Trump's Paradox: Might Uncertainty Encourage International Trade?

PRELIMINARY AND WORK IN PROGRESS.

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ABSTRACT

Common wisdom dictates that uncertainty impedes trade. Yet, recent uncertainty about the future of global trade has coincided with an export frenzy among some trading partners. This prompts us to explore a standard, two-country Armington model of trade, and ask how mutual uncertainty about a trading partner's exports affects the volume of trade. We discover that, just like an uncertain return might prompt precautionary savings, trade uncertainty can induce precautionary exporting. However, the logic differs. Precautionary exporting does not rely on $u''' > 0$. Instead, exports rise because when all firms are uncertain, all firms get better terms of trade in states where consumption is most valuable. Trade creates value, in part, by offering a mechanism to share risk. This risk sharing is most effective when both parties are uninformed. We derive general conditions on preferences such that when mutual uncertainty rises, the trade-promoting improvement in the mean terms of trade outweighs the trade-reducing rise in variance. With CES preferences, when goods are not very substitutable, uncertainty undermines trade. But with empirically plausible elasticities of substitution, uncertainty encourages more trade.

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In any discussion of the frictions to cross-border trade, inevitably one that arises is some discussion about information and uncertainty. Portes and Rey (2005) show that the volume of phone calls between two countries predicts how much they trade. Gould (1994) and Rauch and Trindade (2002) argue that immigrants trade more with their home countries. Recently, trade has become particularly uncertain with the rise of governments openly hostile to existing tariff agreements. The argument that information frictions create uncertainty about foreign economies, and that this uncertainty deters risk-averse potential exporters is compelling. In partial-equilibrium settings, the effect of an increase in uncertainty is to reduce the certainty-equivalent expected return, which acts like a tax on transactions. Yet, in the face of substantial uncertainty, global trade remains strong.

We explore the role of uncertainty in the most standard, simple general equilibrium trade model—a two-good, two-country Armington model. We introduce cross-country uncertainty in the most obvious way: Each country experiences a random shock that affects its export choice. Home firms observe home shocks perfectly. Foreigners observe foreign shocks perfectly. But each group observes the other's shocks imperfectly, with a noisy signal. Then every firm chooses how much to export to an international market. The international relative price clears that market, goods are immediately shipped to their destination country, and agents consume. The paradox we uncover is that trade uncertainty can fuel more trade.

One obvious reason that uncertainty might encourage more trade is that agents have precautionary motives to trade. Agents who export, not knowing how much of the foreign good they will get in return, might export more to make sure they get enough of the foreign good back. For preferences with the right type of curvature, precautionary exporting emerges. But even when preferences do not normally induce precautionary behavior, we show that equilibrium movements in the terms of trade can induce countries to export more in the face of more mutual uncertainty. Just like borrowing constraints can change interest rate dynamics to induce precautionary behavior in a savings problem (Huggett and Ospina, 2001), equilibrium movements in the terms of trade can induce precautionary exporting in trade models with a wide range of non-precautionary preferences.

To understand the mechanism, consider the following example: Suppose that a Mexican truck manufacturer is building trucks, uncertain about the future rate of exchange of Mexican trucks for American corn. By itself, more random variation in the terms of trade of trucks for corn would cause the risk-averse truck maker to scale back: there is no pure precautionary motive. But movements in the terms of trade are not just random. The truck maker knows that states of the world where trucks exchange for many pounds of corn are also states of the world in which Mexico is particularly desperate for corn. Conversely, states where the price of trucks is low are states where Mexican resources are abundant. In these states, getting lots of corn is less imperative. In other words, the terms of trade vary, but they move in such a way as to share risk between countries (Cole and Obstfeld, 1991). When uncertainty is low, the terms of trade vary less and pose less risk to the exporter. But terms of trade that are not variable cannot hedge risk effectively. As uncertainty rises, and the terms of trade are less predictable, they also covary more negatively with endowments, so as to hedge each country's risk. This is what makes trade more attractive.

This logic applies to existing trading relationships only. Our argument does not apply when two countries are new trading partners and many new trading relationships are potentially being formed. The reason is that new trading relationships surely involve fixed costs to set up. Uncertainty affects the willingness to bear those fixed costs in a way that is not captured by this model. However, much of the world's trade takes place between trading partners that are already established, like the U.S. and Mexican car manufacturers. The question there is not whether to start exporting, but how much to trade within an existing relationship. This question is a natural starting point because the setting is simpler, but also because the answer is more surprising.

The effect of information on the risk and the expected return from exporting permeates a broad class of general equilibrium trade models. However, how risk and return affect the incentives to export and which effect dominates, depends on preferences and their parameters. While our analysis ultimately identifies fundamental features of preferences that cause information to affect trade volumes one way or another, we begin with a specific, but commonly-used form of preferences to identify these forces in a well-understood setting and build intuition for how and why they arise. With constant elasticity of substitution (CES) preferences, if goods are highly substitutable, the risk effect dominates and information frictions decrease trade. When goods are have a degree of substitutability typically used in the literature, the rise in the expected terms of trade more than offsets the increase in risk, and firms choose to export more when information is less precise. In other words, information frictions facilitate trade.

With other preferences outside the CES class, the same forces are at play, but may result in different net effects on trade volume. One possibility is that the increase in the terms of trade can reduce exports. The logic is that if I'm expecting to get lots of the foreign good back in return for my exports, and I like a balanced consumption bundle, then I should export less when the relative price of my good rises. Otherwise, I'll have too much of the foreign good to consume. Another possibility is that when the terms of trade become more uncertain, an agent chooses to export more for purely precautionary reasons. Our general results characterize preferences where substitution or precautionary effects dominate. We can distinguish these well-understood effects from our equilibrium terms of trade effect that induces precautionary exporting with standard and commonly-used preferences.

Our analysis proceeds in several steps. First, we consider the effects of uncertainty on the terms of trade. The key insight is that—in general equilibrium— uncertainty affects not only the volatility, but also the expected terms of trade. Mathematically, the mechanism is that uncertainty impairs home agents' ability to condition their exporting behavior on the foreign country's state (and vice-versa). As a result, home and foreign exports covary less. In equilibrium, the terms of trade depend on the ratio of home and foreign exports. If home and foreign exports are always proportional, the terms of trade are constant. Less coordination creates more volatile terms of trade. The mechanism encodes the conventional wisdom that uncertainty deters risk-averse exporters from exporting. However, this conventional wisdom is incomplete. A fall in export covariance makes the numerator and denominator of the terms of trade covary less, while always remaining positive. This results in a terms of trade that occasionally reaches a very high level, but never falls below zero. When such a positive ratio varies more, its mean increases. Thus, high uncertainty, which results in a more volatile terms of trade, also increases the expected level of the terms of trade, making exporting more lucrative. This change in the expected terms of trade is where the benefits of risk-sharing show up. We point out how the change in the covariance, which is risk sharing, affects the mean return to trade. How uncertainty actually affects trade volume depends on which of these forces—the increase in risk or increase in return—dominate.

The final part of the paper derives general conditions on preferences to determine which of these forces—the increase in risk or increase in return—dominate and which direction each pushes exports.

Section 3.4 relaxes the assumption that there are no financial instruments or contracts that share risk. We describe the average amount of trade in settings where some agents can write fully state-contingent contracts and others cannot. We find that allowing more risk-sharing works just like reducing uncertainty. If you can condition exports on the realized price, then it is just like knowing the price. Both reduce the average amount of trade.

Related papers There is a spate of recent papers modeling and measuring information frictions in trade. The most closely related is Steinwender (2014), where exporters in one country learn about exogenous market prices in another country. More precise information decreases uncertainty and increases the expected profits, trade volume and welfare. Our paper is similar because agents learn about aggregate economic conditions in another country and then choose exports. But instead of one trader facing an exogenous price, our model features equilibrium two-country trade. It is exactly the equilibrium price movements that reverses the partial equilibrium results.

Other papers look at different types of information frictions. In Allen (2013), Petropoulou

(2011), Rauch and Watson (2004) and Eaton, Eslava, Krizan, Kugler, and Tybout (2011), producers are uncertain about firm- or match-specific variables such as the location of the best trading partner, the quality of their match or local demand for their specific product. These are undoubtedly important information frictions. But if these frictions inhibit foreign trade more than domestic trade, there must be some country component to them that is known at home, but not abroad. As such, our model complements these theories by filling in that missing piece, the role of uncertainty about a foreign economy.

The effect of uncertain terms of trade on risk sharing is similar to the effect of allowing international borrowing (Brunnermeier and Sannikov (2014)). Both reduced uncertainty and imperfect borrowing undermine risk-sharing, but ours has the opposite predictions for trade volumes.

In financial markets, lower uncertainty also frequently inhibits risk-sharing. The Hirshleifer (1971) effect arises when information precludes trade in assets whose payoffs are contingent on an outcome revealed by the information. Our effect is distinct because 1) our signals are not public, 2) the existence of two distinct consumption goods matters, and 3) our mechanism works through changes in the international relative price. We discuss the importance of each of these differences when we explore risk sharing in Section 3.3.

1. A Benchmark Model of Trade Under Uncertainty

In order to explain the Trump paradox, we need to understand how cross-border uncertainty affects trade. To do this, we write down a simple model with two countries, an endowment economy and a cross-border information friction. The first two ingredients constitute a standard equilibrium model of trade. The uncertainty comes from the assumption that agents in each country know their own country's aggregate endowment, but have imperfect information about the other country's endowment. The friction could have been that each country is uncertain about the quality of the others' goods. In an appendix, we show that it turns out to be isomorphic. In the Trump example, much of uncertainty is about tariffs, quotas or government demand for foreign goods (buy America schemes). While we do not model trade policy or demand shocks explicitly, they would create the kind of uncertainty about terms of trade that is central to this model. Finally, uncertainty could have to do with firm-specific conditions. But if these conditions are really idiosyncratic, then any aggregate effect on trade has more to do with model aggregation properties than with the uncertainty itself.

Agents do not inherently care about the endowments of foreigners. Uncertainty about a variable that is irrelevant to the decision maker typically has no effect. But what agents do care about is the terms of trade for their good. Terms of trade govern how much foreign good arrives in return for one unit of exported home good. That return does affect consumption and utility. Since foreign endowments affect foreigners exports, the supply of exports affects the relative price of home and foreign goods, and this relative price affects utility, this is why uncertainty matters for firms' choices. So for endowment uncertainty to be relevant, it must be that the uncertainty about endowments creates uncertainty about the terms of trade. For there to be any uncertainty about the terms of trade, it is essential that the relative price of goods is not known at the time when exports are chosen. Terms of trade might not be known because of a shipping delay, coupled with a demand shock or sudden policy change. Our timing assumptions capture the idea that exporting is risky. It is the essence of an information asymmetry: Something about the benefits of trade is known when one sells domestically that is unknown when selling abroad. Our question is what happens when exporting become more uncertain.

At first, this model will, intentionally, have no financial markets that agents could use to hedge their productivity shocks or those of the foreign country. A key insight of this model is that the extent of uncertainty about each others' productivity changes how they share risk. Equilibrium movements in international prices are a primary source of such risk-sharing (Cole and Obstfeld, 1991). A model with no risk-sharing instruments provides the clearest view of the information effect on risk sharing through the terms of trade. After we explore this mechanism, Section 3.3 adds back risk-sharing instruments.

This is a repeated static model with the following economic environment.

Preferences: There are 2 countries and a continuum of agents within each country. We denote individual variables with lower case and aggregates with upper case. The problems are symmetric across countries, so we only describe the problem for the domestic country. Agents like to consume two goods, x and y and their utility flow each period is

$$\mathbb{E}\left[U(c_x, c_y)\right].$$

where for now we only restrict U to be increasing and concave in both goods.

Endowments: Each agent in the domestic country has an idiosyncratic endowment of z_x units of good x, where $\ln z_x \sim \mathcal{N}(\mu_x, \sigma_x^2)$. Each agent in the foreign country has an idiosyncratic endowment z_y units of good y, where $\ln z_y \sim \mathcal{N}(\mu_y, \sigma_y^2)$. The means of these distributions are themselves independent random variables: $\mu_x \sim \mathcal{N}(m_x, s_x^2)$ and $\mu_y \sim \mathcal{N}(m_y, s_y^2)$. Because they are average endowment around which all individual endowments are distributed, μ_x and μ_y are aggregate shocks. If we let Φ denote the cumulative density (cdf) of a standard normal variable, then the conditional cdf's of z_x and z_y are $F(ln(z_x)|\mu_x) = \Phi((ln(z_x) - \mu_x)/\sigma_x)$ and $F(ln(z_y)|\mu_y) = \Phi((ln(z_y) - \mu_y)/\sigma_y)$.

Information: At the beginning of the period, firms in country x observe their own endowment z_x and the mean of their country's endowment μ_x. Likewise, agents in country y observe z_y and μ_y. Furthermore, we assume that both countries know the distribution from which mean productivities are drawn and the cross-sectional variance of firm outcomes. In other words, $m_x, m_y, s_x, s_y, \sigma_x$ and σ_y are common knowledge.

In addition, agents in country x observe a signal about the y-endowment $\tilde{m}_y = \mu_y + \eta_y$ where $\eta_y \sim N(0, \tilde{s}_y^2)$. Similarly, agents in country y observe a signal about the x-endowment $\tilde{m}_x = \mu_x + \eta_x$ where $\eta_x \sim N(0, \tilde{s}_x^2)$. Let $\mathcal{I}_x$ denote the information set of an agent in the home country and let $\mathcal{I}_y$ denote the information set of a foreign agent. All home country choices will be a function of the three random variables in the home agents' information set: $\mathcal{I}_x = \{z_x, \mu_x, \tilde{m}_y\}$. Likewise, foreign choices depend on $\mathcal{I}_y = \{z_y, \mu_y, \tilde{m}_x\}$.

Signals are not essential to the problem. We could simply compare economies where countries know either others' endowments, with economies where endowments are not known. Introducing signals allows us to vary uncertainty, in a continuous way, and learn more about its effects.

Bayesian updating Agents in each country combine their prior knowledge of the distribution of the others' productivity and their signal to form posterior beliefs. By Bayes' law, the posterior probability distribution is normal with mean $\hat{m}$ and variance $\hat{s}^2$ given by

$$F(\mu_y|\mathcal{I}_x) = \Phi\left(\frac{\mu_y - \hat{m}_y}{\hat{s}_y}\right) \quad \text{where} \quad \hat{m}_y = \frac{s_y^{-2} m_y + \tilde{s}_y^{-2} \tilde{m}_y}{s_y^{-2} + \tilde{s}_y^{-2}}, \quad \hat{s}_y^2 = \frac{1}{s_y^{-2} + \tilde{s}_y^{-2}} \tag{1}$$

$$F(\mu_x|\mathcal{I}_y) = \Phi\left(\frac{\mu_x - \hat{m}_x}{\hat{s}_x}\right) \quad \text{where} \quad \hat{m}_x = \frac{s_x^{-2} m_x + \tilde{s}_x^{-2} \tilde{m}_x}{s_x^{-2} + \tilde{s}_x^{-2}}, \quad \hat{s}_x^2 = \frac{1}{s_x^{-2} + \tilde{s}_x^{-2}} \tag{2}$$

In order to forecast prices, agents will need to forecast the other country's export choices. Since others' export choices depend on their forecasts of one's own endowment, actions will also depend on beliefs about the beliefs of others. According to Bayes' law, these second-order beliefs are

$$F(\hat{m}_x|\mathcal{I}_x) = \Phi\left(\frac{\hat{m}_x - \hat{\hat{m}}_x}{\hat{\hat{s}}_x}\right) \quad \text{where} \quad \hat{\hat{m}}_x = \frac{s_x^{-2} m_x + \tilde{s}_x^{-2} \mu_x}{s_x^{-2} + \tilde{s}_x^{-2}}, \quad \hat{\hat{s}}_x^2 = \frac{\tilde{s}_x^{-2}}{(s_x^{-2} + \tilde{s}_x^{-2})^2} \tag{3}$$

$$F(\hat{m}_y|\mathcal{I}_y) = \Phi\left(\frac{\hat{m}_y - \hat{\hat{m}}_y}{\hat{\hat{s}}_y}\right) \quad \text{where} \quad \hat{\hat{m}}_y = \frac{s_y^{-2} m_y + \tilde{s}_y^{-2} \mu_y}{s_y^{-2} + \tilde{s}_y^{-2}}, \quad \hat{\hat{s}}_y^2 = \frac{\tilde{s}_y^{-2}}{(s_y^{-2} + \tilde{s}_y^{-2})^2} \tag{4}$$

Note that there is a one-to-one mapping between signals $\tilde{m}$ and posterior beliefs $\hat{m}$. Instead of using signals as a state variable, we will use posterior beliefs, for simplicity and without loss of generality. Thus, we write $\mathcal{I}_x = \{z_x, \mu_x, \hat{m}_y\}$ and $\mathcal{I}_y = \{z_y, \mu_y, \hat{m}_x\}$.[1]

[1] In fact, all higher orders of beliefs can matter for export choices. But, because there are only two shocks observed by each country, the first two orders of beliefs are sufficient to characterize the entire hierarchy.

Price and budget set: Each agent chooses how much to export, t_x or t_y. The relative price p is the number of units of y good required to purchase one unit of x good on the international market. In equilibrium, this price clears the market. An agent who exports t_x units of x goods receives pt_x units of y, for immediate consumption (there is no secondary resale market). We restrict exports and consumption to be non-negative. Therefore the country x budget set is:

$$c_x \quad \epsilon \quad [0, z_x - t_x] \tag{5}$$

$$c_y \quad \epsilon \quad [0, pt_x] \tag{6}$$

and the country y budget set is:

$$c_x \quad \epsilon \quad \left[0, \frac{t_y}{p}\right] \tag{7}$$

$$c_y \quad \epsilon \quad [0, z_y - t_y] \tag{8}$$

When the export decision is made, firms do not know the price p. It is a random variable whose realization depends on their own (known) aggregate state, on the foreign (unknown) aggregate state, on home beliefs about the foreign state and on foreign beliefs about the home state.

Equilibrium An equilibrium is given by export policy functions for domestic $t_x(z_x, \mu_x, \hat{m}_y)$ and foreign $t_y(z_y, \mu_y, \hat{m}_x)$ countries, aggregate exports $T_x(\mu_x, \hat{m}_y), T_y(\mu_y, \hat{m}_x)$, perceived price functions $\tilde{p}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)$ for each country and an actual price function $p(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)$ such that:

1. Given perceived price functions $\tilde{p}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)$, export policies maximize expected consumption of every firm in each country. Substituting the budget sets (5) to (8) into utility $\mathbb{E}[U(c_x, c_y)]$, we can write this problem as

$$t_x(z_x, \mu_x, \hat{m}_y) = \arg\max \mathbb{E}\left[U\left(z_x - t_x, \tilde{p}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)t_x\right) \middle| \mathcal{I}_x\right] \tag{9}$$

$$t_y(z_y, \mu_y, \hat{m}_x) = \arg\max \mathbb{E}\left[U\left(\frac{t_y}{\tilde{p}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)}, z_y - t_y\right) \middle| \mathcal{I}_y\right] \tag{10}$$

   Using the conditional densities (1), (2), (4) and (3), we can compute expectations as

$$t_x(z_x, \mu_x, \hat{m}_y) = \arg\max \int\int U\left(z_x - t_x, \tilde{p}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)t_x\right) dF(\mu_y|\mathcal{I}_x)dF(\hat{m}_x|\mathcal{I}_x) \tag{11}$$

$$t_y(z_y, \mu_y, \hat{m}_x) = \arg\max \int\int U\left(\frac{t_y}{\tilde{p}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)}, z_y - t_y\right) dF(\mu_x|\mathcal{I}_y)dF(\hat{m}_y|\mathcal{I}_y) \tag{12}$$

2. The relative price p clears the international market. Since every unit of x-good exported

must be sold and paid for with y exports, and conversely, every unit of y exports must be sold and paid for with x exports, the only price that clears the international market is the ratio of aggregate exports:

$$p(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) = \frac{T_y(\mu_y, \hat{m}_x)}{T_x(\mu_x, \hat{m}_y)} \tag{13}$$

where aggregate exports in each country are

$$T_x(\mu_x, \hat{m}_y) = \int t_x(z_x, \mu_x, \hat{m}_y) dF(z_x|\mu_x) \tag{14}$$

$$T_y(\mu_y, \hat{m}_x) = \int t_y(z_y, \mu_y, \hat{m}_x) dF(z_y|\mu_y). \tag{15}$$

3. The perceived and actual price functions coincide:

$$\tilde{p}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) = p(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) \qquad \forall (\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)$$

2. Trade and Uncertainty with Constant Elasticity (CES) Preferences

This section illustrates the main argument of the paper in the context of a specific utility function. Specific utility makes our arguments less abstract and allows us to illustrate results graphically, to complement the mathematics. In particular, we start with a form of CES preferences and make the following argument. First, we show how uncertainty inhibits coordination in export behavior. This result is important because it implies that uncertainty inflates both the mean and variance of the terms of trade. Second, we show how the volume of trade depends on the mean and variance in the terms of trade. Thus, we are able to show how uncertainty affects trade. The next section shows that this example is not a special case. After we outline the argument in the simplest way, we generalize the principles at work for arbitrary utility.

Throughout this section, we work with the following CES utility function

$$\mathbb{E}\left[C_x^\theta + C_y^\theta \middle| \mathcal{I}_x \right], \ \theta < 1. \tag{16}$$

The restriction $\theta < 1$ is required for the function to be concave in both goods.[2] Expectations are conditional on the information set of the home country, which contains its own realization of the aggregate shock and the signal about the realization abroad.

[2] How to interpret this utility? Consider a consumer with CRRA utility $\mathbb{E}\left[\frac{C^{1-\sigma}}{1-\sigma}\right]$ and a consumption bundle given by an aggregator $C = \left(C_x^\theta + C_y^\theta\right)^{1/\theta}$. Then our CES-like case is a special case where $\sigma + \theta = 1$. If $0 < \theta < 1$, then $\sigma < 1$ and it is a risk-averse agent. If $\theta < 0$, then σ could be above one and be a risk-lover agent. Recall that the elasticity of intertemporal substitution is $1/\sigma$.

2.1. Information and the Terms of Trade

This section describes how uncertainty affects the stochastic properties of the terms of trade. We proceed in three steps: how uncertainty affects the covariance of exports, how the covariance affects the expected terms of trade, and how the covariance affects the volatility in the terns of trade.

Proposition 1 establishes that less uncertainty (more precise information about the other country's endowment) increases the covariance between aggregate exports.

Result 1 ***Uncertainty reduces the covariance of aggregate exports.*** *In a neighborhood around complete uncertainty ($\tilde{s}_x^2+\tilde{s}_y^2=0$), more uncertainty moves the covariance between aggregate exports toward zero.*

No agent in any setting can condition their action on a variable that is not known to them. In this context, firms can not make their export choices covary with foreign exports if the foreign state is unknown. It's not a feasible (measurable) strategy. When signal precision is zero, therefore, the covariance of exports must be zero. In contrast, when information is perfect and each country knows the other's productivity, higher exports by the foreign country imply that foreign goods will be abundant and thus cheap. Thus home goods will be relative expensive and the return to exporting home goods will be high. This expectation of high returns to exports incentivizes home firms to export more in states when foreign exports are also high.

In less extreme cases, when signal precision is neither zero nor infinite, the degree of coordination in exports depends on the precision of the signal. The more precisely home country agents can condition their exporting behavior on the foreign country's state, the more their exports will covary with the exports of the foreign country. With CES preferences, agents in both countries desire both goods (with that desire modulated by θ). This desire for balanced consumption bundles underlies the motivation to coordinate exports.

Figure 1 illustrates this point. The top panel illustrates the case when there is perfect information. And one sees that exports are highly correlated across countries. When one country exports a lot, the other country has a strong desire to export a lot as well. The bottom panel illustrates the opposite extreme. Here, neither, country has any information about the other country and thus, their exports are independent.

The fact that information allows exports to covary underpins the results that follow. Specifically, it governs the mean and variance of the terms of trade.

The terms of trade is the price that clears the international export market. The only price that clears that market is the ratio of exports. When home and foreign exports covary, the numerator and denominator of the terms of trade covary. Imagine an extreme case where home and foreign

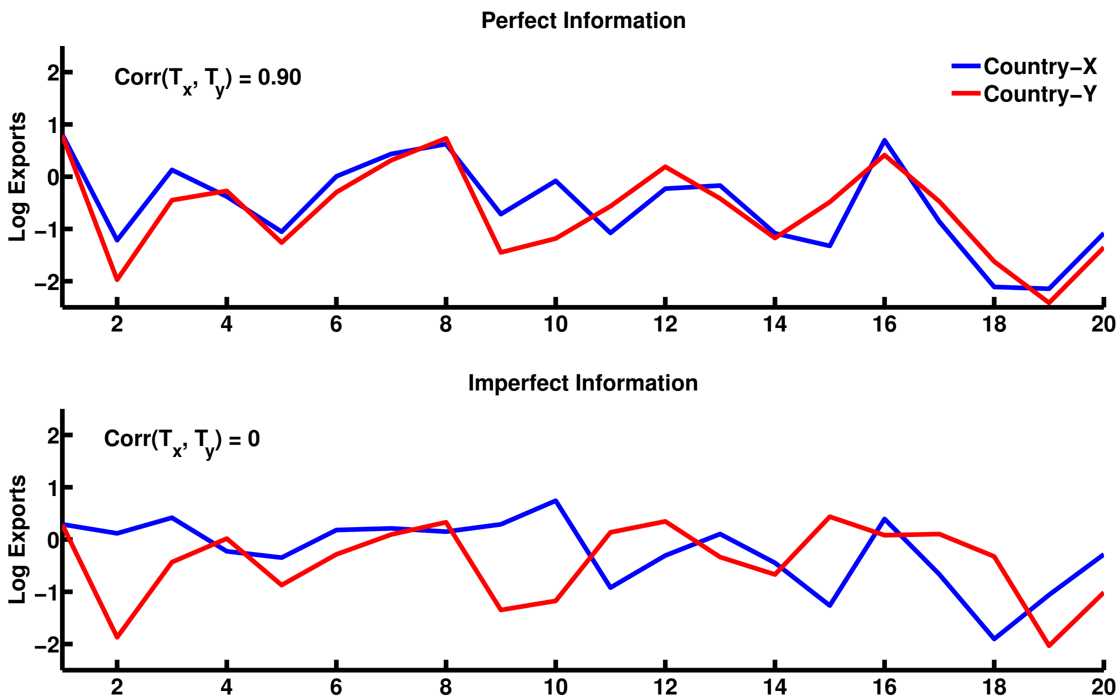

Figure 1: Information and Export Covariance

exports had perfect correlation. Home exports were exactly proportional to foreign exports, in every state. Then the ratio of home and foreign exports would be constant. That would imply a constant terms of trade, with zero variance. When home and foreign exports covary less, the terms of trade becomes more volatile. This is the logic formalized in the following result.

Result 2 ***Lower covariance between exports increases the unconditional volatility of terms of trade*** *If the unconditional expectations and variances of aggregate exports are kept constant, a decrease in the correlation between aggregate exports increases the volatility of the terms of trade for both countries.*

Figure 2 illustrates the link between the reduction in export covariance and the volatility of the terms of trade. As the figure shows, more uncertainty means less covariance in exports and, thus, the terms of trade are much more volatile (compare the red to the blue line).

Figure 2 also shows the link between uncertainty and the average terms of trade. Greater uncertainty reduces export covariance, which makes the numerator and denominator of the terms of trade covary less, while always remanding positive. This high-uncertainty case is the red line. Notice that the high-uncertainty terms of trade occasionally spikes. These are states where home exports are quite low, and therefore scarce and valuable. With a sufficiently low productivity state, exports can become arbitrarily low, which makes the terms of trade arbitrarily high. An yet, the terms of trade never fall below zero. By its nature, the process for

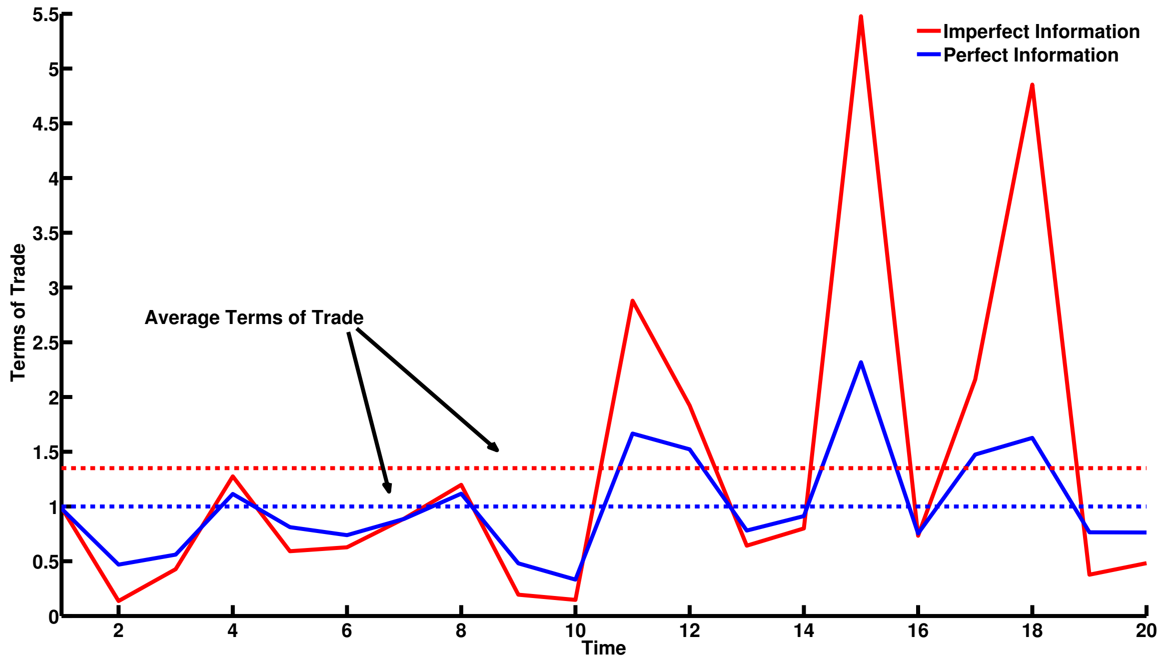

Figure 2: Information Reduces Terms of Trade Volatility and Reduces the Mean

the terms of trade is asymmetric. That is not to say that this is an asymmetry that systematically favors one country on the other. When information is scarce, *both countries simultaneously have high expected terms of trade*. Indeed, a high terms of trade for one country implies a low terms of trade for the other. But a high expected terms of trade does not imply the that expectation of the inverse terms of trade is low.

In short, information frictions increase the expected terms of trade, for both countries. This is the argument formalized in the proposition that follows.

Result 3 ***Lower covariance between exports increases the unconditional expected terms of trade.*** *If the unconditional expectations and variances of aggregate exports are kept constant, a decrease in the correlation between aggregate exports increases the expected terms of trade for both countries.*

Furthermore, if countries are symmetric, the average terms of trade can be expressed as

$$\mathbb{E}\left[p\right] = 1 + CV^2[T_x]\left(1 - corr[T_x, T_y]\right) \tag{17}$$

$CV^2[z] = \frac{\mathbb{V}[z]}{\mathbb{E}[z]^2}$ *is the (constant) squared coefficient of variation.*

Importantly, results 2 and 3 are proven, without reference to the preference specification. The relationship between export covariance and the properties of the terms of trade is a statement about the statistical properties of the ratio of two lognormal random variables. These statistical properties are independent of preferences. Therefore, these two results will carry over when we discuss the model with general preferences at the end. The first result about information enabling correlation is not general because of its sign. In some cases, preferences will make agents want to coordinate their exports negatively. It is always true that the only feasible level of coordination with no information is zero covariance. But less uncertainty might enable either positive or negative export covariance strategies, depending on preferences.

The previous results showed how uncertainty affected the mean and variance in the terms of trade through a coordination motive. The next section connects these forces to firms' decisions of how much to export.

2.2. How the Terms of Trade Distribution Affects the Volume of Trade.

The second part of our argument links the expected terms of trade and its variance to the volume of trade. There are no surprises here. Our main point is not that agents react to changes in the mean and variance of the terms of trade in some strange way. With CES preferences, firms export more when the return to exporting is higher and export less when exporting is risky. So conventional wisdom about uncertainty deterring trade is correct in the CES model. The unexpected part of the relationship between uncertainty and trade came from the previous section, where uncertainty about others' exports raised the expected returns to exporting one's own good.

The next result show that, as one would expect, an increase in the expected terms of trade, which is the return to exporting, increases the average level of exports. It also shows that higher variance in the terms of trade deters exports, because agents are averse to the risky return of exporting.

Proposition 1 *Suppose the terms of trade mean and variance change in* $\frac{d\mathbb{E}_x[p]}{\mathbb{E}_x[p]}$ *and* $\frac{d\mathbb{V}_x[p]}{\mathbb{V}_x[p]}$*, respectively. Then the sign of the change in exports is equal to the sign of the following expression:*

$$\theta\,\frac{d\mathbb{E}_x[p]}{\mathbb{E}_x[p]} + \frac{\mathbb{CV}_x^2[p]}{2}\left(\theta(1-\theta)(2-\theta)\frac{d\mathbb{E}_x[p]}{\mathbb{E}_x[p]} - \theta(1-\theta)\frac{d\mathbb{V}_x[p]}{\mathbb{V}_x[p]}\right) \tag{18}$$

Proposition 1 tell us how much exports will rise or fall, from a given percentage change in the expected mean or variance of the terms of trade. It reveals many facets of the relationship between the terms of trade p and trade volume. First, it tells us that an improvement in the expected terms of trade, holding other moments equal, causes firms to export more. This is true, for elasticity of substitution $0 < \theta \leq 1$ or for $\theta \geq 2$. The elasticity θ governs the size of the trade volume effect.

The variance in the terms of trade also changes with uncertainty. More variance—or risk—in the terms of trade deters trade. And this is true, for any level of the elasticity of substitution. Combined with Proposition 3, this result means that uncertainty deters trade through risk. This result is the "conventional wisdom." That is, noisier signals increase uncertainty, and that this force deters exports.

We have identified two competing forces. Consistent with conventional wisdom, uncertainty creates risk and this deters trade. However, uncertainty also raises the return on trade, encouraging more trade. Which force wins?

For CES preferences with an elasticity of substitution, ... , the mean effect always wins. WHY?

The relative strength of the mean and variance forces depends on the degree of uncertainty, as well as the elasticity of substitution. The mean effect is always positive, meaning that increases in the mean terms of trade always increase average exports. But this effect is weakest when there is a low degree of substitutability between home and foreign goods. In a low substitutability world, quantities consumed of each good are relatively unresponsive to changes in prices. That is why a change in the mean price, or terms of trade, has less effect. The variance effect is always negative. More volatile terms of trade, by itself, always deters trade. This trade-reducing effect is stronger for low degrees of substitutability. This reflects the idea that as the goods become more complementary, then uncertainty matters more as agents want to ensure balanced consumption bundles across goods. Since the trade-increasing mean effect is stronger and the trade-reducing variance effect is weaker for highly substitutable goods, these high-θ economies are ones where greater uncertainty promotes trade.

Our main question is about how uncertainty affects the volume of trade, on average. But even when uncertainty increases the average amount of trade, there are still states of the world in which trade is higher with information, than it would be without. Appendix **??** explores the effect of uncertainty on trade in high and low-endowment states.

2.3. Is this Simply Precautionary Savings?

The result that an increase in uncertainty increases trade looks very much like a precautionary response. Exporting is a form of savings. If the return on that savings is more uncertain, under some preference specifications, agents save more. But that is not the basis for these results. If this were simply precautionary exporting, then a unilateral reduction in one country's uncertainty about the other's exports should reduce trade. That does not happen in our CES model. That precautionary effect may arise for other preferences, and may give rise to more trade with more uncertainty, but that effect is distinct from the equilibrium effect that produces more trade in this model.

To make clear that this is not just a precautionary savings effect, we do the same experiment of increasing uncertainty, but only for one country. We reduce the precision of one country's signal, holding fixed the other country's signal precision. **This is not what the graph is. Can we make the graph consistent with this description?** In one case, both countries know that home's signal precision is lower. In the second case, only home knows that its signal precision is changing.

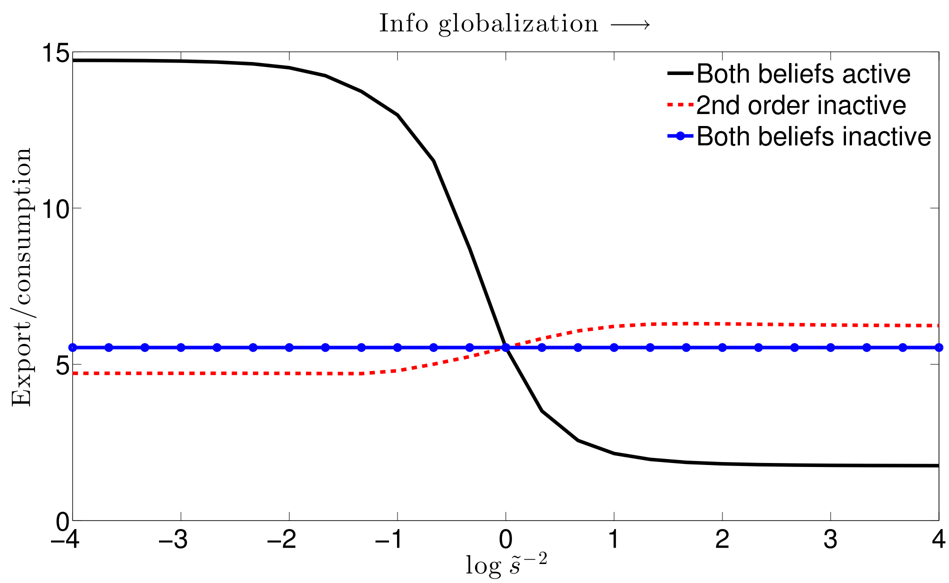

Figure 3: Asymmetric uncertainty increase does not increase trade.

3. The General Case

While CES preferences are commonly-used and useful for illustrating our results and the mechanisms behind them, focusing only one type of preferences leaves open questions: Is CES a special, knife-edge case that generates unusual results? It turns out that CES is not special or knife-edge. A broad class of preferences also have the property that uncertainty promotes trade. But not all preferences have this property. In this last section, we delineate which preferences are like CES in the sense that uncertainty promotes trade. We also offer practical guidance for those who wish to pursue this uncertainty as a trade barrier. Our results reveal what sorts of preferences are required for mutual uncertainty to deter trade. Finally, the general characterization of the forces at work uncovers a new interpretation of our results: that uncertainty about trade raises the returns to trade because it facilitates better international risk-sharing.

3.1. How Export Uncertainty Affects Terms of Trade

The results from Section 2.1, which described the relationship between trade uncertainty and the mean and variance of the terms of trade, did not depend on any preference specification. **Rephrase this. Preferences govern the sign of the action correlations in one of the props.** They used the fact that the terms of trade was the ratio of the two countries exports. The result that countries whose exports have lower covariance have higher expected terms of trade did use the fact that countries' endowments are lognormally distributed. Is this result specific to lognormal variables?

The key to the relationship between trade uncertainty and the expected terms of trade does lie in the distributional assumptions. What is essential is that exports can be arbitrarily close to zero, but can never be negative. If neither country can ever export a negative amount, then the ratio of exports, which in the terms of trade, are bounded below by zero. But if there exist states of the world where either country would choose an export amount arbitrarily close to zero, then the ratio of exports can be arbitrarily large. If the terms of trade are T_y/T_x and T_x can be arbitrarily close to zero, then $p = T_y/T_x$ will occasionally be huge. The point is that the economics of exporting make the distribution of the terms of trade skewed. There is no way this distribution can be symmetric if it is bounded below and unbounded above. Exactly how skewed and what firm the skewness takes, that depends on the distributions and preferences. But the histogram of the terms of trade, for either country, will always have a bigger right tail.

Once we understand that the terms of trade are a skewed distribution, we can see why signals about exports reduce the mean. Think of a skewed distribution as a function of a normal distribution. Left-skewed distributions would be a concave function of a normal – the concavity accentuates the left tail, bad events. In this case, we have a right-skewed distribution. That can be constructed as a convex function of a normal. Lemma prop:convex in the Appendix proves that the distribution of the terms of trade must be a somewhere-convex function of a normal probability density. Now, recall that by the definition of convexity, lotteries of convex functions have higher expected values than the median lottery realization. The more uncertain the lottery, the higher is the expected value. Firms face a terms of trade that is like this convex lottery. The more uncertain the lottery, the higher the expected terms of trade (Figure 4). The higher expected terms of trade is what makes exporting more attractive.

For the CES case, Figure 2 illustrated the same effect in a time series plot. Recall how when information was scarce, the terms of trade were very volatile and this resulted in occasional spikes in the terms of trade, that brought up the average. The analog in Figure 4 is the probability weight placed on the right side of the convex curve, which pulls up the average price $\mathbb{E}[p]$, under high uncertainty. The average terms of trade effect originates in the asymmetry of the terms of trade distribution. This asymmetry arises naturally, whenever exports are required to be non-negative.

What does this convex lottery look like economically? How does the Trump paradox work in practical terms? Trade policy uncertainty means that when you export goods, you may get very little in return. But the least you can get is $\epsilon > 0$. But there is also a possibility that your good is relatively scarce and earns an enormous rate of return. A firm that exports more in the face

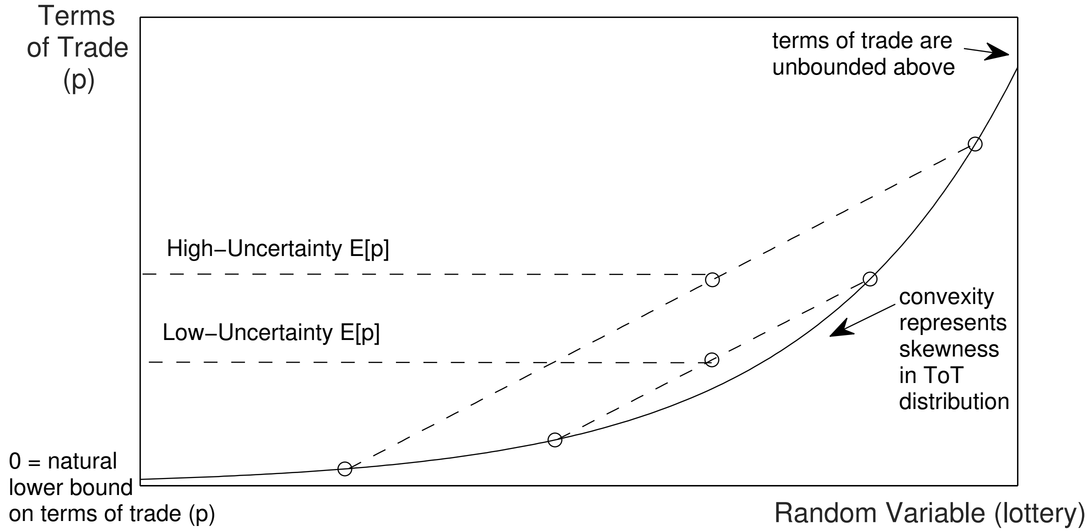

Figure 4: Because Terms of Trade are Skewed, Uncertainty Increases the Mean.

of trade uncertainty is gambling on the possibility that they are one of the few units that gets into the foreign country. If they are, they earn enormous rents on their scarce good. This is a risky lottery and firms dislike risk. But they understand that the odds are stacked in their favor. The more uncertain the trade policy, the greater the possibility of winning an enormous rate of return.

3.2. How Terms of Trade Moments Affect Export Volume: Sufficient Statistics for Preferences

With more general preferences, the uncertainty and trade relationship can work either way because firms may export more or less when the expected terms of trade rise, may export more or less when the variance increases, or because mean effects and variances effects trade off in a different way. We classify preferences, according to a few sufficient statistics, that allow us to say how firms with these preferences will react to trade uncertainty and why.

In a multi-good setting, risk aversion and its related higher-order risk preferences, need to reflect the interaction of preferences for the two goods. We encode risk attitudes with the following coefficients, which turn out to be the sufficient statistics for determining whether our export paradox holds or not:

- $\tilde{\rho}_y^{(1)} = \rho_y^{(1)} \left(1 - \frac{U_{xy}}{p\,U_{yy}}\right)$ where $\rho_y^{(1)} \equiv -\frac{C_y U_{yy}}{U_y}$ relative risk aversion;
- $\tilde{\rho}_y^{(2)} = \rho_y^{(2)} \left(1 - \frac{U_{xyy}}{p\,U_{yyy}}\right)$ where $\rho_y^{(2)} \equiv -\frac{C_y U_{yyy}}{U_{yy}}$ relative prudence; and
- $\tilde{\rho}_y^{(3)} = \rho_y^{(3)} \left(1 - \frac{U_{xyyy}}{p\,U_{yyyy}}\right)$ where $\rho_y^{(3)} \equiv -\frac{C_y U_{yyyy}}{U_{yyy}}$ relative temperance.

The coefficients without tildes are the standard single-good expressions for risk aversion, prudence and temperance. The coefficients with tildes are adjusted by cross-good derivatives, to reflect the fact that there are two goods.

Before we proceed, it is useful to interpret each of these statistics, so that we understand what economic forces we are talking about. Start with risk aversion. Note that $\rho_y \equiv -\frac{C_y U_{yy}}{U_y}$ is the standard coefficient of relative risk aversion (RRA) for the risky good y. Why adjust relative risk aversion for the two goods? If agents do not have any preference for correlated consumption of the two goods ($U_{xy} \leq 0$), then consumption of y is a hedge for the risk of low consumption of x. If agents prefer correlated consumption of both goods, then utility is very high when both goods are abundant and very low when both are scarce. This increases utility risk. Following Kihlstrom and Mirman (1974), when $U_{xy} > 0$ $\tilde{\rho}_y > \rho_y$: the adjusting factor in the RRA amplifies risk aversion compared to a one-good case. .

Prudence is a third derivative of preferences. It is related to the desire for precautionary savings. It governs whether agents want to export more to insure a modicum of foreign consumption or export less when the return to exporting is riskier. Following Eeckhoudt, Rey, and Schlesinger (2007), we adjust prudence for cross-prudence in x. Given a zero-mean δ random variable, an individual is cross-prudent in x if the lottery $[(x, y+\delta); (x-k, y)]$ is preferred to the lottery $[(x, y); (x-k, y+\delta)]$, that is, higher x consumption dampens the detrimental effects of risk in y. Eeckhoudt, Rey, and Schlesinger (2007) shows that cross-prudent preference for x imply that $U_{xyy} > 0$.

Temperance is a negative fourth derivative of utility (see Eeckhoudt and Schlesinger (2006)), which can be interpreted as a preference for risk disaggregation. Consider two zero mean random variables $\varepsilon_1, \varepsilon_2$, then an individual is said to be temperate if the lottery $[\varepsilon_1; \varepsilon_2]$ is preferred to the lottery $[0; \varepsilon_1 + \varepsilon_2]$, where all outcomes of the lotteries have equal probability. A temperate individual prefers risks to be spread across states. With multiple goods, relative temperance also implies that some risk in each good is preferred to concentrating all risk on one good.

Now, we use these sufficient statistics to characterize the relationship between the terms of trade moments and export volume. The first general proposition comes right out of the firm's first-order condition. It says that the marginal rate of substitution of a unit of home good for a unit of foreign good should be equal to the risk-adjusted rate of exchange of the two goods. If this were not true, a firm could improve its utility by exporting less or more.

Proposition 2 *Optimal exports can be approximated as a function of the conditional mean and variance of the terms of trade distribution, $T(f_x, \mathbb{E}_x[p], \mathbb{V}_x[p])$, and are determined by equating the marginal rate* *of substitution, evaluated at the expected terms of trade, with the risk-adjusted expected terms of trade.*

$$\frac{U_x(f_x - T, \mathbb{E}_x[p]T)}{U_y(f_x - T, \mathbb{E}_x[p]T)} = \underbrace{\mathbb{E}_x[p]}_{\textit{expectation}} \left[1 - \underbrace{\rho_y^{(1)}(\mathbb{E}_x[p])}_{\textit{relative risk aversion}} \underbrace{\left(2 - \tilde{\rho}_y^{(2)}(\mathbb{E}_x[p])\right)}_{\textit{adjusted relative prudence}} \underbrace{\frac{\mathbb{CV}_x^2(p)}{2}}_{\textit{variance/expectation}^2} \right] \tag{19}$$

What is useful about this way of expressing the first order condition is that it expresses the risk-adjusted terms of trade in terms of our first two sufficient statistics, and the mean and variance of the terms of trade. So, once we know the mean and variance of the terms of trade, and we know these two features of preferences, we can describe the firms' optimal export condition.

A higher expected terms of trade will increases the desirability of exporting, unless the term in square brackets is negative. If risk aversion and prudence are sufficiently high, then when a firm thinks that it will get more foreign goods back in return for each unit of home exports, it reasons that it can send fewer exports and still have plenty of foreign goods to eat. So it exports less.

A more variable terms of trade raises the coefficient of variation, $\mathbb{CV}_x^2(p)$. This deters exporting, unless the adjusted relative prudence term $2 - \tilde{\rho}_y^{(2)}(\mathbb{E}_x[p])$ is negative. When this prudence term is negative, the firm who faces a more uncertain terms of trade exports more to ensure that they will have enough foreign good to consume, even if the rate of exchange turns out to be low. This is an effect similar to the increase in precautionary savings observed when earnings are more volatile in consumption/savings problems (**CITE**).

The next result simply differentiates (19) with respect to the mean and variance of the terms of trade. The resulting expression clarifies how changes in the terms of trade distribution change the volume of exports.

Proposition 3 *Suppose the terms of trade mean and variance change in $\frac{d\mathbb{E}_x[p]}{\mathbb{E}_x[p]}$ and $\frac{d\mathbb{V}_x[p]}{\mathbb{V}_x[p]}$, respectively. Then the sign of the change in exports is equal to the sign of the following expression:*

$$\underbrace{\left(1 - \tilde{\rho}_y^{(1)}\right)}_{\textit{risk aversion}} \frac{d\mathbb{E}_x[p]}{\mathbb{E}_x[p]} + \frac{\mathbb{CV}_x^2[p]}{2} \left(\rho_y^{(1)} \rho_y^{(2)} \underbrace{\left(3 - \tilde{\rho}_y^{(3)}\right)}_{\textit{temperance}} \frac{d\mathbb{E}_x[p]}{\mathbb{E}_x[p]} - \rho_y^{(1)} \underbrace{\left(2 - \tilde{\rho}_y^{(2)}\right)}_{\textit{prudence}} \frac{d\mathbb{V}_x[p]}{\mathbb{V}_x[p]} \right) \tag{20}$$

What we learn from this is that it clarifies many reasons why exports might rise or fall, in response to clearer mutual information. Information can change risk, it can change whether risk is highest when consumption is low or high, and it can change how whether risk in one consumption good is high when the other is high or low. How a country responds to each of these changes depends on their preferences. Specifically, it depends on their risk aversion, temperance and prudence, as described above.

Why can information frictions boost trade? The main question we're trying to answer is how do cross-border information frictions affect trade volume. We've described some competing effects: Endowment uncertainty raises the mean terms-of-trade, but also raises variance. This leaves the question of why do these competing effects facilitate trade on average?

The mean effect of the terms of trade dominates the variance effect because of preferences. The combination of risk aversion, prudence and temperance is just not strong enough to overcome the higher mean returns to exporting. Under some preferences, higher risk and higher return corresponds to less trade. But under commonly-used preferences – like CES, with elasticities of substitution consistent with other trade facts – the net effect is more trade. This is not an oddity of CES preferences. There is a broad class of preferences that produce the same effect. So, might Trump's, or any one else's trade threats promote trade? Yes, if these threats create uncertainty about the quantity of foreigners' exports and if preferences are not too risk-averse, too prudent or too temperate, this surge in trade is a logical equilibrium outcome.

When is uncertainty a barrier to trade? So far, we have focused on the cases where uncertainty increases trade because these are the most surprising. But in many cases, a researcher might want to build a model where uncertainty is a barrier to trade. What preferences make that possible?

High risk aversion helps. It tempers the reaction to changes in expected terms of trade and amplifies the effect of risk. Low prudence (low $\tilde{\rho}_y^{(2)}$) makes trade volumes more sensitive to changes in terms of trade variance. For positive adjusted risk aversion ($\rho_y^{(1)} > 0$), low prudence implies a lower precautionary motive. Agents to not want to export more in the face of risk to ensure they have some foreign goods to consume. Instead, the export less to expose themselves less to the unknown rate of return. If that stepping away from risk force is strong, then resolving uncertainty about trade will reduce the terms of trade risk and promote more trade.

What sorts of preferences ensure that the mean and variance effect combine to deliver a decrease in trade from a rise in uncertainty? If the information increases both the mean and the variance equally, then the following condition tells us under what conditions this increases exports.

Proposition 4 Uncertainty as a barrier to trade*: A joint increase in average and volatility of terms of trade distribution inhibits exports if*

$$\mathbb{E}\left[U_y\left(1-\tilde{\rho}_y+\frac{\rho_y}{p}(\tilde{\pi}_y-2)\right)\middle|\mathcal{I}_x\right]\leq 0$$

In general, information reduces risk. Conditioning on that information makes random variables less uncertain. That is the nature of information. Depending on agents desire to undertake precautionary savings, the reduction in risk could prompt them to export less or export more. But in a general equilibrium model of trade, that is not the only effect of information. The other effect is to reduce the expected terms of trade. That shift in the terms of trade distribution can also move the desire to export in either direction. But the combination of the mean effect and the variance effect reveals the total impact of information on trade.

Relating the general case and the CES case The CES results we presented earlier for change in trade volume were just a special case of this more general result. Differentiating our CES preferences (16) reveals that the risk aversion term is $1-\tilde{\rho}_y^{(1)} = \theta$; the prudence term is $2-\tilde{\rho}_y^{(2)} = \theta$, and the temperance term is $3-\tilde{\rho}_y^{(3)} = \theta$. Substituting these θ terms into (20) yields the condition for changes in trade volume in (18).

3.3. A Risk-Sharing Interpretation

One way of understanding why it is that uncertainty can facilitate trade is to explore why uncertainty enables better risk-sharing. In our trade model, countries would achieve full risk sharing if each country consistently exported half its endowment. In such a world, both countries would consume the same bundle: half of the home goods produced and half of the foreign goods produced in that period. Consumption in both countries would be perfectly correlated. This full risk-sharing world also achieves the maximum level of average trade. Exporting more than half of one's endowment, on average, never makes sense. So, if full risk sharing implies maximum trade, the question of why uncertainty promotes trade amounts to asking why uncertainty brings the world economy closer to full risk sharing.

The following proposition shows mathematically how uncertainty promotes risk sharing by changing the states in which price is high.

Proposition 5 *For a given level of domestic exports, an increase in foreign uncertainty $\tilde{s}_x^2$ changes the conditional expectation and variance of the terms of trade in the following way:*

$$\frac{d\mathbb{E}_x[p]}{d\tilde{s}_x^2} \approx \frac{1}{T_x}\mathcal{A}\left(\frac{m_x-\mu_x}{s_x^2}\right)$$

where $\mathcal{A} \equiv \frac{\partial T_y}{\partial \hat{m}_x}\frac{\hat{\hat{s}}_x^2}{\tilde{s}_x^2} > 0$ is proportional to the first derivative of the foreign policy function with respect to domestic endowment. Therefore, an increase in foreigners' uncertainty $\tilde{s}_x^2$:

- *increases* $\mathbb{E}_x[p]$ *if the domestic endowment is low, ie.* $\mu_x < m_x$.
- *decreases* $\mathbb{E}_x[p]$ *if the domestic endowment is high, ie.* $\mu_x > m_x$.

By increasing returns to trade (p) when the endowment is low and reducing returns when the endowment is high, uncertainty smooths out the utility of each country's residents. This is uncertainty improving risk sharing. **Still not connected well enough to earlier discussion? Relate to prudence or temperance?**

In finance, the argument for why uncertainty facilitates risk sharing is well-understood and is often called the "Hirschleifer effect." Hirschleifer (CITE) considers the example of two bettors, each with a ticket on an identical, but independent lottery. The bettors can diversify their risk by splitting the two claims, so that if either lottery pays off, both get half the winnings. Now, suppose that both bettors observe noisy signals about the outcome of each lottery. Now, the bettor whose claim is on the lottery with the more favorable signal would want to keep a larger claim on his own lottery. The signal reduces uncertainty about both lottery outcomes, but at the same time, undermines risk-sharing. The only way both bettors will consistently share all their risk is if they know nothing about the lottery outcomes.

The analogy between financial markets and trade is not perfect. The fact that trade involves two or more goods, rather than two lottery tickets that both pay off identical currency units, matters. Risk-sharing in trade takes a different form. There are no ex-ante agreements to share output. Instead, the mechanism for international risk sharing is the movement in the real exchange rate. When one country gets a low endowment, their good is scarce; therefore their good is valuable, and they get lots of foreign goods in return for their exports. The abundance of foreign goods helps to insure the risk of a low endowment.

Information undermines the real exchange rate as a risk sharing mechanism because it allows countries to back away from trade in states when they would prefer not insure their trading partner. For example, when home firms know that foreign firms have little to export, those are states where they would prefer to walk away from full insurance. The full insurance action would be for home to export lots, foreign to export little, and both to consume the same amount. But home agents do not want to export much at those terms. Just like the bettor who no longer wants to share his half ticket in return for one with lower odds, the home country who knows that the foreign endowment is low no longer wants to send lots away for little in return.

3.4. Completing the Contracting Space

So far, we have assumed away all instruments that agents might use to share international risk. Exchange rate futures, international equity holdings, profit-sharing contracts, secondary markets, all could help to share international risk. If we included a complete set of risk-sharing instruments, then agents could hedge the outcomes of all unknown variables and the effects of information asymmetry would disappear. In fact, just allowing firms to write price-contingent export contracts ($\psi^C(p)$ and $\Gamma^C(p)$), would yield outcomes identical to the full-information setting. At every price p, firms would decide how much they would optimally send at that price. Thus, at the realized price p, every firm sends a quantity that is equal to what they would send if they had full information and knew that price in advance. Thus, if we want to have some meaningful role for trade uncertainty, we need to step away from complete contingent export contracts.

To explore an economy with some contingent claims and some meaningful uncertainty, we consider an environment where a fraction α of firms submit price-contingent export plans ($\psi^C(p)$ and $\Gamma^C(p)$) and a fraction $(1-\alpha)$ of firms choose a level of exports ($\psi^N(\mu_x)$ and $\Gamma^N(\mu_y)$) that depends only on their home productivity. This captures the idea that in reality, firms use a variety of contracting arrangements for international transactions that allocate terms of trade risk to different parties. We eliminate any signals about the other country's productivity (the complete uncertainty environment) and study what happens as we change the fraction α of price-contingent exporters.

The equilibrium relative price is still the ratio of foreign exports to home exports:

$$p = e^{\mu_y - \mu_x + \frac{1}{2}(\sigma_y^2 - \sigma_x^2)} \times \left(\frac{\alpha \Gamma^C(p) + (1-\alpha)\Gamma^N(\mu_y)}{\alpha \Psi^C(p) + (1-\alpha)\Psi^N(\mu_x)} \right)$$

However, the total export of each country is now α times the export of the price-contingent firms plus $1-\alpha$ times the export amount chosen by the non-contingent firms. Similarly, we define the export share of the home country to be $\alpha\Psi^C(p) + (1-\alpha)\Psi^N(\mu_x)$.

Having a more complete contracting space and having more information are similar: Both cause the average trade share to fall. When we solve the model with contracts numerically, we see that price contingent exporters export half their production on average, no matter what the composition of other exporters is. The non-price contingent firms export an amount that is declining in the fraction of price-contingent exporters. As the number of price contingent exporters rise, each non-contingent firm is trading against an average foreign firm that is more likely to have chosen a price-contingent quantity. It is like trading against a foreign country that is better informed. Section 2.3 explains why trading with a better informed trading partner makes a firm want to export less.

The price-contingent export share rises, but is flat on a per-firm basis. The non-contingent export share falls in aggregate and because each firm exports less. The net effect is a decline in the trade share.

This model extension teaches us that, yes, introducing complete contingent contracts undermines the effect of asymmetric information. Yet, at the same time, completing the market and reducing information asymmetry work almost identically to reduce trade, for the same reasons. Conditioning exports on the outcome of a random variable and knowing that random variable before exports are chosen are functionally equivalent.

4. Conclusions

Information frictions are often invoked as reasons for low levels of international trade. But in an equilibrium model, the link between information friction and trade volume is not simple. Our model shows how information also changes the expected terms of trade. It also highlights that, in the face of risk, some types of agents may prefer to export more, to ensure they have a sufficient amount of the foreign good to consume. This depends on agents' preferences.

With constant elasticity of substitution (CES) preferences, information frictions impede trade when goods are not very substitutable. The decline in trade arises because the increase in risk from lower precision information deters trade, and that risk effect is stronger than the effect on the mean terms of trade, which encourages exporting. But with empirically plausible elasticity parameters, the opposite is true. Information frictions encourage trade. CES preference are not some special or anomalous case. We derive a a broad class of preferences for which similar effects arise.

The results teach us that, if we believe that information frictions are truly an important barriers to international trade, then we must also buy in to preference assumptions that differ from commonly-used preferences and parameters. If we believe that CES preferences with standard elasticity parameters are a good representation of behavior, then researchers should be searching for new forms of trade frictions.

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A. Proofs and Solution Details

1.1. Preliminaries for CES model

Notation: For each firm, we define the utility-and-trade-friction adjusted price aggregator as:

$$Q(z,\mu,\hat{m};p,s) \equiv \left[\mathbb{E}\left[\lambda(z,\mu,\hat{m})^{\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta-1}} + \mathbb{E}\left[(\lambda(z,\mu,\hat{m})sp)^{\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta-1}}\right]^{\frac{\theta-1}{\theta}}$$

where $(z,\mu,\hat{m})$ is the state of the firm, $\mathcal{I}$ is her information set, $p > 0$ are the country's terms of trade, $s \in [0,1]$ is a trade cost and $\lambda(z,\mu,\hat{m}) \equiv c(z,\mu,\hat{m})^{\frac{1-\theta}{\theta}}$ is a measure of firm's utility. Also denote $\bar{\lambda}(z,\mu,\hat{m}) \equiv \mathbb{E}\left[\lambda(z,\mu,\hat{m})^{\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta}}$. Each firm will aggregate prices using this function evaluated at the corresponding terms of trade and trade costs faced by its country. Note that this price index is firm specific.

Optimal exports of home country The maximization problem for the x country and its FOC are:

$$V(z_x,\mu_x,\hat{m}_y) = \max_{t_x} \mathbb{E}\left[(1-\sigma)^{-1}\left((z_x-t_x)^{\theta} + \left(\frac{t_x}{(1+\tau)q}\right)^{\theta}\right)^{(1-\sigma)/\theta}\middle|\mathcal{I}_x\right]$$

$$E\left[\frac{1}{\theta}\left((z_x-t_x)^{\theta} + \left(\frac{t_x}{(1+\tau)q}\right)^{\theta}\right)^{(1-\sigma-\theta)/\theta}\left(-\theta(z_x-t_x)^{\theta-1} + \theta\frac{t_x^{\theta-1}}{(1+\tau)^{\theta}q^{\theta}}\right)\middle|\mathcal{I}_x\right] = 0$$

Let $\lambda(z_x,\mu_x,\hat{m}_y) \equiv c(z_x,\mu_x,\hat{m}_y)^{(1-\theta)/\theta}$, write first term as $\lambda(z_x,\mu_x,\hat{m}_y)^{\theta} = c(z_x,\mu_x,\hat{m}_y)^{1-\theta}$ and break the expectation:

$$E\left[\frac{t_x^{\theta-1}}{(1+\tau)^{\theta}q^{\theta}}\lambda(z_x,\mu_x,\hat{m}_y)^{\theta}\middle|\mathcal{I}_x\right] = (z_x-t_x)^{\theta-1}E\left[\lambda(z_x,\mu_x,\hat{m}_y)^{\theta}\middle|\mathcal{I}_x\right]$$

Pulling out the non random term t_x we get:

$$\left(\frac{t_x}{z_x-t_x}\right)^{\theta-1} = \frac{E\left[\lambda(z_x,\mu_x,\hat{m}_y)^{\theta}\middle|\mathcal{I}_x\right]}{E\left[\left(\frac{\lambda(z_x,\mu_x,\hat{m}_y)}{(1+\tau)q}\right)^{\theta}\middle|\mathcal{I}_x\right]}$$

Rearranging and using the definition of the price aggregator, we get an implicit expression for optimal exports t_x:

$$
\begin{aligned}
t_x(z_x,\mu_x,\hat{m}_y) &= \frac{\mathbb{E}\left[\lambda(z_x,\mu_x,\hat{m}_y)^{\theta}\middle|\mathcal{I}_x\right]^{\frac{1}{\theta-1}}}{\mathbb{E}\left[\lambda(z_x,\mu_x,\hat{m}_x)^{\theta}\middle|\mathcal{I}_x\right]^{\frac{1}{\theta-1}}+\mathbb{E}\left[\left(\frac{\lambda(z_x,\mu_x,\hat{m}_x)}{(1+\tau)q}\right)^{\theta}\middle|\mathcal{I}_x\right]^{\frac{1}{\theta-1}}} \\
&= \left(\frac{\left[\mathbb{E}\left[\lambda(z_x,\mu_x,\hat{m}_x)^{\theta}\middle|\mathcal{I}_x\right]^{\frac{1}{\theta-1}}+\mathbb{E}\left[\left(\frac{\lambda(z_x,\mu_x,\hat{m}_x)}{(1+\tau)q}\right)^{\theta}\middle|\mathcal{I}_x\right]^{\frac{1}{\theta-1}}\right]^{\frac{\theta-1}{\theta}}}{\mathbb{E}\left[\lambda(z_x,\mu_x,\hat{m}_y)^{\theta}\middle|\mathcal{I}_x\right]^{\frac{1}{\theta}}}\right)^{\frac{\theta}{1-\theta}} \\
&= z_x\left(\frac{Q\left(z_x,\mu_x,\hat{m}_x;\frac{1}{q},\frac{1}{1+\tau}\right)}{\bar{\lambda}(z_x,\mu_x,\hat{m}_x)}\right)^{\frac{\theta}{1-\theta}}
\end{aligned}
$$

Note that with perfect information: $Q\left(z_x,\mu_x,\hat{m}_x;\frac{1}{q},\frac{1}{1+\tau}\right)=\lambda(z_x,\mu_x,\hat{m}_x)\left(1+\left(\frac{1}{(1+\tau)q}\right)^{\frac{\theta}{\theta-1}}\right)^{\frac{\theta-1}{\theta}}$ and $\bar{\lambda}(z_x,\mu_x,\hat{m}_x)=\lambda(z_x,\mu_x,\hat{m}_x)$, then the export expression simplifies to a linear function of z_x:

$$
t_x = z_x\left(\frac{1}{1+((1+\tau)q)^{\frac{\theta}{1-\theta}}}\right)
$$

Analogously, one can show that firm optimal exports in the foreign country are given by:

$$
t_y(z_y,\mu_y,\hat{m}_x) = z_y\left(\frac{Q\left(z_y,\mu_y,\hat{m}_x;q,\frac{1}{1+\tau}\right)}{\bar{\lambda}(z_y,\mu_y,\hat{m}_x)}\right)^{\frac{\theta}{1-\theta}}
$$

and with perfect information they reduce to:

$$
t_y = z_y\left(\frac{1}{1+\left(\frac{1+\tau}{q}\right)^{\frac{\theta}{1-\theta}}}\right)
$$

1.2. Lemma 1: Exports are proportional to firm productivity

Proof. Guess a solution $t(z,\mu,\hat{m}) = z\Psi(\mu,\hat{m})$. First we show that the composite good is also proportional to z:

$$\begin{aligned} c(z,\mu,\hat{m}) &= \Big((z - t(z,\mu,\hat{m}))^{\theta} + (t(z,\mu,\hat{m})sp))^{\theta}\Big)^{1/\theta} \\ &= \Big(z^{\theta}(1-\Psi(\mu,\hat{m}))^{\theta} + z^{\theta}\left(\Psi(\mu,\hat{m})sp\right)^{\theta}\Big)^{1/\theta} \\ &= z\Big((1-\Psi(\mu,\hat{m}))^{\theta} + (\Psi(\mu,\hat{m})sp)^{\theta}\Big)^{1/\theta} \\ &= z\Psi_2(\mu,\hat{m}) \end{aligned}$$

where $\Psi_2 \equiv \Big((1-\Psi)^{\theta} + (\Psi sp)^{\theta}\Big)^{1/\theta}$.

Second, we substitute the composite consumption in $\bar{\lambda}$ and we obtain a separable function between idiosyncratic and aggregate variables:

$$\begin{aligned} \bar{\lambda}(z,\mu,\hat{m}) &\equiv \mathbb{E}\left[\lambda(z,\mu,\hat{m})^{\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta}} \\ &= \mathbb{E}\left[c(z,\mu,\hat{m})^{1-\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta}} \\ &= \mathbb{E}\left[z^{1-\theta}\Psi_2(\mu,\hat{m})^{1-\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta}} \\ &= z^{(1-\theta)/\theta}\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta}} \end{aligned}$$

Third, substitute λ in Q and again we obtain a separable function:

$$\begin{aligned} Q(z,\mu,\hat{m};p,s) &\equiv \left[\mathbb{E}\left[\lambda(z,\mu,\hat{m})^{\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta-1}} + \mathbb{E}\left[(\lambda(z,\mu,\hat{m})sp)^{\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta-1}}\right]^{\frac{\theta-1}{\theta}} \\ &= \left[\mathbb{E}\left[c(z,\mu,\hat{m})^{1-\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta-1}} + \mathbb{E}\left[c(z,\mu,\hat{m})^{1-\theta}(sp)^{\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta-1}}\right]^{\frac{\theta-1}{\theta}} \\ &= \left[\mathbb{E}\left[z^{1-\theta}\Psi_2(\mu,\hat{m})^{1-\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta-1}} + \mathbb{E}\left[z^{1-\theta}\Psi_2(\mu,\hat{m})^{1-\theta}(sp)^{\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta-1}}\right]^{\frac{\theta-1}{\theta}} \\ &= \left[z^{-1}\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta-1}} + z^{-1}\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}(sp)^{\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta-1}}\right]^{\frac{\theta-1}{\theta}} \\ &= z^{(1-\theta)/\theta}\left[\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta-1}} + \mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}(sp)^{\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta-1}}\right]^{\frac{\theta-1}{\theta}} \end{aligned}$$

Finally, substituting all the elements above in the implicit function that defines the export policy we get that terms with z inside Q and $\bar{\lambda}$ cancel out:

$$\begin{aligned}
t(z,\mu,\hat{m}) &= z\left(\frac{Q\left(z,\mu,\hat{m};p,s\right)}{\bar{\lambda}(z,\mu,\hat{m})}\right)^{\frac{\theta}{1-\theta}} \\
&= z\left(\frac{z^{(1-\theta)/\theta}\left[\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}\middle|\mathcal{I}\right]^{1/(\theta-1)}+\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}(sp)^{\theta}\middle|\mathcal{I}\right]^{1/(\theta-1)}\right]^{(\theta-1)/\theta}}{z^{(1-\theta)/\theta}\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}\middle|\mathcal{I}\right]^{1/\theta}}\right)^{\theta/(1-\theta} \\
&= z\left(\frac{\left[\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta-1}}+\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}(sp)^{\theta}\middle|\mathcal{I}\right]^{1/(\theta-1)}\right]^{(\theta-1)/\theta}}{\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}\middle|\mathcal{I}\right]^{1/\theta}}\right)^{\theta/(1-\theta)} \\
&= z\Psi(\mu,\hat{m})
\end{aligned}$$

Therefore, we have verified that the export policy is indeed linear in idiosyncratic productivity z.

Furthermore, the component that depends on aggregate shocks is given by:

$$\Psi(\mu,\hat{m})=\left(\frac{\left[\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta-1}}+\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}(sp)^{\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta-1}}\right]^{\frac{\theta-1}{\theta}}}{\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta}}}\right)^{\frac{\theta}{1-\theta}}$$

where

$$\Psi_2(\mu,\hat{m})=\Big((1-\Psi(\mu,\hat{m}))^{\theta}+(\Psi(\mu,\hat{m})sp)^{\theta}\Big)^{1/\theta}$$

■

1.3. Derivation of fixed point problems

Using Lemma 1, we can aggregate the exports of domestic and foreign firms:

$$T_x(\mu_x,\hat{m}_y)=\int z_x\Psi(\mu_x,\hat{m}_y)dF(z_x|\mu_x)=e^{\mu_x+\frac{1}{2}\sigma_x^2}\Psi(\mu_x,\hat{m}_y)$$

$$T_y(\mu_y,\hat{m}_x)=\int z_y\Gamma(\mu_y,\hat{m}_x)dF(z_y|\mu_y)=e^{\mu_y+\frac{1}{2}\sigma_y^2}\Gamma(\mu_y,\hat{m}_x)$$

where the last equality holds because z_x and z_y are lognormal. Given the iceberg cost, only a fraction $\frac{1}{1+\tau}$ of exports arrive to the international markets. The actual equilibrium price will be given by:

$$q(\mu_x,\mu_y,\hat{m}_x,\hat{m}_y) = \frac{T_x(\mu_x,\hat{m}_y)/1+\tau}{T_y(\mu_y,\hat{m}_x)/1+\tau} = \frac{e^{\mu_x+\frac{1}{2}\sigma_x^2}\Psi(\mu_x,\hat{m}_y)}{e^{\mu_y+\frac{1}{2}\sigma_y^2}\Gamma(\mu_y,\hat{m}_x)} = f\frac{\Psi(\mu_x,\hat{m}_y)}{\Gamma(\mu_y,\hat{m}_x)}$$

where $f \equiv e^{(\mu_x-\mu_y)}e^{\frac{1}{2}(\sigma_x^2-\sigma_y^2)}$ are the relative fundamentals.

Rearranging Ψ and Γ and impose consistency of beliefs (the actual price function is used by agents to form their beliefs), we get that equilibrium is given by three functions Ψ, Γ and q such that they solve the following fixed point problems:

$$\begin{aligned}
\Psi(\mu_x,\hat{m}_y) &= \frac{1}{1+(1+\tau)^{\frac{\theta}{1-\theta}}\left(\frac{\mathbb{E}_{\mu_y,\hat{m}_x}\left[\Psi_2(\mu_x,\mu_y,\hat{m}_x,\hat{m}_y)^{1-\theta}\middle|\mathcal{I}_x\right]}{\mathbb{E}_{\mu_y,\hat{m}_x}\left[\Psi_2(\mu_x,\mu_y,\hat{m}_x,\hat{m}_y)^{1-\theta}q(\mu_x,\mu_y,\hat{m}_x,\hat{m}_y)^{-\theta}\middle|\mathcal{I}_x\right]}\right)^{\frac{1}{1-\theta}}} \\
\Gamma(\mu_y,\hat{m}_x) &= \frac{1}{1+(1+\tau)^{\frac{\theta}{1-\theta}}\left(\frac{\mathbb{E}_{\mu_x,\hat{m}_y}\left[\Gamma_2(\mu_y,\mu_x,\hat{m}_y,\hat{m}_x)^{1-\theta}\middle|\mathcal{I}_y\right]}{\mathbb{E}_{\mu_x,\hat{m}_y}\left[\Gamma_2(\mu_y,\mu_x,\hat{m}_y,\hat{m}_x)^{1-\theta}q(\mu_x,\mu_y,\hat{m}_x,\hat{m}_y)^{\theta}\middle|\mathcal{I}_y\right]}\right)^{\frac{1}{1-\theta}}} \\
q(\mu_x,\mu_y,\hat{m}_x,\hat{m}_y) &= f\frac{\Psi(\mu_x,\hat{m}_y)}{\Gamma(\mu_y,\hat{m}_x)}
\end{aligned}$$

where the auxiliary functions are:

$$\begin{aligned}
\Psi_2(\mu_x,\mu_y,\hat{m}_x,\hat{m}_y) &= \left((1-\Psi(\mu_x,\hat{m}_y))^\theta + \left(\frac{\Psi(\mu_x,\hat{m}_y)}{(1+\tau)q(\mu_x,\mu_y,\hat{m}_x,\hat{m}_y)}\right)^\theta\right)^{1-\sigma/\theta} \\
\Gamma_2(\mu_y,\mu_x,\hat{m}_y,\hat{m}_x) &= \left((1-\Gamma(\mu_y,\hat{m}_x))^\theta + \left(\frac{\Gamma(\mu_y,\hat{m}_x)q(\mu_y,\mu_x,\hat{m}_x,\hat{m}_y)}{(1+\tau)}\right)^\theta\right)^{1-\sigma/\theta}
\end{aligned}$$

This system can be expressed compactly as:

$$\begin{aligned}
\Psi &= g_1(\mathcal{I}_x,\Psi,\Gamma) \\
\Gamma &= g_2(\mathcal{I}_y,\Psi,\Gamma) \\
q &= fg_3(\Psi,\Gamma)
\end{aligned}$$

1.4. Proofs

Proof of Result 1: Information Creates Covariance Proof.

Part 1: From derivative to covariance. The first step is to connect the derivative $\frac{dT_x}{dT_y}$ with the covariance $\mathbb{C}[T_x, T_y|\mathcal{I}_x]$. Note that T_y and μ_x are the only random variables for the agent in country x. A first order approximation of the policy function $T_x(T_y, \mu_x)$ yields

$$T_x(\mu_x, T_y) \approx T_x(\mu_x, \mathbb{E}[T_y|\mathcal{I}_x]) + \beta\,(T_y - \mathbb{E}[T_y|\mathcal{I}_x]) + \gamma\,(\mu_x - m_x) = \alpha + \beta T_y + \gamma \mu_x$$

where α gathers all the constants. From an ex-ante perspective, T_x is a random variable. With this approximation, the covariance with T_y is given by:

$$\mathbb{C}[T_x, T_y] \approx \mathbb{C}(\alpha + \beta T_y + \gamma \mu_x, T_y) = \beta \mathbb{V}(T_y)$$

i.e. the own aggregate shock does not induce covariance with other countries exports. Therefore, the slope is given by

$$\beta = \frac{\mathbb{C}[T_x, T_y]}{\mathbb{V}(T_y)} = \frac{dT_x}{dT_y}\Big|_{T_y = \mathbb{E}[T_y]}$$

With no information $\beta = 0$. With perfect information and condition M, $\frac{dT_x}{dT_y} > 0 \ \forall T_y$, and therefore, $\beta > 0$. We have established that

$$sign\left(\frac{dT_x}{dT_y}\right) = sign\left(\mathbb{C}(T_x, T_y)\right) = sign(\beta)$$

Part 2: Continuity of the covariance in the amount of information. If the conditional distribution of terms of trade p is a continuous function of the signal and its precision, then the continuity of Bayesian updating together with the continuity of the integral operator ensure that any conditional expectation is continuous as well. Since the covariance is an expectation, it is also a continuos function of the signal precision. By (i) for no information (zero precision) the covariance is zero, and for perfect information (infinity precision) the covariance is positive. By the continuity established in (ii), there exists an interval for precision between 0 and infinity for which the covariance is increasing in precision. Therefore, more information increases the covariance of aggregate exports.

■

Proof of Result 2: Variance of terms of trade **Proof.** A first order approximation of $p = \frac{T_y}{T_x}$ around the expectation of aggregate exports $(\mathbb{E}[T_x], \mathbb{E}[T_y])$:

$$\frac{T_y}{T_x} \approx \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]} + \frac{1}{\mathbb{E}[T_x]}(T_y - \mathbb{E}[T_y]) - \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]^2}(T_x - \mathbb{E}[T_x])$$

Now take expectation on both sides and cancel the first order terms:

$$\mathbb{E}\left[\frac{T_y}{T_x}\right] \approx \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]}$$

Subtract the two previous expressions to compute the variance:

$$\begin{aligned}\mathbb{V}\left[\frac{T_y}{T_x}\right] &= \mathbb{E}\left[\left(\frac{T_y}{T_x} - \mathbb{E}\left[\frac{T_y}{T_x}\right]\right)^2\right] = \mathbb{E}\left[\left(\frac{1}{\mathbb{E}[T_x]}(T_y - \mathbb{E}[T_y]) - \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]^2}(T_x - \mathbb{E}[T_x])\right)^2\right] \\ &= \frac{1}{\mathbb{E}[T_x]^2}\left[\mathbb{V}[T_y] + \frac{\mathbb{E}[T_y]^2}{\mathbb{E}[T_x]^2}\mathbb{V}[T_x] - 2\frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]}\mathbb{C}[T_x, T_y]\right]\end{aligned}$$

Keeping the variance of exports constant, the larger covariance decreases the variance of the terms of trade. By symmetry, $\mathbb{E}[T_y] = \mathbb{E}[T_x]$ and $\mathbb{V}[T_y] = \mathbb{V}[T_x]$, we can simplify to:

$$\mathbb{V}\left[\frac{T_y}{T_x}\right] = \frac{2}{\mathbb{E}[T_x]^2}\left(\mathbb{V}[T_x] - \mathbb{C}[T_x, T_y]\right)$$

Again, using the definition of coefficient of variation and the correlation coefficient, and symmetry:

$$\frac{\mathbb{V}\left[p\right]}{2} = CV^2[T_x]\left(1 - corr([T_x, T_y])\right)$$

The proof is analogous for the foreign country. ■

Proof of Result 3: Expected terms of trade **Proof.** A second order approximation of $p = \frac{T_y}{T_x}$ around the unconditional expectation of aggregate exports $(\mathbb{E}[T_x], \mathbb{E}[T_y])$ yields:

$$\frac{T_y}{T_x} \approx \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]} + \frac{1}{\mathbb{E}[T_x]}(T_y - \mathbb{E}[T_y]) - \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]^2}(T_x - \mathbb{E}[T_x]) + \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]^3}(T_x - \mathbb{E}[T_x])^2 - \frac{1}{\mathbb{E}[T_x]^2}(T_x - \mathbb{E}[T_x])(T_y - \mathbb{E}[T_y])$$

Taking expectations on both sides, which makes the first order terms equal to zero, yields the result:

$$\mathbb{E}\left[\frac{T_y}{T_x}\right] \approx \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]} + \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]^3}\mathbb{V}[T_x] - \frac{1}{\mathbb{E}[T_x]^2}\mathbb{C}[T_x, T_y] \tag{21}$$

By symmetry, $\mathbb{E}[T_y] = \mathbb{E}[T_x]$ and $\mathbb{V}[T_y] = \mathbb{V}[T_x]$, we can simplify to:

$$\mathbb{E}\left[\frac{T_y}{T_x}\right] = 1 + \frac{\mathbb{V}[T_x]}{\mathbb{E}[T_x]^2} - \frac{\mathbb{C}[T_x, T_y]}{\mathbb{E}[T_x]^2}$$

Furthermore, using the definition of coefficient of variation $CV^2[z] = \frac{\mathbb{V}[z]}{\mathbb{E}[z]^2}$ and the correlation coefficient, together with symmetry across countries, we obtain:

$$\mathbb{E}\left[p\right] = 1 + CV^2[T_x]\left(1 - corr([T_x, T_y])\right)$$

The proof is analogous from the foreign country's perspective, using an approximation of $1/p$.

■

Proof of Proposition 1 This is a special case of Proposition 3, proven below.

Proof of Lemma 1

Lemma 1 *Let $g(\cdot)$ be the probability density function of the terms of trade and $\phi(\cdot)$ be a normal probability density. Suppose that $g(\cdot) = h(\cdot) * \phi(\cdot)$, where h is continuous. Then the function h must be somewhere convex.*

Proof. The terms of trade is a ratio of two non-negative stochastic variables: T_x/T_y. Both T_x and T_y are proportional to a log-normal variable. As such, they can take on any positive value with strictly positive probability density. Thus, the ratio of the two variables has positive density over the positive real line. Thus, the function $g(p)$ takes on value zero for all $p < 0$.

We can thus deduce three properties on the function h: 1) If $g(p)$ takes on value zero for all $p < 0$, and the normal density is positive-valued over the whole real line, then $h(p)$ must be zero for all $p < 0$. 2) For g to be a probability density, it must be that $h(p)$ does not fall below zero. 3) h cannot be a constant function. Since we know it takes on value zero, it would then be zero everywhere. If that were true, g would be zero everywhere, which is not a probability density because it does not integrate to one.

From these three properties, we can deduce that h must be somewhere convex. Suppose not. If the function h is nowhere convex, then it is globally, weakly concave. Since it is not a constant function, there exists some x^* such that $h'(x^*) \neq 0$. Let m be a linear function with the slope $h'(x^*)$ and that passes x^*. Then for all $x \in \mathbb{R}$, $h(x) \leq m(x)$. Since g is a linear function with non-zero slope, there exists p^* such that $m(p^*) < 0$. This means that $h(p^*) < 0$, which violates the assumption that $h(p)$ does not fall below zero. This contradiction proves that under the assumptions stipulated, h must be somewhere convex.

■

Proof of Proposition 2: Sign of change in exports for CES **Proof.** Given the domestic country state—endowment μ_x and signal about foreign endowment $\tilde{m}_y$—the FOC of the maximization problem yields

$$\mathbb{E}_x[w(p)] = 0 \qquad \text{with} \qquad w(p) = pU_y(f_x - T_x(p), pT_x(p)) - U_x(f_x - T_x(p), pT_x(p))$$

where the expectation operator is conditional on its information set $\mathbb{E}_x[\cdot] = \mathbb{E}[\cdot|\mu_x, \tilde{m}_y]$ (equal to its state), $w(p)$ is the marginal utility of exports, and $f_x = e^{\mu_x}$ is the country's aggregate endowment.

A second order approximation of $w(p)$ around the terms of trade conditional expectation $\mathbb{E}_x[p]$ gives:

$$\begin{aligned} 0 = \mathbb{E}_x[w(p)] &\approx w(\mathbb{E}_x[p]) + w'(\mathbb{E}_x[p])\mathbb{E}_x[p - \mathbb{E}_x[p]] + \frac{1}{2}w''(\mathbb{E}_x[p])\mathbb{E}_x[p - \mathbb{E}_x[p])^2] \\ 0 &= w(\mathbb{E}_x[p]) + \frac{\mathbb{V}_x[p]}{2}w''(\mathbb{E}_x[p]) \\ 0 &= U_y\left(\mathbb{E}_x[p]\right)\left\{\mathbb{E}_x[p] - \rho_y^{(1)}(\mathbb{E}_x[p])\left(\frac{2-\tilde{\rho}_y^{(2)}(\mathbb{E}_x[p])}{2}\right)\frac{\mathbb{V}_x[p]}{\mathbb{E}_x[p]}\right\} - U_x\left(\mathbb{E}_x[p]\right) \\ 0 &= \varphi(T, \mathbb{E}_x[p], \mathbb{V}_x[p]) \end{aligned}$$

The expression $\varphi(\cdot) = 0$ determines optimal exports as a function of conditional moments of the terms of trade. Rearranging the expression in terms of the marginal rate we obtain the result. ■

Proof of Proposition 3 **Proof.** By Implicit Function Theorem applied to $\varphi(T, \mathbb{E}_x[p], \mathbb{V}_x[p]) = 0$ we have that

$$\begin{aligned} \frac{\partial\varphi}{\partial T_x}\left(\frac{\partial T_x}{\partial\mathbb{E}_x[p]}d\mathbb{E}_x[p] + \frac{\partial T_x}{\partial\mathbb{V}_x[p]}d\mathbb{V}_x[p]\right) + \frac{\partial\varphi}{\partial\mathbb{E}_x[p]}d\mathbb{E}_x[p] + \frac{\partial\varphi}{\partial\mathbb{V}_x[p]}d\mathbb{V}_x[p] = 0 \\ \frac{\partial T_x}{\partial\mathbb{E}_x[p]}d\mathbb{E}_x[p] + \frac{\partial T_x}{\partial\mathbb{V}_x[p]}d\mathbb{V}_x[p] = -\left(\frac{\partial\varphi}{\partial\mathbb{E}_x[p]}d\mathbb{E}_x[p] + \frac{\partial\varphi}{\partial\mathbb{V}_x[p]}d\mathbb{V}_x[p]\right)\Big/\frac{\partial\varphi}{\partial T_x} \end{aligned}$$

Since the denominator is negative (utility is concave in exports), the sign of the derivative is given by the numerator.[3]

$$\begin{aligned} num &= \frac{\partial\varphi}{\partial\mathbb{E}_x[p]}d\mathbb{E}_x[p] + \frac{\partial\varphi}{\partial\mathbb{V}_x[p]}d\mathbb{V}_x[p] \\ &= \left(w'(\mathbb{E}_x[p]) + \frac{\mathbb{V}_x(p)}{2}w'''(\mathbb{E}_x[p])\right)d\mathbb{E}_x[p] + \frac{1}{2}w''(\mathbb{E}_x[p])d\mathbb{V}_x[p] \\ &= U_y(\mathbb{E}_x[p])\left[\left(\left(1-\tilde{\rho}_y^{(1)}\right) + \frac{\rho_y^{(1)}\rho_y^{(2)}}{\mathbb{E}_x[p]^2}\frac{\mathbb{V}_x[p]}{2}(3-\tilde{\rho}_y^{(3)})\right)d\mathbb{E}_x[p] - \frac{\rho_y^{(1)}}{\mathbb{E}_x[p]}(2-\tilde{\rho}_y^{(2)})\frac{d\mathbb{V}_x[p]}{2}\right] \end{aligned}$$

where the new term $w'''(p)$ is equal to $w'''(p) = U_y\frac{\rho_y^{(1)}\rho_y^{(2)}}{p^2}(\tilde{\rho}_y^{(3)} - 3)$, where $\tilde{\rho}_y^{(3)} = \rho_y^{(3)}\left(1 - \frac{U_{xyyy}/p}{U_{yyyy}}\right)$ and $\rho_y^{(3)} \equiv -\frac{C_y U_{yyyy}}{U_{yyy}}$ is the coefficient of relative temperance. If this expression is positive, then an increase in the conditional mean and variance of the terms of trade increase exports. ■

[3]To do: prove denom is negative. When computing derivatives, we ignore changes in coefficients of relative risk (aversion, prudence, and temperance). For the preferences we consider, this coefficients are constant. In more general cases, these changes are quantitatively small.

Proof of Proposition 4 **Proof.** Let η be a random variable with mean $\mathbb{E}_\eta[\eta] = \bar{\eta}$ and variance $\mathbb{V}_\eta[\eta] = \sigma_\eta$. Define a new function $m(\eta) \equiv \mathbb{E}[g(T, p + \eta)|\mathcal{I}_x]$, which is the expected marginal utility of additional exports, given a level of exports T and a price $p + \eta$, where $g(\cdot)$ is given by (**??**) and the expectation is taken over the distribution of the terms of trade p. Fix an η realization. Combining the Implicit Function Theorem and the definition of m yields

$$\frac{dT}{d\eta} = -\frac{\partial \mathbb{E}[g|\mathcal{I}_x]/\partial p}{\partial \mathbb{E}[g|\mathcal{I}_x]/\partial T} = -\frac{\partial m(\eta)/\partial \eta\Big|_{\eta=0}}{\partial m(\eta)/\partial T\Big|_{\eta=0}} \tag{22}$$

Since expected marginal utility is decreasing in exports, the denominator is always negative. Then we will characterize the sign of the numerator $\partial m(\eta)/\partial \eta\Big|_{\eta=0}$. We do a second order approximation of $m(\eta)$ around $\eta = 0$ and then take its expectation across η to obtain:

$$\begin{aligned}
\mathbb{E}_\eta[m(\eta)] &\approx m(0) + m'(0)\mathbb{E}_\eta[\eta] + \frac{1}{2}m''(0)\mathbb{E}_\eta[\eta^2] \\
&= \mathbb{E}[g(T,p)|\mathcal{I}_x] + \bar{\eta}\mathbb{E}\left[\frac{\partial g(T,p)]}{\partial p}\Big|\mathcal{I}_x\right] + \frac{(\sigma_\eta + \bar{\eta}^2)}{2}\mathbb{E}\left[\frac{\partial^2 g(T,p)}{\partial p^2}\Big|\mathcal{I}_x\right]
\end{aligned}$$

where in the last line we use the definition of m to evaluate $m(0)$, $m'(0)$, and $m''(0)$.

We can isolate the effect of changes in the mean by taking the derivative of the previous expression with respect to $\bar{\eta}$ and then evaluate at $\bar{\eta} = 0$

$$\frac{\partial \mathbb{E}[m(\eta)]}{\partial \bar{\eta}}\Big|_{\bar{\eta}=0} = \frac{\partial \mathbb{E}[g(T,p)]}{\partial p} + \bar{\eta}\frac{\partial^2 \mathbb{E}[g(T,p)]}{\partial p^2} = \frac{\partial \mathbb{E}[g(T,p)]}{\partial p}$$

Analogously, we can isolate the effect of changes in the variance by taking the derivative with respect to σ_η and then evaluate at $\sigma_\eta = 0$

$$\frac{\partial \mathbb{E}[m(\eta)]}{\partial \sigma_\eta}\Big|_{\sigma_\eta=0} = \frac{1}{2}\frac{\partial^2 \mathbb{E}[g(T,p)]}{\partial p^2}$$

Finally, to get the total effect from joint changes in the mean and variance we take the total derivative with respect to both mean and variance:

$$\begin{aligned}
d\mathbb{E}[m(\eta)] &= (1+\bar{\eta})d\bar{\eta}\ \mathbb{E}\left[\frac{\partial g(T,p)]}{\partial p}\Big|\mathcal{I}_x\right] + d\sigma_\eta\ \mathbb{E}\left[\frac{1}{2}\frac{\partial^2 g(T,p)}{\partial p^2}\Big|\mathcal{I}_x\right] \\
&= (1+\bar{\eta})d\bar{\eta}\ \underbrace{\mathbb{E}\left[U_y\left(1-\tilde{\rho}_y\right)\Big|\mathcal{I}_x\right]}_{\text{Condition M}} + d\sigma_\eta \underbrace{\mathbb{E}\left[\frac{U_y\rho_y}{2p}\left(\tilde{\pi}_y - 2\right)\Big|\mathcal{I}_x\right]}_{\text{with Condition V}}
\end{aligned}$$

In the particular case where the variance and the mean are linearly related (as it happens in our

GE model) as $d\sigma_\eta = 2d\bar{\eta}$, the condition simplifies to:

$$d\mathbb{E}[m(\eta)] \quad = \quad d\bar{\eta}\left[(1+\bar{\eta})\underbrace{\mathbb{E}\left[U_y\left(1-\tilde{\rho}_y\right)\Big|\mathcal{I}_x\right]}_{\geq 0 \text{ Condition M}}+\underbrace{\mathbb{E}\left[\frac{U_y\rho_y}{p}\left(\tilde{\pi}_y-2\right)\Big|\mathcal{I}_x\right]}_{\geq 0 \text{ with Condition V}}\right]$$

Evaluating at $\bar{\eta} = 0$, we obtain Condition T:

$$\frac{d\mathbb{E}[m(\eta)]}{d\bar{\eta}}\Big|_{\bar{\eta}=0} \quad = \quad \underbrace{\mathbb{E}\left[U_y\left((1-\tilde{\rho}_y)+\frac{\rho_y}{p}\left(\tilde{\pi}_y-2\right)\right)\Big|\mathcal{I}_x\right]}_{\text{Condition T}}$$

■

Proof of Proposition 5 **Proof.** For fixed domestic exports, the change in the expected terms of trade and the change in foreign exports are proportional. Using our approximation to foreign exports, we take derivatives with respect to foreign $\tilde{s}_x^2$:

$$\frac{d\mathbb{E}_x[T_y]}{d\tilde{s}_x^2} \approx \frac{\partial T_y\left(\hat{m}_y, \hat{\hat{m}}_x, \hat{\hat{\hat{m}}}_y\right)}{\partial \tilde{s}_x^2} = \frac{\partial T_y}{\partial \hat{m}_x}\frac{\partial \hat{\hat{m}}_x}{\partial \tilde{s}_x^2} = \frac{\partial T_y}{\partial \hat{m}_x}\frac{\hat{\hat{s}}_x^2}{\tilde{s}_x^2}\left(\frac{m_x-\mu_x}{s_x^2}\right) = \mathcal{A}\left(\frac{m_x-\mu_x}{s_x^2}\right)$$

where $\mathcal{A} \equiv \frac{\partial T_y}{\partial \hat{m}_x}\frac{\hat{\hat{s}}_x^2}{\tilde{s}_x^2}$. This expression is positive in states with a low domestic endowment, i.e. $\mu_x < m_x$

For the variance,

$$\begin{aligned}\frac{d\mathbb{V}_x[T_y]}{d\tilde{s}_x^2} \quad &\approx \quad \frac{\partial \hat{\hat{s}}_x^2}{\partial \tilde{s}_x^2}\left(\frac{\partial T_y}{\partial \hat{m}_x}\right)^2 + 2\frac{\partial \hat{\hat{m}}_x}{\partial \tilde{s}_x^2}\left[\left(\frac{\partial T_y}{\partial \mu_y}\right)\left(\frac{\partial^2 T_y}{\partial \mu_y \hat{\hat{m}}_x}\right)\hat{s}_y^2 + \left(\frac{\partial T_y}{\partial \hat{m}_x}\right)\left(\frac{\partial^2 T_y}{\partial \hat{m}_x \hat{\hat{m}}_x}\right)\hat{\hat{s}}_x^2 + \left(\frac{\partial T_y}{\partial \hat{\hat{m}}_y}\right)\left(\frac{\partial^2 T_y}{\partial \hat{\hat{m}}_y \hat{\hat{m}}_x}\right)\hat{\hat{s}}\right. \\ &= \quad \mathcal{B}\left(\frac{m_x-\mu_x}{s_x^2}\right) + \mathcal{C}\left(\frac{\hat{\hat{s}}_x^2}{\tilde{s}_x^2} - \frac{1}{2}\right)\end{aligned}$$

where

$$\begin{aligned}\mathcal{B} \quad &\equiv \quad 2\frac{\hat{s}_x^2}{\tilde{s}_x^2}\left[\left(\frac{\partial T_y}{\partial \mu_y}\right)\left(\frac{\partial^2 T_y}{\partial \mu_y \hat{\hat{m}}_x}\right)\hat{s}_y^2 + \left(\frac{\partial T_y}{\partial \hat{m}_x}\right)\left(\frac{\partial^2 T_y}{\partial \hat{m}_x \hat{\hat{m}}_x}\right)\hat{\hat{s}}_x^2 + \left(\frac{\partial T_y}{\partial \hat{\hat{m}}_y}\right)\left(\frac{\partial^2 T_y}{\partial \hat{\hat{m}}_y \hat{\hat{m}}_x}\right)\hat{\hat{s}}_y^2\right] > 0 \\ \mathcal{C} \quad &\equiv \quad 2\frac{\hat{s}_x^2}{\tilde{s}_x^2}\left(\frac{\partial T_y}{\partial \hat{m}_x}\right)^2 > 0\end{aligned}$$

Note that the term $\left(\frac{\hat{\hat{s}}_x^2}{\tilde{s}_x^2} - \frac{1}{2}\right)$ can be written as $\left(\frac{s_x^2}{\tilde{s}_x^2} - 1\right)$. Therefore, for the variance to increase, we require a small signal noise, i.e. $\tilde{s}_x^2 < s_x^2$. *Still need to show: $\mathcal{A}, \mathcal{B}, \mathcal{C} > 0$ ■

A. Details of Computational Algorithm: Not for Publication

1.1. Polynomial approximation to policy functions

Functional Basis Let $\{\Phi_k\}_{k=1}^{M}$ be a basis of polynomials with support $x \in [a, b]$. We use linear splines and uniform nodes for the 2 states of each country.

1. **Grid for state 1: Own productivity**:

   - In x country it is distributed $\mu_x \sim \mathcal{N}(m_x, s_x^2)$, where m_x, s_x are parameters. We construct uniform nodes $\{\mu_x^i\}_{i=1}^{N}$ in the support $[m_x - 4s_x, m_x + 4s_x]$.

   - In y country it is distributed $\mu_y \sim \mathcal{N}(m_y, s_y^2)$, where m_y, s_y are parameters. We construct uniform nodes $\{\mu_y^i\}_{i=1}^{N}$ in the support $[m_y - 4s_y, m_y + 4s_y]$.

2. **Grid for state 2: Posterior mean of foreign productivity**:

   - In x country, the posterior mean of foreign productivity is $\hat{m}_y \sim \mathcal{N}(m_y, \bar{s}_y^2)$ where $\bar{s}_y^2 = \frac{s_y^4}{s_y^2 + \tilde{s}_y^2}$. However, to use a fixed grid that does not change with the precision of information, we construct the nodes $\{\hat{\mu}_y^j\}_{j=1}^{N}$ over the support $[m_y - 4s_y, m_y + 4s_y]$.

   - In y country, the posterior mean of foreign productivity is $\hat{m}_x \sim \mathcal{N}(m_x, \bar{s}_x^2)$ where $\bar{s}_x^2 = \frac{s_x^4}{s_x^2 + \tilde{s}_x^2}$. Analogously, we construct the nodes $\{\hat{m}_x^j\}_{j=1}^{N}$ over the support $[m_x - 4s_x, m_x + 4s_x]$.

Approximating functions We approximate four conditional expectations with polynomials:

$$
\begin{aligned}
\mathbb{E}_{\mu_y, \hat{m}_x}\left[\Psi_2(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)^{1-\theta} \middle| \mathcal{I}_x\right] &\approx g^1(\mu_x, \hat{m}_y) \\
\mathbb{E}_{\mu_y, \hat{m}_x}\left[\Psi_2(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)^{1-\theta} q(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)^{-\theta} \middle| \mathcal{I}_x\right] &\approx g^2(\mu_x, \hat{m}_y) \\
\mathbb{E}_{\mu_x, \hat{m}_y}\left[\Gamma_2(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y)^{1-\theta} \middle| \mathcal{I}_y\right] &\approx h^1(\mu_y, \hat{m}_x) \\
\mathbb{E}_{\mu_x, \hat{m}_y}\left[\Gamma_2(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y)^{1-\theta} q(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)^{\theta} \middle| \mathcal{I}_y\right] &\approx h^2(\mu_y, \hat{m}_x)
\end{aligned}
$$

where the polynomials are constructed using the basis for each dimension evaluated at the nodes described above:

$$
\begin{aligned}
g^1(\mu_x^i, \hat{m}_y^j) &\equiv \sum_{k,k' \in K \times K'} g^1_{k,k',i,j} \Phi_k(\mu_x^i) \Phi_{k'}(\hat{m}_y^j) \\
g^2(\mu_x^i, \hat{m}_y^j) &\equiv \sum_{k,k' \in K \times K'} g^2_{k,k',i,j} \Phi_k(\mu_x^i) \Phi_{k'}(\hat{m}_y^j) \\
h^1(\mu_y^i, \hat{m}_x^j) &\equiv \sum_{k,k' \in K \times K'} h^1_{k,k',i,j} \Phi_k(\mu_y^i) \Phi_{k'}(\hat{m}_x^j), \\
h^2(\mu_y^i, \hat{m}_x^j) &\equiv \sum_{k,k' \in K \times K'} h^2_{k,k',i,j} \Phi_k(\mu_y^i) \Phi_{k'}(\hat{m}_x^j)
\end{aligned}
$$

1.2. Computing expectations

For each country, we have two random variables, foreign productivity and second order beliefs, for which we will evaluate expectations using Gaussian Quadrature method. For this, we must define a set of nodes $\{x_a\}_{a=1}^{N_q}$ and weights $\{w_a\}_{a=1}^{N_q}$ such that

$$\mathbb{E}[f(X)] = \sum_{a=1}^{N_q} w_a f(x_a)$$

and further moments conditions are satisfied.

- **Grid for random variable 1: foreign productivity**: The distribution of foreign aggregate productivity depends on the second state, the posterior mean $\hat{m}$.

  - In x country, for each value of the second state (the posterior mean) we have that foreign productivity is Normal with mean equal to the posterior mean $\hat{m}_y^j$ and variance equal to the posterior variance $\hat{s}_y^2 = \left(s_y^{-2} + \tilde{s}_y^{-2}\right)^{-1} = \frac{1}{\frac{1}{s_y^2}+\frac{1}{\tilde{s}_y^2}}$

  $$\mu_y^j \sim \mathcal{N}(\hat{m}_y^j, \hat{s}_y^2)$$

  Then for each $j = 1, ..., N$, Gaussian Quadrature procedure constructs nodes of foreign productivity $\{\mu_y^{j,b}\}_b = 1, ..., N_q$ and corresponding weights $\{\omega^b\}_b = 1, ..., N_q$. Note that the weights do not depend on j.

  - In y country, for each value of the second state (the posterior mean $\hat{m}_x^j$) we have that foreign productivity is Normal with mean equal to the posterior mean $\hat{m}_x^j$ and variance equal to the posterior variance $\hat{s}_x^2 = (s_x^{-2} + \tilde{s}_x^{-2})^{-1} = \frac{1}{\frac{1}{s_x^2}+\frac{1}{\tilde{s}_x^2}}$

  $$\mu_x^j \sim \mathcal{N}(\hat{m}_x^j, \hat{s}_x^2)$$

  Then for each $j = 1, ..., N$, Gaussian Quadrature procedure constructs nodes of foreign productivity $\{\mu_x^{j,b}\}_b = 1, ..., N_q$ and corresponding weights $\{\omega^b\}_b = 1, ..., N_q$.

  Extreme cases

  - Perfect Info: As $\tilde{s}_y \to 0$, $\mu_y^j \sim \mathcal{N}(\hat{m}_y^j, 0) = \mathcal{N}(\mu_y^j, 0)$. The grid degenerates to a single point for each j: $\mu_y^{j,b} = \mu_y^j$.
  - No Info: As $\tilde{s}_y \to \infty$, $\mu_y^j \sim \mathcal{N}(\hat{m}_y^j, s_y^2) = \mathcal{N}(m_y, \bar{s}_y^2)$ which is equal the distribution of the posterior mean (the second state). Clearly, $\bar{s}_y^2 = \frac{s_y^4}{s_y^2+\tilde{s}_y^2} \to s_y^2$ as well, which makes the distribution of foreign productivity equal to the prior. However, in the code we

have fixed grids for the states so that they do not depend on signal precision. Therefore, as we reduce signal precision, the grid will not converge to the prior. However, the simulations take care of it.

- **Grid for random variable 2: second order beliefs**: From the perspective of the domestic country, the second order beliefs about the posterior mean (this is, what the domestic country thinks the posterior mean of the foreign country is) is a Normal random variable that depends on the first state, the domestic aggregate productivity μ.

  - In the x country, for each value of the first state (aggregate productivity μ_x^i), we have that the second order belief is Normal with mean and variance as follows:

$$\hat{m}_x^i \sim \mathcal{N}(\hat{\hat{m}}_x^i, \hat{\hat{s}}_x^2) \qquad \text{with} \qquad \hat{\hat{m}}_x^i \equiv \frac{s_x^{-2} m_x + \tilde{s}_{p_x}^{-2} \mu_x^i}{s_x^{-2} + \tilde{s}_{p_x}^{-2}}, \qquad \hat{\hat{s}}_x^2 \equiv \tilde{s}_{p_x}^{-2} (s_x^{-2} + \tilde{s}_{p_x}^{-2})^{-2} = \frac{1}{(\frac{\tilde{s}_{px}}{s_x^2} + \frac{1}{\tilde{s}_{px}})^2}$$

  where $\tilde{s}_{p_x}$ is the foreign signal noise as perceived by the domestic country. With known information structures $\tilde{s}_{p_x} = \tilde{s}_x$, but with unknown information structures $\tilde{s}_{p_x} \neq \tilde{s}_x$.

  Then for each $i = 1, ..., N$, Gaussian Quadrature procedure constructs nodes for second order beliefs $\{\mu_x^{i,a}\}_a = 1, ..., N_q$ and corresponding weights $\{\gamma^a\}_a = 1, ..., N_q$.

  - In the y country, we have that for each value of the first state μ_y^i the second order belief is distributed as:

$$\hat{m}_y^i \sim \mathcal{N}(\hat{\hat{m}}_y^i, \hat{\hat{s}}_y^2) \qquad \text{with} \qquad \hat{\hat{m}}_y^i \equiv \frac{s_y^{-2} m_y + \tilde{s}_{p_y}^{-2} \mu_y^i}{s_y^{-2} + \tilde{s}_{p_y}^{-2}}, \qquad \hat{\hat{s}}_y^2 \equiv \tilde{s}_{p_y}^{-2} (s_y^{-2} + \tilde{s}_{p_y}^{-2})^{-2} = \frac{1}{(\frac{\tilde{s}_{py}}{s_y^2} + \frac{1}{\tilde{s}_{py}})^2}$$

  Extreme cases

  - Perfect Info: As $\tilde{s}_{p_x} \to 0$, then the distribution becomes degenerate at the true realizations: $\hat{m}_x^i \sim \mathcal{N}(\mu_x^i, 0) \quad \forall i$ and the grid becomes: $\hat{m}_x^{i,a} = \mu_x^i, \; a = 1, ..., N_q$
  - Imperfect Info: As $\tilde{s}_{p_x} \to \infty$, then the distribution becomes degenerate at the prior means $\hat{m}_x^i \sim \mathcal{N}(m_x, 0) \quad \forall i$ and the grid becomes $\hat{m}_x^{i,a} = m_x, \; a = 1, ..., N$

Computational algorithm We solve the fixed point problem by iterating on the export policy functions Ψ and Γ which are approximated using linear splines. For each country we define grids for their two states: aggregate productivity and posterior mean of foreign productivity. We also define grids for foreign productivity and second order beliefs that countries use to evaluate their perceived price function. Expectations with respect to foreign productivity and second order beliefs are computed using Gaussian quadrature. Once we have solved the fixed point problem, we simulate the repeated economy for T=100,000 periods and compute average statistics across simulations.

1.3. Finding the fixed point

1. For reference, we organize the states as follows. For x - country: $(\mu_x, \hat{m}_y)$ and for y - country: $(\mu_y, \hat{m}_x)$. For the price and other economy wide variables, we make the following convention: $q(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)$.

2. Guess initial set of coefficients for polynomials $\{g^1_{k,k',i,j}, g^2_{k,k',i,j}, h^1_{k,k',i,j}, h^2_{k,k',i,j}\}$.

   - We start by solving the perfect information case and approximate the policies with the polynomials to get the first set of coefficients. Since with perfect information $\hat{m}_y = \mu_y$ and $\hat{m}_x = \mu_x$, we have the following system of equations:

$$\begin{aligned}
\Psi^{PI}(\mu_x, \mu_y) &= \frac{1}{1 + [(1+\tau)q^{PI}(\mu_x, \mu_y)]^{\frac{\theta}{1-\theta}}} \\
\Gamma^{PI}(\mu_y, \mu_x) &= \frac{1}{1 + \left[\frac{(1+\tau)}{q^{PI}(\mu_x,\mu_y)}\right]^{\frac{\theta}{1-\theta}}} \\
q^{PI}(\mu_x, \mu_y) &= f\frac{\Psi^{PI}(\mu_x, \mu_y)}{\Gamma(\mu_y, \mu_x)}
\end{aligned}$$

   Thus q^{PI} solves the following equation[4]:

$$q^{PI} - f\frac{\frac{1}{1+[(1+\tau)q^{PI}(\mu_x,\mu_y)]^{\frac{\theta}{1-\theta}}}}{\frac{1}{1+\left[\frac{(1+\tau)}{q^{PI}(\mu_x,\mu_y)}\right]^{\frac{\theta}{1-\theta}}}} = 0$$

   Once we have the price, we recover the policies and construct the first guess of coefficients and approximating functions.

3. For the X - country:

   - For each state $(\mu^i_x, \hat{m}^j_y)$, approximate Ψ using the polynomials g^1 and g^2 evaluated at the state:

$$\Psi(\mu^i_x, \hat{m}^j_y) \approx \frac{1}{1 + (1+\tau)^{\frac{\theta}{1-\theta}}\left(\frac{g^1(\mu^i_x,\hat{m}^j_y)}{g^2(\mu^i_x,\hat{m}^j_y)}\right)^{\frac{1}{1-\theta}}}$$

   - For each quadrature node $(\mu^a_y, \hat{m}^b_x)$ approximate Γ using the polynomials h^1 and h^2 evaluate at the nodes $\{\mu^a_y\}_{a=1}^{N_q}, \{\hat{m}^b_x\}_{b=1}^{N_q}$

$$\Gamma(\mu^a_y, \hat{m}^b_x) \approx \frac{1}{1 + (1+\tau)^{\frac{\theta}{1-\theta}}\left(\frac{h^1(\mu^a_y,\hat{m}^b_x)}{h^2(\mu^a_y,\hat{m}^b_x)}\right)^{\frac{1}{1-\theta}}}$$

[4]Notice that without the trade cost τ, the price is: $q^{PI}(\mu_x, \mu_y) = f^{1-\theta} \quad \forall(\mu_y, \mu_x)$.

- Construct q and Ψ_2 in 4 dimensions using $\Psi(\mu_x^i, \hat{m}_y^j)$ and $\Gamma(\mu_y^a, \hat{m}_x^b)$:

$$q(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b) \approx e^{(\mu_x^i - \mu_y^a)} e^{\frac{1}{2}(\sigma_x^2 - \sigma_y^2)} \frac{\Psi(\mu_x^i, \hat{m}_y^j)}{\Gamma(\mu_y^a, \hat{m}_x^b)}$$

$$\Psi_2(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b) = \left((1 - \Psi(\mu_x^i, \hat{m}_y^j))^\theta + \left(\frac{\Psi(\mu_x^i, \hat{m}_y^j)}{(1+\tau) q(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b)} \right)^\theta \right)^{\frac{1}{\theta}}$$

- Compute the conditional expectations of $\Psi_2^{1-\theta}$ and $\Psi_2^{1-\theta} q^{-\theta}$ that integrate out the two random variables $(\mu_y, \hat{m}_x)$ as the weighted sum of the functions evaluated at the quadrature nodes, using the quadrature weights $\{\omega^a\}_{a=1}^{N_q}$ and $\{\gamma^b\}_{b=1}^{N_q}$:

$$\begin{aligned}
&\mathbb{E}_{\mu_y, \hat{m}_x} \left[\Psi_2^{1-\theta}(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x) \middle| \mathcal{I}_x \right] \\
= &\int_{\mu_y} \int_{\hat{m}_x} \Psi_2^{1-\theta}(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x) \quad \phi\left(\frac{\mu_y - \hat{m}_y}{\hat{s}_y} \right) \phi\left(\frac{\hat{m}_x - \hat{\hat{m}}_x}{\hat{\hat{s}}_x} \right) d\mu_y d\hat{m}_x \\
\approx &\sum_{a=1}^{N_q} \sum_{b=1}^{N_q} \omega^a \gamma^b \Psi_2^{1-\theta}(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b)
\end{aligned}$$

$$\begin{aligned}
&\mathbb{E}_{\mu_y, \hat{m}_x} \left[\Psi_2^{1-\theta}(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x) q^{-\theta}(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x) \middle| \mathcal{I}_x \right] \\
= &\int_{\mu_y} \int_{\hat{m}_x} \Psi_2^{1-\theta}(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x) q^{-\theta}(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x) \phi\left(\frac{\mu_y - \hat{m}_y}{\hat{s}_y} \right) \phi\left(\frac{\hat{m}_x - \hat{\hat{m}}_x}{\hat{\hat{s}}_x} \right) d\mu_y d\hat{m}_x \\
\approx &\sum_{a=1}^{N_q} \sum_{b=1}^{N_q} \omega^a \gamma^b \Psi_2^{1-\theta}(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b) q^{-\theta}(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b)
\end{aligned}$$

4. For the Y- country, we do analogous calculations.

   - For each state $(\mu_y^i, \hat{m}_x^j)$, approximate Γ using the polynomials h^1 and h^2 evaluated at the state:

$$\Gamma(\mu_y^i, \hat{m}_x^j) \approx \frac{1}{1 + (1+\tau)^{\frac{\theta}{1-\theta}} \left(\frac{h^1(\mu_y^i, \hat{m}_x^j)}{h^2(\mu_y^i, \hat{m}_x^j)} \right)^{\frac{1}{1-\theta}}}$$

   - For each quadrature node $(\mu_x^a, \hat{m}_y^b)$ approximate Ψ using the polynomials g^1 and g^2 evaluate at the nodes $\{\mu_x^a\}_{a=1}^{N_q}$, $\{\hat{m}_y^b\}_{b=1}^{N_q}$

$$\Psi(\mu_x^a, \hat{m}_y^b) \approx \frac{1}{1 + (1+\tau)^{\frac{\theta}{1-\theta}} \left(\frac{g^1(\mu_x^a, \hat{m}_y^b)}{g^2(\mu_x^a, \hat{m}_y^b)} \right)^{\frac{1}{1-\theta}}}$$

- Construct q and Γ_2 in 4 dimensions using $\Gamma(\mu_y^i, \hat{m}_x^j)$ and $\Psi(\mu_x^a, \hat{m}_y^b)$ (note that the state for the price is in the same order as for the X-country):

$$q(\mu_x^a, \hat{m}_y^b, \mu_y^i, \hat{m}_x^j) \approx e^{(\mu_x^a - \mu_y^i)} e^{\frac{1}{2}(\sigma_x^2 - \sigma_y^2)} \frac{\Psi(\mu_x^a, \hat{m}_y^b)}{\Gamma(\mu_y^i, \hat{m}_x^j)}$$

$$\Gamma_2(\mu_y^i, \hat{m}_x^j, \mu_x^a, \hat{m}_y^b) = \left((1 - \Gamma(\mu_y^i, \hat{m}_x^j))^\theta + \left(\frac{\Gamma(\mu_y^i, \hat{m}_x^j) q(\mu_x^a, \hat{m}_y^b, \mu_y^i, \hat{m}_x^j)}{(1+\tau)} \right)^\theta \right)^{\frac{1}{\theta}}$$

- Compute the conditional expectations of $\Gamma_2^{1-\theta}$ and $\Gamma_2^{1-\theta} q^\theta$ that integrate out the two random variables $(\mu_x, \hat{m}_y)$. This is just the weighted sum of the functions evaluated at the quadrature nodes, using the quadrature weights $\{\omega^a\}_{a=1}^{N_q}$ and $\{\gamma^b\}_{b=1}^{N_q}$:

$$\begin{aligned}
&\mathbb{E}_{\mu_x, \hat{m}_y} \left[\Gamma_2^{1-\theta}(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y) \middle| \mathcal{I}_y \right] \\
= &\int_{\mu_y} \int_{\hat{m}_x} \Gamma_2^{1-\theta}(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y) \quad \phi\left(\frac{\mu_x - \hat{m}_x}{\hat{s}_x} \right) \phi\left(\frac{\hat{m}_y - \hat{\hat{m}}_y}{\hat{\hat{s}}_y} \right) d\mu_x d\hat{m}_y \\
\approx &\sum_{a=1}^{N_q} \sum_{b=1}^{N_q} \omega^a \gamma^b \Gamma_2^{1-\theta}(\mu_y^i, \hat{m}_x^j, \mu_x^a, \hat{m}_y^b)
\end{aligned}$$

$$\begin{aligned}
&\mathbb{E}_{\mu_x, \hat{m}_y} \left[\Gamma_2^{1-\theta}(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y) q^\theta(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x) \middle| \mathcal{I}_y \right] \\
= &\int_{\mu_y} \int_{\hat{m}_x} \Gamma_2^{1-\theta}(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y) q^\theta(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x) \phi\left(\frac{\mu_x - \hat{m}_x}{\hat{s}_x} \right) \phi\left(\frac{\hat{m}_y - \hat{\hat{m}}_y}{\hat{\hat{s}}_y} \right) d\mu_x d\hat{m}_y \\
\approx &\sum_{a=1}^{N_q} \sum_{b=1}^{N_q} \omega^a \gamma^b \Gamma_2^{1-\theta}(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b) q^\theta(\mu_x^a, \hat{m}_y^b, \mu_y^i, \hat{m}_x^j)
\end{aligned}$$

5. Update coefficients by i) fitting polynomials to approximate the conditional expectations and ii) using a linear combination of the new coefficients with the previous guess.

6. Repeat steps until convergence of coefficients.

7. Once convergence is achieved, recover all variables at the firm level and at the aggregate level.

   Recall the definitions of domestic, foreign and relative fundamentals:

$$f_x \equiv e^{\mu_x + \frac{1}{2}\sigma_x^2}, \qquad f_y \equiv e^{\mu_y + \frac{1}{2}\sigma_y^2} \qquad f \equiv e^{(\mu_x - \mu_y)} e^{\frac{1}{2}(\sigma_x^2 - \sigma_y^2)}$$

(a) Price function:

$$q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) = f\frac{\Psi(\mu_x, \hat{m}_y)}{\Gamma(\mu_y, \hat{m}_x)}$$

(b) Firms' export policy and consumptions in x country:

$$\begin{aligned}
t_x(z_x, \mu_x, \hat{m}_y) &= z_x\Psi(\mu_x, \hat{m}_y) \\
c_x(z_x, \mu_x, \hat{m}_y) &= z_x(1 - \Psi(\mu_x, \hat{m}_y)) \\
c_y(z_x, \mu_x, \hat{m}_y, \mu_y, \hat{m}_x) &= \frac{t_x(z_x, \mu_x, \hat{m}_y)}{(1+\tau)q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)} \\
c(z_x, \mu_x, \hat{m}_y, \mu_y, \hat{m}_x) &= z_x\Psi_2(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)
\end{aligned}$$

(c) Firms' export policy and consumptions in y country:

$$\begin{aligned}
t_y(z_y, \mu_y, \hat{m}_x) &= z_y\Gamma(\mu_y, \hat{m}_x) \\
c_y^*(z_y, \mu_y, \hat{m}_x) &= z_y(1 - \Gamma(\mu_y, \hat{m}_x)) \\
c_x^*(z_y, \mu_y, \hat{m}_x, \mu_x, \hat{m}_y) &= \frac{t_y(z_y, \mu_y, \hat{m}_x)q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)}{(1+\tau)} \\
c^*(z_x, \mu_x, \hat{m}_y, \mu_y, \hat{m}_x) &= z_y\Gamma_2(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y)
\end{aligned}$$

(d) Aggregate variables in x country:

$$\begin{aligned}
T_x(\mu_x, \hat{m}_y) &= f_x\Psi(\mu_x, \hat{m}_y) \\
C_x(\mu_x, \hat{m}_y) &= f_x(1 - \Psi(\mu_x, \hat{m}_y)) \\
C_y(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x) &= \frac{T_x(\mu_x, \hat{m}_y)}{(1+\tau)q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)} \\
C(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x) &= f_x\Psi_2(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)
\end{aligned}$$

(e) Aggregate variables in y country:

$$\begin{aligned}
T_y(\mu_y, \hat{m}_x) &= f_y\Gamma(\mu_y, \hat{m}_x) \\
C_y^*(\mu_y, \hat{m}_x) &= f_y(1 - \Gamma(\mu_y, \hat{m}_x)) \\
C_x^*(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y) &= \frac{T_y(\mu_y, \hat{m}_x)q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)}{(1+\tau)} \\
C^*(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y) &= f_y\Gamma_2(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y)
\end{aligned}$$