Discussion: Rational Inattention via Ignorance Equivalence

Armenter, Müller-Itten & Stangebye

Isaac Baley
UPF, CREi and Barcelona GSE

2nd Winter Workshop of the Catalan Economic Society,
December 2019

Rational Inattention: Background

- Choice under costly information acquisition
  - Long history, goes back to Simon in 50's, Stigler in 60's
- **Rational Inattention**: [Sims (2003)]
  - Use concepts from *Information Theory*
  - Agents choose structure and quantity of info, e.g. noisy signals
  - Cost of acquiring info proportional to informational gains

$$\text{cost of info} = \lambda \times \underbrace{\text{Shannon's entropy}}_{\text{log distance btw prior and posterior}}$$

- Early applications in Macro (price-setting, portfolio choice...)
  - Limited due to difficulty
  - Special cases: Gaussian + Quadratic, static

Rational Inattention: Background (...)

- Renewed interest in Micro/Decision Theory/Behavioral
- **Discrete choice**
  - Finite states: $\omega \in \Omega$
  - Discrete actions: $\mathcal{A} = \{a_1, a_2, \ldots, a_N\}$
  - Preferences: $u(a, \omega)$, λ
- **Solution: Stochastic choice**
  - Choose directly probability over actions given state $\Pr(a|\omega)$
  - **Consideration set**: $\mathcal{B} = \{a| \Pr(a) > 0\} \subseteq \mathcal{A}$
    - Used in psychology (heuristics), marketing ("evoked" set)

Rational Inattention: Some Background (...)

- **Necessary Condition:** [Matejka & McKay, 2015]
  - Multinominal logit: "twist" mass towards most favorable outcomes

$$\Pr(a|\omega) = \frac{\Pr(a) e^{\frac{u(a,\omega)}{\lambda}}}{\sum_{a' \in \mathcal{B}} \Pr(a') e^{\frac{u(a',\omega)}{\lambda}}}, \qquad \mathcal{B} \ \underline{\text{exogenous}}$$

- **Necessary + Sufficient Conditions:** [Caplin, Dean & Leahy, 2019]
  - Determine set of chosen/ignored actions
  - $\mathcal{B}$ endogenous, determined by λ, u and prior beliefs
  - RI agents reduce alternatives before gathering information!
- **Continuous state** [Jung, et.al, 2015]
  - Even with continuous $\mathcal{A}$, consideration set $\mathcal{B}$ is lower dimensional!

This Paper: Theory

- **Equivalence**

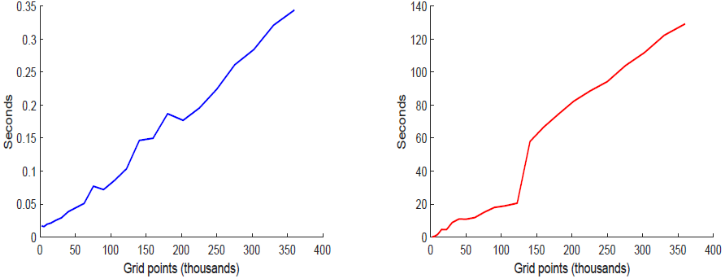
  Rational Inattention (RI) $\iff$ Geometric Attention Problem (GAP)

- **Key new concept:** Ignorance equivalent $\tilde{a}$
  - Fictional alternative chosen unconditionally (blindly, no learning!)
  - Analogous to certainty equivalent

- Exploit **simpler structure of GAP** and convex optimization to...
  1. Derive necessary and sufficient conditions for GAP
  2. Transform RI into EU maximization over modified menu
  3. Tailor fast algorithm to estimate $\mathcal{B}$ and bound its cardinality $\#(\mathcal{B})$

Application I: Matejka (2015)

- Pricing problem of RI monopolist under stochastic demand elasticity
- Monopolist puts positive probability only on few "reference" prices
- Proposed algorithm: Speed + Scalibility

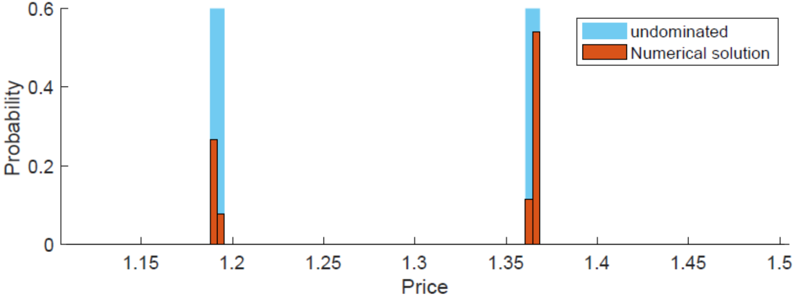


(a) GAP-SQP algorithm (b) Blahut-Arimoto algorithm

Figure 4: Running times across grid precision

Application I: Matejka (2015)

- Where is the consideration set $\mathcal{B}$?
- True solution lies within 95% cover $\Rightarrow$ algorithm performs well

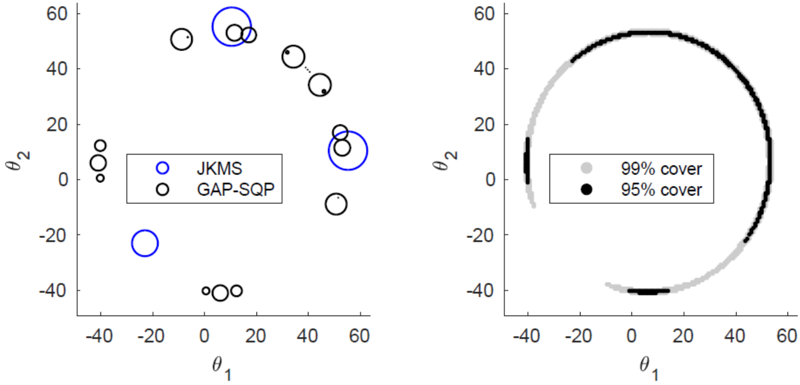


Application II: Jung *et.al.* (2019)

- Investor with CARA utility chooses portfolio weights $(\theta_0, \theta_1, \theta_2)$
  - one riskless asset
  - two risky assets (unlearnable + learnable noise)
- Jung *et. al.* show numerically (**but not analytically!**) that solution's support is finite
- New method yields **more points** than those found by Jung *et.al.*!
  - Consideration set looks like circle $\rightarrow$ action space may be continuous
  - Illustrates power of algorithm

Application II: Jung et. al (2019)

Solution in this example might be continuous!



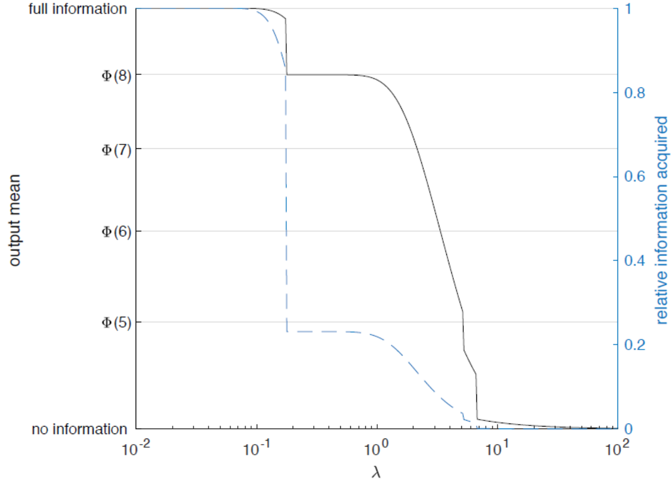
(a) estimated portfolio choice (b) partial covers

Application III: Task assignment

- New example to show application in environments with finite actions
- Manager allocates 10 workers across three tasks:
  - Task 0 is never critical (dismissal)
  - Task 1 or 2 may be critical (unknown to manager beforehand)
- Output determined by number of workers in critical task
  - One (unknown) slacker prevents others from producing
  - 20 states (2 potential critical tasks X 10 potential slackers)
  - Manager has 6124 potential assignments
- Numerically solution is...
  - Very rich, non-linear in information costs, multiple strategies

Application III: Task assignment

Solution to RI problems can be quite complex!



(a) Mean output (solid black) and information acquisition (dashed blue).

Comments and Questions

- Hard paper to read the first time.... and the second (after skimming literature)
  - Perhaps make paper more self-contained
- On the theory: Equivalence of RI to GAP very novel
  - Use of geometric intuition to tailor the algorithm is quite neat
- On the applications: can you explore dynamic settings?
  - Consumption-savings problem (Tutino, 2013)
  - Simple 2-period model (Sims, 2006)

Comments and Questions (...)

- Recent experimental evidence rejects Shannon's model

$$\log\left(\frac{P(\omega|a)}{P(\omega|b)}\right) = \frac{u(a,\omega) - u(b,\omega)}{\lambda}$$

  - Linearity and symmetry are rejected [Caplin, Dean & Neligh]

- Literature proposes alternative "posterior-separable" cost functions

  - Does the equivalence RI $\iff$ GAP still hold under alternative costs?

  - Can the algorithm be extended to consider alternative costs?