mobility.

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