Aggregate Dynamics in Lumpy Economies

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What do we do

- **Lumpiness** in micro adjustment is pervasive
  - Periods of inaction followed by bursts of activity
  - Price-setting, investment, labor adjustment, portfolios, ...
- How does **micro lumpiness** affect **aggregate dynamics**?
- We characterize **new structural relationships** between:

  a) Aggregate dynamics

  b) Steady state cross-sectional moments

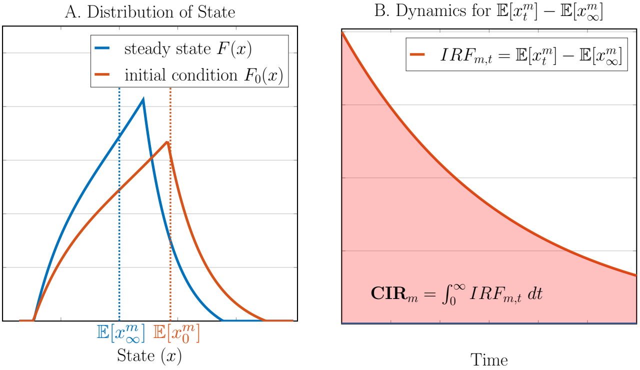
  c) Micro-level data
- Empirical **application to lumpy investment**
  - Reveals key moments for investment dynamics
  - Not computed before, quant. important, missed by standard models

Aggregate Dynamics via CIR

- Continuum of ex-ante identical agents, state x

$$\text{idiosyncratic shocks} \; + \; \text{adjustment friction} \Longrightarrow x \text{ dynamics}$$

- Cumulative Impulse Response (CIR_m)
    - Area between m-th moment at t and its steady state value
    - CIR captures: impact + persistence



Our framework: Preview

- **Three analytical results**:
  1. **Aggregation**:
     - CIR solves a one-agent recursive problem
  2. **Representation**:
     - One-agent problem is represented via steady state moments
  3. **Observation**:
     - Steady state moments are recovered from micro-data

- **Advantages:**
  - Limited assumptions on inaction model
  - Dynamics for any cross-sectional moment of x
  - General (small) perturbations around steady state

- **Limitation:**
  - We use steady state policies along the transition (no GE effects)

Application to Lumpy Investment: Preview

- **State:** $x = \text{capital gap} = \log(k_t/k_t^*)$
  - k_t, k_t^* : current capital, optimal static capital (not observed)
  - Between adjustments: $dx_t = \nu dt + \sigma^2 dW_t, \quad W_t \sim \text{Wiener}$
  - Random adjustment cost

- **Aggregation + Representation:**

$$\text{CIR}_1 = \frac{\mathbb{V}ar(x) - \nu\ \mathbb{C}ov(x, age)}{\sigma^2}$$

- **Observation:**

  $\Delta x = \text{investment}, \ \text{freq} = \text{adjustment dates}, \ \text{age} = \text{time since adjustment}$

$$\nu \sim \text{freq} \times \mathbb{E}(\Delta x) \qquad \mathbb{V}ar(x) \sim \mathbb{E}(\Delta x^3)$$
$$\sigma^2 \sim \text{freq} \times \mathbb{E}(\Delta x^2) \qquad \mathbb{C}ov(x, age) \sim \mathbb{C}ov(\text{freq}, \Delta x^2)$$

- **Data:** Plant-level data from Chile and Colombia
  - $\mathbb{C}ov(x, age)$ is
    - key for aggregate dynamics
    - positive and large, first time to be computed
    - missed by benchmark models

Contributions

- **Inaction and aggregate dynamics**: Caplin/Spulber (87), Caplin/Leahy (91), Caballero/Engel (91,93,99), Golosov/Lucas (07), Cooper/Haltiwanger/Willis (15), Bloom (09), Clementi/Palazzo (16), Elsby/Michaels (17)

  Here: structural link between micro-data and aggregate dynamics

- **Framework builds on Alvarez, Le Bihan and Lippi (16)**
  - n-product random menu cost model + driftless innovations
    - Characterize CIR for *first* moment $\sim \frac{1}{6}\frac{\mathbb{K}ur[\Delta p]}{freq[\Delta p]}$
    - Analytical ($n = 1, \infty$) and numerical ($n > 1$) verification of ODEs

  Here: structural link btw steady state and CIR in general environment

- **Measurement of capital-gap:** Hamermesh(89), Caballero/Engel (93), Caballero/Engel/Haltiwanger(97), Cooper/Willis(04), Hsieh/Klenow(09)

  Here: new theory (observation) and application

- **Lumpy Investment**: Kahn/Thomas (08), Bachmann/Caballero/Engel (13), Winberry (16)

  Here: revisit models' prerequisites to fit micro data

Roadmap

1. Benchmark Model of Lumpy Investment
   - Explain theoretical tools with an application
2. Three Properties of the CIR
   - Aggregation + Representation
   - Observation
3. Empirical application
   - Data
   - Implications (model free)
   - Assess lumpy investment model

Environment

- Representative household:
  - Consumes
  - Owns firms, receives dividends
- Continuum of firms indexed with $\omega \in [0, 1]$
  - Produce output using capital
  - Invest subject to adjustment costs
- Production technology: $$y_{\omega,t} = e_{\omega,t}^{1-\alpha} k_{\omega,t}^{\alpha}$$
  - e: idiosyncratic productivity $\quad d\log(e_{\omega,t}) = \mu dt + \sigma dW_{\omega,t}$
  - k: capital (uncontrolled) $\quad d\log(k_{\omega,t}) = -\psi dt$
- Random fixed adjustment cost: $\kappa_{\omega,t} e_{\omega,t}$

$$\kappa_{\omega,t} = \begin{cases} \kappa & \text{with prob} \quad 1 - \lambda dt \\ \xi_{\omega,t} & \text{with prob} \quad \lambda dt \end{cases} \qquad \text{with} \qquad \xi_{\omega,t} \sim H[0, \kappa]$$

Problems and equilibrium

- **Firms' problem:**
  - Maximize expected profits, discounted at Arrow–Debreu price Q_t
  - Choose adjustment dates and investment: $\{\tau_{\omega,i}, \Delta k_{\omega,i}\}_{i=1}^{\infty}$

$$\max_{\{\tau_{\omega,i},\Delta k_{\omega,i}\}_{i=1}^{\infty}} \mathbb{E}\left[\underbrace{\int_0^{\infty} Q_s y_{\omega,s}\, ds}_{\text{production}} - \underbrace{\sum_{i=1}^{\infty} Q_{\tau_{\omega,i}} \left(\Delta k_{\tau_{\omega,i}} + \kappa_{\omega,\tau_{\omega,i}} e_{\omega,\tau_{\omega,i}}\right)}_{\text{investment + adjustment costs}}\right]$$

- **Household's problem:**
  - Maximize utility subject to budget

$$\max_{C_t} \int_0^{\infty} e^{-\rho t} C_t dt \quad s.t. \quad \int_0^{\infty} Q_t \left(C_t - \Pi_t\right) dt = 0$$

  - C_t, Π_t: aggregate consumption and profits

- **Equilibrium:** Optimality + Feasibility Definition
  - Prices are independent of the aggregate state (distribution) More

Capital gaps and policy

- Redefine state as ***capital-gap***:

$$x_{\omega,t} \;\equiv\; \log(k_{\omega,t}/e_{\omega,t}) - \mathbb{E}[\log(k_{\omega,ss}/e_{\omega,ss})]$$

- ***Uncontrolled*** capital-gap:

$$\tilde{x}_{\omega,t} \;=\; \tilde{x}_{\omega,0} + \nu t + \sigma W_{\omega,t}, \qquad \nu \equiv -(\psi + \mu)$$

- ***Controlled*** capital-gap:

$$x_{\omega,t} \;=\; \tilde{x}_{\omega,t} + \sum_{\tau_{\omega,i} \leq t} \Delta x_{\tau_{\omega,i}}$$

  - Stopping policy: $\tau_{\omega,i+1} = \inf\left\{t \geq \tau_{\omega,i} : x_{\omega,t} \notin [\underline{x}, \overline{x}] \text{ } or \text{ } dN^x_{\omega,t} = 1\right\}$
  - Reset point: $x_{\tau_{\omega,i}} \;=\; \hat{x}$
  - Investment: $\Delta x_{\tau_{\omega,i}} \;=\; \hat{x} - x_{\tau^-_{\omega,i}}$
  - $N^x_{\omega,t}$: Poisson counter with arrival rate $\Lambda(x)$

- ***Age*** of capital-gap: $age_{\omega,t} = t - \tau_{\omega,i}$

Aggregate Dynamics

- **CIR's definition for m-th moment**

$$\text{CIR}_m(F_0) \equiv \int_0^\infty \Bigg(\underbrace{\int x^m dF_t(x)}_{\text{moment at } t} - \underbrace{\int x^m dF(x)}_{\text{ergodic moment}} \Bigg) dt$$

  - $F(x)$: ergodic distribution
  - $F_t(x)$: time-t distribution, given initial condition $F_0(x)$

- **Baseline case**
  - Dynamics for **average** state: $m = 1$, normalize $\mathbb{E}[x] = 0$
  - Initial condition as **translation**: $F_0(x) = F(x - \delta)$, with δ small

- **Recursive representation** (from Alvarez, Le Bihan, Lippi (2016))

$$\text{CIR}_1(\delta) \equiv \int_0^\infty \underbrace{\int x dF_t(x)}_{\text{cross-section at } t} \; dt \; = \int \underbrace{\mathbb{E}\Big[\int_0^\tau x_t dt \Big| x_0 = x \Big]}_{\text{stopping time problem}} \; \underbrace{dF(x - \delta)}_{\text{initial cross-section}}$$

Aggregate Dynamics

- **CIR's definition for m-th moment**

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- **First order approximation + integration by parts**

$$\text{CIR}_1(\delta) \approx \delta \times \int \underbrace{\frac{d}{dx}\mathbb{E}\Big[\int_0^\tau x_t dt \Big| x_0 = x\Big]}_{\text{changes in stopping time}} \underbrace{dF(x)}_{\text{steady state}}$$

Roadmap

1. Benchmark Model of Lumpy Investment
   - Explain theoretical tools through an example

2. **Three Properties of the CIR**
   - Aggregation + Representation
   - Observation

3. Empirical Application
   - Data
   - Implications (model free)
   - Assess lumpy investment model

Aggregation + Representation

Recall random adjustment costs:

$$\kappa_{\omega,t} = \begin{cases} \kappa & \text{with prob} \quad 1-\lambda dt \\ \xi_{\omega,t} & \text{with prob} \quad \lambda dt \end{cases} \qquad \text{with} \quad \xi_{\omega,t} \sim H[0,\kappa]$$

- If $\kappa \to \infty$ and $\xi_{\omega,t} = 0$, then $\text{CIR}_1(\delta)/\delta \approx \mathbb{E}[age]$
  - $\mathbb{E}[age]$ = speed of agg. adjustment in time-dependent models
- If $\xi_{\omega,t} = 0$, then $\text{CIR}_1(\delta)/\delta \approx \frac{\mathbb{V}ar[x]}{\sigma^2} - \frac{\nu \ \mathbb{C}ov[x,age]}{\sigma^2}$
- In general, $\text{CIR}_1(\delta)/\delta \approx \mathbb{E}[age] + \sum_{j=0}^{\infty} \mathcal{E}_j \mathbb{E}[x^j]$
  - $\sum_{j=0}^{\infty} \mathcal{E}_j \mathbb{E}[x^j]$: provides info about $d\tau/dx_0$
  - $\mathbb{E}[x^j]$: from data; $\mathcal{E}_j$ from data need $\Delta x | \hat{x} + x_0$

Aggregation + Representation

Focus on (most common) case $\xi_{\omega,t} = 0$:

$$\text{CIR}_1(\delta)/\delta \;\approx\; \frac{\mathbb{V}ar[x]}{\sigma^2} - \frac{\nu \; \mathbb{C}ov[x, age]}{\sigma^2}$$

- Ratio $\frac{\mathbb{V}ar[x]}{\sigma^2}$ measures ***responsiveness*** to idiosyncratic shocks
  - Investment dispersion ($\mathbb{V}ar[x]$) vs. productivity dispersion (σ^2)
  - In frictionless limit, responsiveness is total

$$\mathbb{V}ar[x] = 0 \quad \Longrightarrow \quad \text{Immediate convergence to steady state}$$

- Ratio $-\frac{\nu \; \mathbb{C}ov[x,age]}{\sigma^2}$ corrects for dispersion generated by the drift
- Challenge: capital gap x is not observable!

Observation

Use **micro-level data on adjustments** $(\Delta x, \tau)$ to recover:

- **Reset point**

$$\hat{x} = \underbrace{\frac{\mathbb{E}[\Delta x]}{2}\left(1 - \mathbb{CV}^2[\tau]\right)}_{\text{compensates drift}} + \underbrace{\frac{\mathbb{C}ov[\tau, \Delta x]}{\mathbb{E}[\tau]}}_{\text{compensates asymmetry}}$$

  - Ensures that distribution is centered, i.e. $\mathbb{E}[x] = 0$

- **Drift and volatility**

$$\nu = -\frac{\mathbb{E}[\Delta x]}{\mathbb{E}[\tau]}, \qquad \sigma^2 = \frac{\mathbb{E}[\Delta x^2]}{\mathbb{E}[\tau]} + 2\nu\hat{x}$$

  - Drift = frequency × average investment
  - Volatility = frequency × investment dispersion (adjusted for drift)

Observation (cont...)

Use micro-level panel data panel on $(\Delta x, \tau)$ to recover:

- **Ergodic variance:**

$$\mathbb{V}[x] = \frac{\hat{x}^3 - \mathbb{E}[(\hat{x} - \Delta x)^3]}{3\mathbb{E}[\Delta x]}$$

  - Reflects asymmetry in investment rates

- **Average age:**

$$\mathbb{E}[age] = \mathbb{E}[\tau] \left(\frac{1 + \mathbb{CV}^2[\tau]}{2} \right)$$

  - Reflects duration's average and dispersion (renewal theory)

- **Covariance:**

$$\mathbb{C}ov[x, age] = \frac{1}{2\nu} \left(\frac{\mathbb{E}\left[\tau \left(\hat{x} - \Delta x\right)^2\right]}{\mathbb{E}[\tau]} - \mathbb{V}[x] - \sigma^2 \mathbb{E}[age] \right)$$

  - Reflects covariance between investment dispersion and duration

Discussion about three properties

- Together, provide **tight link** between:

  aggregate dynamics $\sim$ **steady state** $\sim$ **micro data**

- **Key assumptions**
  - Markovian states and policies
  - Small perturbations around steady state ($\delta \approx 0$)
  - Full adjustment upon action
  - Gaussian shocks

- **Extensions (in paper)**
  - Higher moments $m > 1$ (e.g. dynamics of variance)
  - More general initial conditions F_0 (e.g. mean-preserving spreads)
  - Mean-reversion

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Data

- Establishment-level annual panel data
  - Chile: *Encuesta Nacional Industrial Anual*
  - Colombia: *Encuesta Anual Manufacturera*

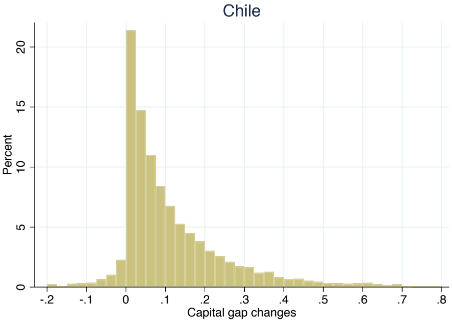
	Chile	**Colombia**
Period	1995-2007	1995-2016
Establishments (per year)	3,470	5,615
Size (avg. workers)	87	92

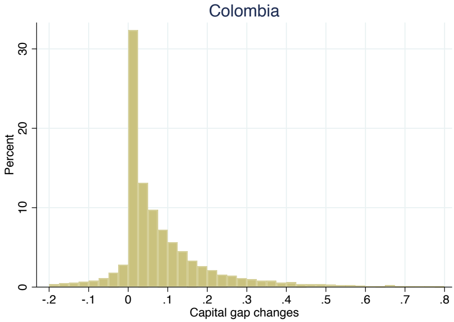
- Balanced panel
- Capital: buildings + machinery and equipment + transport

$$\Delta x_t = \begin{cases} \log(1 + i_t) & \text{if} \quad |i_t| > 1\% \\ 0 & \text{if} \quad |i_t| < 1\%, \end{cases} \quad , \quad i_t = \frac{I_t}{K_t}$$

Distribution of investment rates Δx

- Lots of inaction, spikes and asymmetries Details





Inputs from data

- Investment frequency ~ 5 quarters, highly dispersed
- Average investment ~ 7-10%, highly dispersed
- Covariances close to zero

	Chile	**Colombia**
Frequency		
$\mathbb{E}[\tau]$	1.392	1.223
$\mathbb{CV}^2[\tau]$	0.692	0.497
Investment rates		
$\mathbb{E}[\Delta x]$	0.108	0.071
$\mathbb{E}[\Delta x^2]$	0.033	0.022
$\mathbb{E}[(\hat{x} - \Delta x)^3]$	-0.009	0.006
Covariances		
$\mathbb{C}ov[\tau, \Delta x]$	0.001	0.009
$\mathbb{C}ov[\tau(\hat{x} - \Delta x)^2]$	0.033	0.027

Outputs from Theory (Model free)

- Implied drift and volatility comparable to literature

Parameters	**Chile**	**Colombia**
ν	-0.078	-0.058
σ	0.146	0.122
$\hat{x}$	0.017	0.025

- Large positive covariance and ex-post/ex-ante dispersion ratio

SS moments	**Chile**	**Colombia**
$\mathbb{V}[x]$	0.027	0.022
$\mathbb{C}ov[x, age]$	0.132	0.119

- Aggregate: Implied multiplier around 2

Agg. Dynamics	**Chile**	**Colombia**
$\mathbb{V}[x]/\sigma^2$	1.315	1.463
$-\nu\mathbb{C}ov[x, age]/\sigma^2$	0.480	0.461
$\text{CIR}(\delta)/\delta$	1.796	1.923

Can the random adj. cost model generate the data?

- Calibrate the random lumpy model to Chile's data
  - Discount $\rho = 0.04$, share $\alpha = 0.58$, deprec $\psi = 0.062$ externally
  - Data: Productivity process $(\mu, \sigma) = (0.016, 0.146)$
  - Adj cost $\kappa = 0.45$ and arrival rate $\lambda = 0.71$ match $\mathbb{E}[\tau]$ and $\mathbb{V}[x]$

- Model generates **negative** covariance between capital gap and age

Moments missed	**Chile**	**Colombia**	**Model**
$\mathbb{C}ov[\tau, \Delta x]$	0.001	0.009	0.193
$\mathbb{C}ov[x, age]$	0.132	0.119	-0.082

- By missing $\mathbb{C}ov[x, age]$, misses aggregate dynamics!

Agg. Dynamics	**Chile**	**Colombia**	**Model**
$\mathbb{V}[x]/\sigma^2$	1.315	1.463	1.268
$-\nu\mathbb{C}ov[x, age]/\sigma^2$	0.480	0.461	-0.300
$\text{CIR}(\delta)/\delta$	1.796	1.923	0.968

Final thoughts

- If the random adj cost model is true, formula for CIR holds
- Problem: Model misses $\mathbb{C}ov[x, age]$
  - As a consequence, predicts a lower CIR
  - Finding is robust to additional features:
    - Labor, persistent productivity, free small investments
- **Next steps:**
  - Data work: heterogeneity (sectors, firms), time aggregation,...
  - Bring general characterization to the data
    - To estimate $\mathcal{E}_j$, we need from data: $\Delta x|\hat{x} + x_0$
    - VAT shocks in Karadi/Reiff (14)
    - Corporate tax changes in Zwick/Mahon (17)

APPENDIX

Equilibrium Return

Definition

Given shock process for productivity and fixed adjustment costs $\{W_{\omega,t}, N_{\omega,t}\}$, an **equilibrium** consists of processes for prices $\{Q_t\}$; household's policy $\{C_t\}$, firms' policy $\{\tau_{\omega,i}, \Delta K_{\omega,i}\}$ and value $V_{\omega,t}(K)$ such that:

1. Given $\{Q_t\}$, $\{C_t\}$ solve the household problem.
2. Given $\{Q_t, W_{\omega,t}, N_{\omega,t}\}$, $\{\tau_{\omega,i}, K_{\omega,i}\}$ solve the firms' problem with value $V_{\omega,t}(K)$.
3. Goods market clears.

$$\underbrace{\mathbb{E}\left[E_{\omega,t}^{1-\alpha} K_{\omega,t}^{\alpha}\right]}_{Y_t} = C_t + \underbrace{\mathbb{E}\left[\mathbb{1}_{\{\tau,t\}}\left[\kappa_{\omega,t} E_{\omega,t} + \Delta K_{\omega,t}\right]\right]}_{I_t}.$$

GE feedback to individual policies Return

- Firms' states:
  - Idiosyncratic state: capital (K) and productivity (E)
  - Aggregate state: distribution of (K, E)
- In our GE framework, **prices** are independent of aggregate state
- Aggregate state **does depend** on distribution of (K, E)
- Other examples:
  - Golosov/Lucas(07) (monetary shocks)
  - Kehoe/Midrigan(08), Carvalho/Nechio(11) (RER dynamics)
  - Bachman et. al. (13) (pro-cyclicality of investment)
  - Winberry(16) (policy stimulus)
- If GE important, tools are...
  - Convenient way to summarize role of inaction (Blanco/Cravino,18)

Data: Summary Statistics Return

- Establishment-level annual panel data
  - Chile: *Encuesta Nacional Industrial Anual*
  - Colombia: *Encuesta Anual Manufacturera*
  - US I = Cooper/Haltiwanger (06) plants
  - US II = Zwick/Mahon (17) tax records

	Chile	**Colombia**	**US I**	**US II**
Period	1995-2007	1995–2016	1972–1988	1993–2010
Establishments (per year)	3,470	5,615	7,000	128,151
Size (avg. workers)	87	92		
Average investment	9.68	8.8	12.2	10.4
Positive fraction	65.1	68.7	81.5	
Negative fraction	3.9	9.2	10.4	
Inaction rate	31.0	22.1	8.1	23.7
Spike rate	17.1	16.0	20.4	14.4
Positive spikes	16.2	14.4	18.6	
Negative spikes	0.8	1.6	1.8	