Advanced Macroeconomics II

Lecture 7
Consumption: Non-Separable Preferences

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Roadmap: Non-Separable Preferences

1. Habit Formation

2. Durables

3. Hyperbolic Discounting
Non-Separable Preferences

- So far we have assumed that preferences are separable over time ⇒ consumption expenditure in a given period does not have any effect in the marginal utility of any other period.

- There are three simple ways to depart from time separability:
  
  ▶ Habit formation: Utility comes from current consumption AND past consumption (or consumptions of others).
  
  ▶ Durable goods: Certain types of goods yield utility flows many periods after its purchase, for example: fridge, a TV set, a car, ... 
  
  ▶ Hyperbolic discounting: present is disproportionately more discounted than future periods.
Roadmap: Non-Separable Preferences

1. Habit Formation

2. Durables

3. Hyperbolic Discounting
Habit Formation (1): Idea

- Households care about current consumption relative to past consumption.

- Preferences with habit formation break the link between
  a) preferences over consumption at different points in time and
  b) consumption over different states of nature.

- **Internal habits:** care about consumption relative to past consumption.
  - get less utility from consuming the same as before
  - hard problem to solve

- **External habits:** care about consumption relative to other people (average).
  - get less utility from consuming less than the rest
  - Easier to solve: with many agents, own consumption choice does not affect average consumption.
Habit Formation (2): Subtractive model

- Include past consumptions summarised via a habit stock $h_t$ in utility:
  $$u(c_t, h_t)$$

- Early models assumed an additive form, where utility is derived from the difference between current consumption and the habit stock:
  $$u(c_t, h_t) = u(c_t - \varphi h_t)$$
  - Simplest case: habit stock equal to last period's consumption $h_t = c_{t-1}$
    $$u(c_t, c_{t-1}) = u(c_t - \varphi c_{t-1})$$

- More general law of motion for $h_t$:
  $$h_t = \lambda h_{t-1} + (1 - \lambda) c_{t-1} = (1 - \lambda) \sum_{j=1}^{\infty} \lambda^{j-1} c_{t-j}, \quad \lambda \in [0, 1]$$
  - $\lambda$ measures the persistence of the habit process.
  - High $\lambda$ indicates that consumption in the distant past is hard to forget.
Habit Formation (3): Additive model and CRRA

- For the outer utility function \( u(x) \) we require:
  - \( u_h < 0 \) and \( u_{ch} > 0 \) (adjacent complementarity)
  - standard properties \( u_c > 0 \) and \( u_{cc} < 0 \).

- **Example 1:** Problem set with quadratic utility.

- **Example 2:** CRRA specification:
  \[
  u(c_t, h_t) = \frac{(c_t - \varphi h_t)^{1-\theta} - 1}{1 - \theta}, \quad \theta > 0, \varphi > 0
  \]
  \[
  h_t = \lambda h_{t-1} + (1 - \lambda) c_{t-1}, \quad \lambda \in [0, 1]
  \]
  - \( u_h = -\varphi (c_t - \varphi h_t)^{-\theta} < 0 \) which requires \( \varphi > 0 \)
  - \( u_{ch} = \theta \varphi (c_t - \varphi h_t)^{-(1+\theta)} > 0 \) which requires \( \theta > 0 \)
  - \( \varphi \) usually between 0 and 1, measures the relative weight of habits
  - What does \( \theta \) measure in this case?

- Problem with CRRA: potential for negative argument inside \( u(\cdot) \)!
Habit Formation (4): Multiplicative habits

- Multiplicative form introduced by Abel (1990) and Galí (1994):

\[ u(c_t, h_t) = u \left( \frac{c_t}{h_t^{\varphi}} \right) \]

\[ h_t = \lambda h_{t-1} + (1 - \lambda) c_{t-1}, \quad \lambda \in [0, 1] \]

- No more problems with assuming \( u \) to be CRRA.
  - If \( c_t > 0 \ \forall t \), then \( h_t > 0 \ \forall t \).

- We can write as: \( u \left( \left( \frac{c_t}{h_t^{\varphi}} \right)^{\varphi} c_t^{1-\varphi} \right) \)
  - If \( \varphi = 0 \), the model collapses to standard model without habits.
  - If \( \varphi = 1 \), consumers only care about consumption relative to habits, not level.

- Regarding the law of motion of habit stock:
  - If \( \lambda = 1 \), \( h_t = h \) is a multiplicative factor on \( u \), back to standard model.
  - If \( \lambda = 0 \), habits equal to only past consumption \( c_{t-1} \).
Habit Formation (5): Problem with multiplicative habits

- The sequential problem with multiplicative habits:

\[
V_0 = \max_{\{c_t\}_{t=0}} \sum_{t=0}^{\infty} \beta^t \frac{\left(\frac{c_t}{h_t^\varphi}\right)^{1-\theta}}{1-\theta} - 1
\]

subject to:

\[
a_{t+1} = Ra_t - c_t + y_t, \quad a_0 \text{ given}
\]

\[
h_{t+1} = \lambda h_t + (1 - \lambda) c_t, \quad h_0 \text{ given}
\]

- The recursive problem with multiplicative habits:

\[
V(a, h) = \max_{a'} \left(\frac{Ra+y-a'}{h^\varphi}\right)^{1-\theta} - 1 + \beta \mathbb{E}[V(a', \lambda h + (1 - \lambda) (Ra + y - a'))]
\]

- This is a hard problem to solve analytically, need to go numerically.

- Why?
Habit Formation (6): Euler equation

- Denote derivatives as $\frac{\partial f}{\partial x}(x_t) = f_t^x$
- The FOC:
  $$u_t^c = \beta \mathbb{E}_t[V_{t+1}^a - (1 - \lambda)V_{t+1}^h]$$
- Envelope conditions:
  $$V_t^a = Ru_t^c + \beta R(1 - \lambda)\mathbb{E}_t[V_{t+1}^h]$$
  $$V_t^h = u_t^h + \beta \lambda \mathbb{E}_t[V_{t+1}^h]$$
- Euler equation:
  $$u_t^c = \beta \mathbb{E}_t[Ru_{t+1}^c + \beta R(1 - \lambda)\mathbb{E}_t[V_{t+2}^h] - (1 - \lambda) (u_{t+1}^h + \beta \lambda \mathbb{E}_{t+1}[V_{t+2}^h])]$$
- Use Law of Iterated Expectations and simplify as:
  $$u_t^c = \beta \mathbb{E}_t[Ru_{t+1}^c - (1 - \lambda)u_{t+1}^h] - \beta^2 (R - \lambda)(1 - \lambda) \mathbb{E}_t[V_{t+2}^h]$$
- Note that if $\lambda = 1$, then we have standard Euler Equation.
Habit Formation (7): Euler equation

• We can substitute recursively $V_{t+j}^{h}$ and get:

$$u^c_t = \beta \mathbb{E}_t[Ru^c_{t+1}] - (1 - \lambda)\beta \sum_{j=1}^{\infty} [\beta (R - \lambda)]^{j-1} \mathbb{E}_t[u^h_{t+j}]$$

• In the CRRA case with multiplicative habits, we get that:

$$u^h = - \varphi u^c = (ch^{-\varphi})^{-\theta} h^{-\varphi} > 0$$

• Substituting back we obtain:

$$u^c_t = \beta \mathbb{E}_t[Ru^c_{t+1}] + \varphi (1 - \lambda)\beta \sum_{j=1}^{\infty} [\beta (R - \lambda)]^{j-1} \mathbb{E}_t[u^c_{t+j}]$$

- standard

- full sequence of future marginal utilities $> 0$

• Since $u^c > 0$, we get $u^c_t > \beta R \mathbb{E}_t[u^c_{t+1}]$ which implies:

$$c_t < \beta R \mathbb{E}_t[c_{t+1}] \quad \text{(familiar?)}$$
Habit Formation (8): Asset pricing

- One of the applications of habits is in asset pricing (details tomorrow).

- Intertemporal elasticity of substitution is the inverse of relative risk aversion.

- Problem of normal CRRA utility function for asset pricing: \( u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta} \)
  
  - Plausible (small) values for \( \theta \)
  
  - Imply a too high IES: \( \frac{1}{\theta} \)

- Can habits reconcile low \( \theta \) and low IES?
Habit Formation (9): Risk aversion and IES

- **Additive CRRA case:**

\[
u(c_t, h_t) = \frac{(c_t - \varphi h_t)^{1-\theta}}{1-\theta}
\]
\[
u_c(c_t, h_t) = (c_t - \varphi h_t)^{-\theta}
\]
\[
u_{cc}(c_t, h_t) = -\theta (c_t - \varphi h_t)^{-(1+\theta)}
\]

- **Relative risk aversion:**

\[
\gamma(c_t, h_t) \equiv -\frac{c_t u_{cc}}{u_c} = \theta \left( \frac{c_t}{c_t - \varphi h_t} \right) > \theta
\]

And it is increasing in habits

\[
\frac{\partial \gamma(c_t, h_t)}{\partial h_t} = \frac{\theta \varphi c_t}{(c_t - \varphi h_t)^2} > 0
\]

- Habits make you act *as if* you are more risk averse: subtracting \(\varphi h_t\) brings you closer to the really curved region of the utility function.
Habit Formation (10): Risk aversion and IES

- The IES is the inverse of the coefficient of relative risk aversion:

\[
IES(c_t, h_t) = -\frac{u_c}{c_t u_{cc}} = \frac{1}{\theta} \frac{c_t - \varphi h_t}{c_t} = \frac{1}{\theta} \left( 1 - \varphi \frac{h_t}{c_t} \right)
\]

- With habit formation, what matters for intertemporal substitution is ‘effective’ consumption \(c_t - \varphi h_t\).
  - If \(\varphi h_t\) close to \(c_t\), IES becomes low even for low \(\theta\).

- Macro: Habits are useful to get ”hump shaped” impulse-responses to shocks, since it introduces persistence.

- Microfoundations are weak: very little empirical evidence and unclear how such a model aggregates
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Durables (1)

- Consumption of durable goods are very volatile ⇒ important role for business cycles.
- Evidence that households do not continuously adjust their stock of durables.
- Approaches in the literature:
  1. PIH model with accumulation of durables.
  2. Convex adjustment.
  3. Non-convex adjustment costs.
Durables (2): PIH model

• Utility depends on nondurable consumption $c_t$ as well as on the flow of services from durables, stock $D_t$.

\[ u(c_t, D_t) \]

• Bellman Equation with 4 state variables $(a_t, D_t, y_t, p_t)$:

\[ V(a_t, D_t, y_t, p_t) = \max_{c_t, e_t} \left\{ u(c_t, D_t) + \beta \mathbb{E}_t \left[ V(a_{t+1}, D_{t+1}, y_{t+1}, p_{t+1}) \right] \right\} \]

subject to wealth and durable accumulation:

\[ a_{t+1} = R_t (a_t + y_t - c_t - p_t e_t) \]
\[ D_{t+1} = (1 - \delta) D_t + e_t \]

▷ $e_t$ = expenditures on durable goods ($\sim$ investment).
▷ $\delta \in [0, 1]$ = depreciation rate.
▷ $p_t$ = relative price of durable goods.

• Durables bought in current period yield services starting in the next period.
Durables (3): PIH model

- FOCs:
  \[
  \frac{\partial u(c_t, D_t)}{\partial c_t} - \beta R_t \mathbb{E}_t \left[ \frac{\partial V(a_{t+1}, D_{t+1}, y_{t+1}, p_{t+1})}{\partial a_{t+1}} \right] = 0 \quad (c_t)
  \]
  \[
  \beta \mathbb{E}_t \left[ \frac{\partial V(a_{t+1}, D_{t+1}, y_{t+1}, p_{t+1})}{\partial D_{t+1}} - p_t R_t \frac{\partial V(a_{t+1}, D_{t+1}, y_{t+1}, p_{t+1})}{\partial a_{t+1}} \right] = 0 \quad (e_t)
  \]

- Envelope conditions:
  \[
  \frac{\partial V(a_t, D_t, y_t, p_t)}{\partial a_t} = \beta R_t \mathbb{E}_t \left[ \frac{\partial V(a_{t+1}, D_{t+1}, y_{t+1}, p_{t+1})}{\partial a_{t+1}} \right]
  \]
  \[
  \frac{\partial V(a_t, D_t, y_t, p_t)}{D_t} = \frac{\partial u(c_t, D_t)}{\partial D_t} + \beta (1 - \delta) \mathbb{E}_t \left[ \frac{\partial V(a_{t+1}, D_{t+1}, y_{t+1}, p_{t+1})}{\partial D_{t+1}} \right]
  \]

- Euler equation for consumption still holds, but now utility of non-durables may be affected by durables (if \( u_{c,D} \neq 0 \)):
  \[
  \frac{\partial u(c_t, D_t)}{\partial c_t} = \beta R_t \mathbb{E}_t \left[ \frac{\partial u(c_{t+1}, D_{t+1})}{\partial c_{t+1}} \right]
  \]
Durables (4): PIH model

- For durables’ Euler equation, we need an expression for \( \frac{\partial V(a_t, D_t, y_t, p_t)}{D_t} \).

- Combine the FOC for \( e_t \) and the envelope condition for \( D_t \):

\[
\frac{\partial V(a_t, D_t, y_t, p_t)}{D_t} = \frac{\partial u(c_t, D_t)}{\partial D_t} + (1 - \delta) p_t \beta R_t \mathbb{E}_t \left[ \frac{\partial V(a_{t+1}, D_{t+1}, y_{t+1}, p_{t+1})}{\partial a_{t+1}} \right]
\]

\[
= \frac{\partial u(c_t, D_t)}{\partial D_t} + (1 - \delta) p_t \frac{\partial u(c_t, D_t)}{\partial c_t}
\]

- Substitute the previous into the envelope condition for \( D_t \) (evaluated at \( t \) on the LHS and forwarded to \( t + 1 \) in the RHS) and rearrange to obtain:

\[
p_t \frac{\partial u(c_t, D_t)}{\partial c_t} = \beta \mathbb{E}_t \left[ \frac{\partial u(c_{t+1}, D_{t+1})}{\partial D_{t+1}} + (1 - \delta) p_{t+1} \frac{\partial u(c_{t+1}, D_{t+1})}{\partial c_{t+1}} \right]
\]

\[\text{cost of purchasing a unit of durable} \]

\[\text{direct utility of durable} \]

\[\text{sell undepreciated durable and consume} \]
Durables (5): Homework

- Consider consumption of ONLY durable goods: \( u(D_t) \) (no need for \( p_t \) now)
- The problem is given by:

\[
V(a_t, D_t, y_t) = \max_{e_t} \quad u(D_t) + \beta E_t \left[ V(a_{t+1}, D_{t+1}, y_{t+1}) \right]
\]

subject to wealth and durable accumulation:

\[
\begin{align*}
    a_{t+1} &= R_t (a_t + y_t - e_t) \\
    D_{t+1} &= (1 - \delta) D_t + e_t
\end{align*}
\]

- This problem yields a standard Euler equation for the stock of durables:

\[
u'(D_t) = \beta RE_t [u'(D_{t+1})]
\]

- With quadratic utility, stock of durables becomes a martingale.
- But purchases \( e_t \) are an ARMA(1,1) process.
Durables (6): Empirical Tests

• Mankiw (1982) studies exactly the previous problem, and tests the ARMA(1,1) prediction for purchases of durables:

\[ e_{t+1} = a_0 + a_1 e_t + \varepsilon_{t+1} - (1 - \delta)\varepsilon_t \]

where AR coefficient is \( a_1 = \beta R \) and MA piece is parametrized by \( 1 - \delta \).

• Note that if \( \delta = 1 \) (non-durable), we have random walk.

• What does the MA component do?
  ▶ Motivates or discourages purchases depending on past shocks.
  ▶ Positive shock in the past, current purchases will be pushed down.
  ▶ **Volatile purchases smooths the stock of durables!**
Durables (7): Failure of PIH and other models

- The previous model is rejected by the data.
  - Mankiw estimates lagged MA coefficient of -0.1, which implies $\delta \approx 0.9$.
  - This is not consistent with data on depreciation of durables.
- Also, households do not adjust the stock of durable every period, but they do lumpy purchases (infrequent and large), because of:
  1. irreversibilities (due to imperfect information about quality of used durables)
  2. discrete nature of goods
  3. nature of adjustment costs
- Literature has explored durables with adjustment costs (convex and non-convex) as well as irreversibilities, as with investment.
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3. Hyperbolic Discounting
Hyperbolic Discounting (1): Motivation

- Consider the following experiment.

- One hour before the exam, I give you two options:
  
  - Option A: I give you 1 week more to study, but the exam will be 5% longer (more difficult).
  
  - Option B: you take the exam in one hour as it is.
Hyperbolic Discounting (2): Motivation

• Now consider a new experiment.

• The final exam is in exactly one year.

  ▶ Option A: I give you 1 week more to study (the exam will be in 365+7 days), but the exam will be 5% longer (more difficult).

  ▶ Option B: you take the exam in 365 days.
Hyperbolic Discounting (3): Motivation

• The standard model implies that you should choose the same!

• The standard consumption model features additive separability and **exponential discounting:**

\[
U(c_0, c_1, c_2, \ldots) = \delta(0) u(c_0) + \delta(1) E_0 u(c_1) + \delta(2) E_0 (c_2) + \ldots
\]

where the discount is given by:

\[
\delta(t) = \beta^t = \exp(t \log \beta) = \exp\left(\hat{\beta} t\right)
\]

• Constant discount rate:

\[
- \frac{\partial \delta(t)/\partial t}{\delta(t)} = - \frac{\partial \beta^t/\partial t}{\beta^t} = - \ln \beta
\]

• It implies that \(MRS(t, t + 1) = MRS(t + k, t + k + 1)\)

\[
\frac{\beta u'(c_{t+1})}{u'(c_t)} = \frac{\beta u'(c_{t+k+1})}{u'(c_{t+k})}
\]
Hyperbolic Discounting (4): Specification

- Now let's consider hyperbolic discounting:

\[ \delta(t) = \frac{1}{(1 + \alpha t)^{\gamma / \alpha}} \]

- The discount rate is given by:

\[
\frac{\partial \delta(t)}{\partial t} \bigg/ \delta(t) = \frac{\partial \left[(1 + \alpha t)^{-\gamma / \alpha}\right]}{\partial t} = \frac{\gamma}{(1 + \alpha t)}
\]

- Discount falls with time:
  - In the short run, discount is \( \gamma \).
  - In the limit \( t \to \infty \), discount is zero.

- Trade-off today vs. tomorrow is less heavily discounted than same trade-off in future periods.
Hyperbolic Discounting (5): Approximation

- A simple approximation in called quasi-hyperbolic is the following:

\[
\delta(t) = \delta \beta^t \text{ for } t \geq 1, \text{ where } \beta < 1 \text{ and } \delta < 1
\]

\[
U(c_t, c_{t+1}, \ldots) = u(c_0) + \delta \beta \mathbb{E}_0 u(c_1) + \delta \beta^2 \mathbb{E}_0 (c_2) + \ldots
\]

- This is often called \( \delta - \beta \) model.

- It captures the basic property of the sharp short-run drop in valuations.


- Experimental data confirms hyperbolic discounting.
Hyperbolic Discounting: (6) Different discounts

Figure 1: Exponential and hyperbolic discount functions

- **Exponential**: $\delta^t$, with $\delta = .944$.
- **Hyperbolic**: $(1+\alpha)^{-\gamma t}$, with $\alpha = 4$ and $\gamma = 1$.
- **Quasi-hyperbolic**: $\{1, \beta \delta, \beta \delta^2, \beta \delta^3, \ldots\}$, with $\beta = .7$ and $\delta = .957$.

Harris and Liabson (2001)
Hyperbolic Discounting (7): Time inconsistency

• Main result: time inconsistency.
  
  ▶ Example: I am too lazy to exercise today, because I discount the benefit of get fit heavily. But today I want myself to exercise tomorrow.

• Time consistent solution implies that my choices today take into account that I want my future self to behave the way I want (not necessarily the optimal choice for him).
  
  ▶ Gym subscription
  ▶ Sign up to present at a student seminar even though I have nothing (yet) to present.
  ▶ Procrastination.
  ▶ Drug abuse.
Hyperbolic Discounting (8): Macroeconomic Implications


• Studies hyperbolic consumers in macro.

  ▶ Buy illiquid assets (houses).

  ▶ Borrow on credit cards.

  • Too much short term debt at high interest rates.

  ▶ Little liquid wealth, unable to smooth consumption ⇒

    • High comovement between income and consumption (especially around retirement, when labour income falls).