Advanced Macroeconomics II

Lecture 7

Consumption: Non-Separable Preferences

Isaac Baley
UPF & Barcelona GSE

February 4, 2016

Roadmap: Non-Separable Preferences

1. Habit Formation
2. Durables
3. Hyperbolic Discounting

Non-Separable Preferences

- So far we have assumed that preferences are separable over time $\Rightarrow$ consumption expenditure in a given period does not have any effect in the marginal utility of any other period.

- There are three simple ways to depart from time separability:

  - **Habit formation**: Utility comes from current consumption AND past consumption (or consumptions of others).

  - **Durable goods**: Certain types of goods yield utility flows many periods after its purchase, for example: fridge, a TV set, a car, ...

  - **Hyperbolic discounting**: present is disproportionately more discounted than future periods.

Roadmap: Non-Separable Preferences

1. **Habit Formation**
2. Durables
3. Hyperbolic Discounting

Habit Formation (1): Idea

- Households care about current consumption **relative** to past consumption.
- Preferences with habit formation break the link between
  a) preferences over consumption at different points in time and
  b) consumption over different states of nature.
- **Internal habits:** care about consumption relative to past consumption.
  - get less utility from consuming the same as before
  - hard problem to solve
- **External habits:** care about consumption relative to other people (average).
  - get less utility from consuming less than the rest
  - Easier to solve: with many agents, own consumption choice does not affect average consumption.

Habit Formation (2): Subtractive model

- Include past consumptions summarised via a habit stock h_t in utility:

$$u(c_t, h_t)$$

- Early models assumed an additive form, where utility is derived from the difference between current consumption and the habit stock:

$$u(c_t, h_t) = u(c_t - \varphi h_t)$$

  - Simplest case: habit stock equal to last periods consumption $h_t = c_{t-1}$

$$u(c_t, c_{t-1}) = u(c_t - \varphi c_{t-1})$$

- More general law of motion for h_t:

$$h_t = \lambda h_{t-1} + (1-\lambda) c_{t-1} = (1-\lambda) \sum_{j=1}^{\infty} \lambda^{j-1} c_{t-j}, \quad \lambda \in [0,1]$$

  - λ measures the persistence of the habit process.
  - High λ indicates that consumption in the distant past is hard to forget.

Habit Formation (3): Additive model and CRRA

- For the outer utility function $u(x)$ we require:
  - $u_h < 0$ and $u_{ch} > 0$ (adjacent complementarity)
  - standard properties $u_c > 0$ and $u_{cc} < 0$.
- **Example 1:** Problem set with quadratic utility.
- **Example 2:** CRRA specification:

$$
\begin{aligned}
u(c_t, h_t) &= \frac{(c_t - \varphi h_t)^{1-\theta} - 1}{1-\theta}, \qquad \theta > 0,\ \varphi > 0 \\
h_t &= \lambda h_{t-1} + (1-\lambda)\, c_{t-1}, \qquad \lambda \in [0,1]
\end{aligned}
$$

  - $u_h = -\varphi\,(c_t - \varphi h_t)^{-\theta} < 0$ which requires $\varphi > 0$
  - $u_{ch} = \theta\varphi\,(c_t - \varphi h_t)^{-(1+\theta)} > 0$ which requires $\theta > 0$
  - φ usually between 0 and 1, measures the relative weight of habits
  - What does θ measure in this case?
- Problem with CRRA: potential for negative argument inside $u(\cdot)$!

Habit Formation (4): Multiplicative habits

- Multiplicative form introduced by Abel (1990) and Galí(1994):

$$
\begin{aligned}
u(c_t, h_t) &= u\left(\frac{c_t}{h_t^{\varphi}}\right) \\
h_t &= \lambda h_{t-1} + (1-\lambda)\, c_{t-1}, \qquad \lambda \in [0,1]
\end{aligned}
$$

- No more problems with assuming u to be CRRA.
  - If $c_t > 0\ \forall t$, then $h_t > 0\ \forall t$.
- We can write as: $u\left(\left(\frac{c_t}{h_t}\right)^{\varphi} c_t^{1-\varphi}\right)$
  - If $\varphi = 0$, the model collapses to standard model without habits.
  - If $\varphi = 1$, consumers only care about consumption relative to habits, not level.
- Regarding the law of motion of habit stock:
  - If $\lambda = 1$, $h_t = h$ is a multiplicative factor on u, back to standard model.
  - If $\lambda = 0$, habits equal to only past consumption c_{t-1}.

Habit Formation (5): Problem with multiplicative habits

- The sequential problem with multiplicative habits:

$$V_0 = \max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{\left(\frac{c_t}{h_t^{\varphi}}\right)^{1-\theta} - 1}{1-\theta}$$

subject to:

$$a_{t+1} = Ra_t - c_t + y_t, \qquad a_0 \text{ given}$$

$$h_{t+1} = \lambda h_t + (1-\lambda)\, c_t, \qquad h_0 \text{ given}$$

- The recursive problem with multiplicative habits:

$$V(a,h) = \max_{a'} \frac{\left(\frac{Ra+y-a'}{h^{\varphi}}\right)^{1-\theta} - 1}{1-\theta} + \beta \mathbb{E}[V(a', \lambda h + (1-\lambda)(Ra + y - a'))]$$

- This is a hard problem to solve analytically, need to go numerically.
- Why?

Habit Formation (6): Euler equation

- Denote derivatives as $\frac{\partial f}{\partial x}(x_t) = f_t^x$
- The FOC:

$$u_t^c = \beta \mathbb{E}_t[V_{t+1}^a - (1-\lambda)V_{t+1}^h]$$

- Envelope conditions:

$$\begin{aligned} V_t^a &= R u_t^c + \beta R(1-\lambda)\mathbb{E}_t[V_{t+1}^h] \\ V_t^h &= u_t^h + \beta\lambda\mathbb{E}_t[V_{t+1}^h] \end{aligned}$$

- Euler equation:

$$u_t^c = \beta \mathbb{E}_t[R u_{t+1}^c + \beta R(1-\lambda)\mathbb{E}_t[V_{t+2}^h] - (1-\lambda)\left(u_{t+1}^h + \beta\lambda\mathbb{E}_{t+1}[V_{t+2}^h]\right)]$$

- Use Law of Iterated Expectations and simplify as:

$$u_t^c = \beta \mathbb{E}_t[R u_{t+1}^c - (1-\lambda)u_{t+1}^h] - \beta^2(R-\lambda)(1-\lambda)\mathbb{E}_t[V_{t+2}^h]$$

- Note that if $\lambda = 1$, then we have standard Euler Equation.

Habit Formation (7): Euler equation

- We can substitute recursively V^h_{t+j} and get:

$$u^c_t = \beta \mathbb{E}_t[R u^c_{t+1}] - (1-\lambda)\beta \sum_{j=1}^{\infty} [\beta(R-\lambda)]^{j-1} \mathbb{E}_t[u^h_{t+j}]$$

- In the CRRA case with multiplicative habits, we get that:

$$u^h = -\varphi u^c = \left(ch^{-\varphi}\right)^{-\theta} h^{-\varphi} > 0$$

- Substituting back we obtain:

$$u^c_t = \underbrace{\beta \mathbb{E}_t[R u^c_{t+1}]}_{\text{standard}} + \underbrace{\varphi(1-\lambda)\beta \sum_{j=1}^{\infty} [\beta(R-\lambda)]^{j-1} \mathbb{E}_t[u^c_{t+j}]}_{\text{full sequence of future marginal utilities}>0}$$

- Since $u^c > 0$, we get $u^c_t > \beta R \mathbb{E}_t[u^c_{t+1}]$ which implies :

$$c_t < \beta R \mathbb{E}_t[c_{t+1}] \qquad \text{(familiar?)}$$

Habit Formation (8): Asset pricing

- One of the applications of habits is in asset pricing (details tomorrow).
- Intertemporal elasticity of substitution is the inverse of relative risk aversion.
- Problem of normal CRRA utility function for asset pricing: $u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta}$
  - Plausible (small) values for θ
  - Imply a too high IES: $\frac{1}{\theta}$
- Can habits reconcile low θ and low IES?

Habit Formation (9): Risk aversion and IES

- Additive CRRA case:

$$\begin{aligned} u(c_t, h_t) &= \frac{(c_t - \varphi h_t)^{1-\theta}}{1-\theta} \\ u_c(c_t, h_t) &= (c_t - \varphi h_t)^{-\theta} \\ u_{cc}(c_t, h_t) &= -\theta\,(c_t - \varphi h_t)^{-(1+\theta)} \end{aligned}$$

- Relative risk aversion:

$$\gamma(c_t, h_t) \equiv -\frac{c_t u_{cc}}{u_c} = \theta\left(\frac{c_t}{c_t - \varphi h_t}\right) > \theta$$

  And it is increasing in habits

$$\frac{\partial \gamma(c_t, h_t)}{\partial h_t} = \frac{\theta \varphi c_t}{(c_t - \varphi h_t)^2} > 0$$

- Habits make you act *as if* you are more risk averse: subtracting φh_t brings you closer to the really curved region of the utility function.

Habit Formation (10): Risk aversion and IES

- The IES is the inverse of the coefficient of relative risk aversion:

$$IES(c_t, h_t) = -\frac{u_c}{c_t u_{cc}} = \frac{1}{\theta}\frac{c_t - \varphi h_t}{c_t} = \frac{1}{\theta}\left(1 - \varphi \frac{h_t}{c_t}\right)$$

- With habit formation, what matters for intertemporal substitution is 'effective' consumption $c_t - \varphi h_t$.
  - If φh_t close to c_t, IES becomes low even for low θ.
- Macro: Habits are useful to get "hump shaped" impulse-responses to shocks, since it introduces persistence.
- Microfoundations are weak: very little empirical evidence and unclear how such a model aggregates

Roadmap: Non-Separable Preferences

1. Habit Formation
2. **Durables**
3. Hyperbolic Discounting

Durables (1)

- Consumption of durable goods are very volatile $\Rightarrow$ important role for business cycles.
- Evidence that households do not continuously adjust their stock of durables.
- Approaches in the literature:
  1. PIH model with accumulation of durables.
  2. Convex adjustment.
  3. Non-convex adjustment costs.

Durables (2): PIH model

- Utility depends on nondurable consumption c_t as well as on the flow of services from durables, stock D_t.

$$u(c_t, D_t)$$

- Bellman Equation with 4 state variables (a_t, D_t, y_t,p_t):

$$V(a_t, D_t, y_t, p_t) = \max_{c_t, e_t} \left\{ u(c_t, D_t) + \beta \mathbb{E}_t \left[V(a_{t+1}, D_{t+1}, y_{t+1}, p_{t+1}) \right] \right\}$$

subject to wealth and durable accumulation:

$$\begin{aligned} a_{t+1} &= R_t (a_t + y_t - c_t - p_t e_t) \\ D_{t+1} &= (1-\delta) D_t + e_t \end{aligned}$$

  - e_t = expenditures on durable goods ($\sim$ investment).
  - $\delta \in [0, 1]$ = depreciation rate.
  - p_t = relative price of durable goods.
- Durables bought in current period yield services starting in the next period.

Durables (3): PIH model

- FOCs:

$$\frac{\partial u(c_t, D_t)}{\partial c_t} - \beta R_t \mathbb{E}_t\left[\frac{\partial V(a_{t+1}, D_{t+1}, y_{t+1}, p_{t+1})}{\partial a_{t+1}}\right] = 0 \qquad (c_t)$$

$$\beta \mathbb{E}_t\left[\frac{\partial V(a_{t+1}, D_{t+1}, y_{t+1}, p_{t+1})}{\partial D_{t+1}} - p_t R_t \frac{\partial V(a_{t+1}, D_{t+1}, y_{t+1}, p_{t+1})}{\partial a_{t+1}}\right] = 0 \qquad (e_t)$$

- Envelope conditions:

$$\frac{\partial V(a_t, D_t, y_t, p_t)}{\partial a_t} = \beta R_t \mathbb{E}_t\left[\frac{\partial V(a_{t+1}, D_{t+1}, y_{t+1}, p_{t+1})}{\partial a_{t+1}}\right]$$

$$\frac{\partial V(a_t, D_t, y_t, p_t)}{D_t} = \frac{\partial u(c_t, D_t)}{\partial D_t} + \beta(1-\delta)\mathbb{E}_t\left[\frac{\partial V(a_{t+1}, D_{t+1}, y_{t+1}, p_{t+1})}{\partial D_{t+1}}\right]$$

- Euler equation for consumption still holds, but now utility of non-durables may be affected by durables (if $u_{c,D} \neq 0$):

$$\frac{\partial u(c_t, D_t)}{\partial c_t} = \beta R_t \mathbb{E}_t\left[\frac{\partial u(c_{t+1}, D_{t+1})}{\partial c_{t+1}}\right]$$

Durables (4): PIH model

- For durables' Euler equation, we need an expression for $\frac{\partial V(a_t, D_t, y_t, p_t)}{D_t}$.
- Combine the FOC for e_t and the envelope condition for D_t:

$$\frac{\partial V(a_t, D_t, y_t, p_t)}{D_t} = \frac{\partial u(c_t, D_t)}{\partial D_t} + (1-\delta)\, p_t \underbrace{\beta R_t \mathbb{E}_t \left[\frac{\partial V(a_{t+1}, D_{t+1}, y_{t+1}, p_{t+1})}{\partial a_{t+1}} \right]}_{= \frac{\partial u(c_t, D_t)}{\partial c_t} \quad \text{from FOC for } c_t}$$

$$= \frac{\partial u(c_t, D_t)}{\partial D_t} + (1-\delta)\, p_t \frac{\partial u(c_t, D_t)}{\partial c_t}$$

- Substitute the previous into the envelope condition for D_t (evaluated at t on the LHS and forwarded to $t+1$ in the RHS) and rearrange to obtain:

$$\underbrace{p_t \frac{\partial u(c_t, D_t)}{\partial c_t}}_{\text{cost of purchasing a unit of durable}} = \beta \mathbb{E}_t \left[\underbrace{\frac{\partial u(c_{t+1}, D_{t+1})}{\partial D_{t+1}}}_{\text{direct utility of durable}} + \underbrace{(1-\delta)\, p_{t+1} \frac{\partial u(c_{t+1}, D_{t+1})}{\partial c_{t+1}}}_{\text{sell undepreciated durable and consume}} \right]$$

Durables (5): Homework

- Consider consumption of ONLY durable goods: $u(D_t)$ (no need for p_t now)
- The problem is given by:

$$V(a_t, D_t, y_t) = \max_{e_t} \quad u(D_t) + \beta \mathbb{E}_t [V(a_{t+1}, D_{t+1}, y_{t+1})]$$

subject to wealth and durable accumulation:

$$\begin{aligned} a_{t+1} &= R_t (a_t + y_t - e_t) \\ D_{t+1} &= (1-\delta) D_t + e_t \end{aligned}$$

- This problem yields a standard Euler equation for the stock of durables:

$$u'(D_t) = \beta R \mathbb{E}_t [u'(D_{t+1})]$$

- With quadratic utility, stock of durables becomes a martingale.
- But purchases e_t are an ARMA(1,1) process.

Durables (6): Empirical Tests

- Mankiw (1982) studies exactly the previous problem, and tests the ARMA(1,1) prediction for purchases of durables:

$$e_{t+1} = \underbrace{a_0 + a_1 e_t}_{AR(1)} + \underbrace{\varepsilon_{t+1} - (1-\delta)\varepsilon_t}_{MA(1)}$$

  where AR coefficient is $a_1 = \beta R$ and MA piece is parametrized by $1-\delta$.

- Note that if $\delta = 1$ (non-durable), we have random walk.
- What does the MA component do?
  - Motivates or discourages purchases depending on past shocks.
  - Positive shock in the past, current purchases will be pushed down.
  - **Volatile purchases smooths the stock of durables!**

Durables (7): Failure of PIH and other models

- The previous model is rejected by the data.
  - Mankiw estimates lagged MA coefficient of -0.1, which implies $\delta \approx 0.9$.
  - This is not consistent with data on depreciation of durables.
- Also, households do not adjust the stock of durable every period, but they do lumpy purchases (infrequent and large), because of:
  i) irreversibilities (due to imperfect information about quality of used durables)
  ii) discrete nature of goods
  iii) nature of adjustment costs
- Literature has explored durables with adjustment costs (convex and non-convex) as well as irreversibilities, as with investment.

Roadmap: Non-Separable Preferences

1. Habit Formation
2. Durables
3. **Hyperbolic Discounting**

Hyperbolic Discounting (1): Motivation

- Consider the following experiment.
- One hour before the exam, I give you two options:
  - Option A: I give you 1 week more to study, but the exam will be 5% longer (more difficult).
  - Option B: you take the exam in one hour as it is.

Hyperbolic Discounting (2): Motivation

- Now consider a new experiment.
- The final exam is in exactly one year.
    - Option A: I give you 1 week more to study (the exam will be in 365+7 days), but the exam will be 5% longer (more difficult).
    - Option B: you take the exam in 365 days.

Hyperbolic Discounting (3): Motivation

- The standard model implies that you should choose the same!
- The standard consumption model features additive separability and **exponential discounting:**

$$U\left(c_0, c_1, c_2, ...\right) = \delta\left(0\right) u\left(c_0\right) + \delta\left(1\right) \mathbb{E}_0 u\left(c_1\right) + \delta\left(2\right) \mathbb{E}_0\left(c_2\right) + ...$$

  where the discount is given by:

$$\delta\left(t\right) = \beta^t = \exp\left(t \log \beta\right) = \exp\left(\hat{\beta} t\right)$$

- Constant discount rate:

$$-\frac{\partial \delta(t)/\partial t}{\delta(t)} = -\frac{\partial \beta^t/\partial t}{\beta^t} = -\ln \beta$$

- It implies that $MRS\left(t, t+1\right) = MRS\left(t+k, t+k+1\right)$

$$\frac{\beta u'\left(c_{t+1}\right)}{u'\left(c_t\right)} = \frac{\beta u'\left(c_{t+k+1}\right)}{u'\left(c_{t+k}\right)}$$

Hyperbolic Discounting (4): Specification

- Now lets consider hyperbolic discounting:

$$\delta(t) = \frac{1}{(1+\alpha t)^{\frac{\gamma}{\alpha}}}$$

- The discount rate is given by:

$$\frac{\partial \delta(t)/\partial t}{\delta(t)} = \frac{\partial \left[(1+\alpha t)^{\frac{-\gamma}{\alpha}} \right] / \partial t}{(1+\alpha t)^{\frac{-\gamma}{\alpha}}} = \frac{\gamma}{(1+\alpha t)}$$

- Discount falls with time:
  - In the short run, discount is γ.
  - In the limit $t \to \infty$, discount is zero.
- Trade-off today vs. tomorrow is less heavily discounted than same trade-off in future periods.

Hyperbolic Discounting (5): Approximation

- A simple approximation in called quasi-hyperbolic is the following:

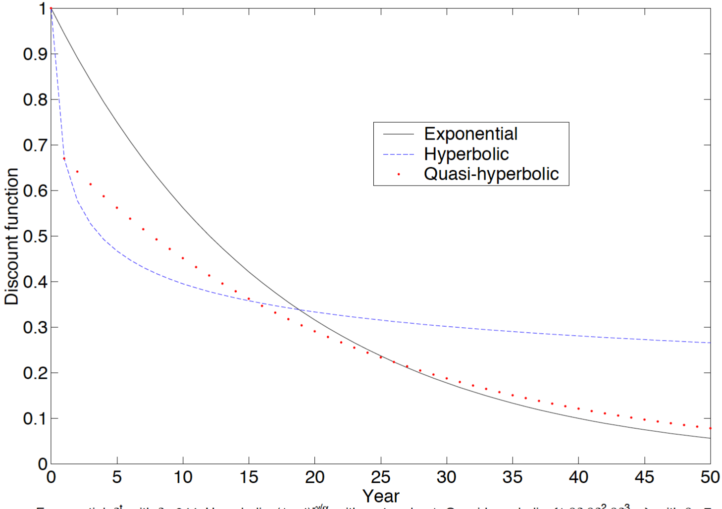
$$\delta(t) = \delta\beta^t \text{ for } t \geq 1, \text{ where } \beta < 1 \text{ and } \delta < 1$$

$$U(c_t, c_{t+1}, ...) = u(c_0) + \delta\beta\mathbb{E}_0 u(c_1) + \delta\beta^2\mathbb{E}_0(c_2) + ...$$

- This is often called $\delta - \beta$ model.
- It captures the basic property of the sharp short-run drop in valuations.
- Laibson (1997) based on Phelps and Pollak (1968).
- Experimental data confirms hyperbolic discounting.

Hyperbolic Discounting: (6) Different discounts

Figure 1: Exponential and hyperbolic discount functions



Exponential: δ^t, with δ=.944. Hyperbolic: $(1+\alpha t)^{-\gamma/\alpha}$, with α=4 and γ=1. Quasi-hyperbolic: $\{1,\beta\delta,\beta\delta^2,\beta\delta^3,...\}$, with β=.7 and δ=.957.

Harris and Liabson (2001)

Hyperbolic Discounting (7): Time inconsistency

- Main result: time inconsistency.
  - Example: I am too lazy to exercise today, because I discount the benefit of get fit heavily. But today I want myself to exercise tomorrow.
- Time consistent solution implies that my choices today take into account that I want my future self to behave the way I want (not necessarily the optimal choice for him).
  - Gym subscription
  - Sign up to present at a student seminar even though I have nothing (yet) to present.
  - Procrastination.
  - Drug abuse.

Hyperbolic Discounting (8): Macroeconomic Implications

- Angeletos et al (JEP, 2001): "The hyperbolic consumption model: Calibration, Simulation, and Empirical Evaluation."
- Studies hyperbolic consumers in macro.
  - Buy illiquid assets (houses).
  - Borrow on credit cards.
    - Too much short term debt at high interest rates.
  - Little liquid wealth, unable to smooth consumption $\Rightarrow$
    - High comovement between income and consumption (especially around retirement, when labour income falls).