Information Globalization

PRELIMINARY AND WORK IN PROGRESS.

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ABSTRACT

How do information frictions affect international prices and trade? In a standard, two-country Armington model of trade, information frictions impede the coordination of exporting behavior across countries. Because the terms of trade depend on relative exports, less coordination leads to more volatile terms of trade. Volatility in the terms of trade has the potential to *reduce* the level of trade by making trade more risky, but it also has the potential to *increase* the level of trade by increasing the expected terms of trade. We derive general conditions on preferences as to which of these forces—the increase in risk or increase in return—dominate. With CES preferences, as long as goods are not too substitutable, information frictions impede trade. With empirically plausible elasticities of substitution, information frictions facilitate trade.

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In any discussion of the frictions to cross-border trade, inevitably one that arises is some discussion about information (or lack there of). Portes and Rey (2005) show that the volume of phone calls between two countries predicts how much they trade. Gould (1994) and Rauch and Trindade (2002) argue that immigrants trade more with their home countries. The argument that information frictions create uncertainty about foreign economies, and that this uncertainty deters risk-averse potential exporters is compelling. In partial-equilibrium settings, the effect of an increase in uncertainty is to reduce the certainty-equivalent expected return, which acts like a tax on transactions.

We explore the role of information frictions in the most standard, simple general equilibrium trade model—a two-good, two-country Armington model. We introduce information asymmetry in the most obvious way: Each country experiences a random endowment shock that its home firms observe perfectly, but foreigners observe imperfectly, with a noisy signal. Each firm chooses what fraction of their endowment to export to an international market. The international relative price clears that market, goods are immediately shipped to their destination country, and agents consume.

Our analysis proceeds in several steps. First, we focus on the effects that information frictions have on the terms of trade. The key result is that—in general equilibrium—information affects both the volatility *and* the expected terms of trade. The mechanism behind this result is that information frictions lead to less coordination in exports across countries. The intuition is that because home country agents can not condition their exporting behavior on the foreign country's state, their exports will not covary with the exports of the foreign country (which similarly know nothing about home).

In equilibrium, because the terms of trade depend on relative exports, less coordination leads to more volatile terms of trade. The mechanism encodes the conventional wisdom information frictions create uncertainty about foreign economies, and that this uncertainty deters risk-averse potential exporters. The conventional wisdom is incomplete, however. Changes in coordination in exports also increases the expected terms of trade or reporting. Thus, how information frictions actually affect the level of trade depends on which of these forces—the increase in risk or increase in return—dominate.

The second step of the paper derives general conditions on preferences to determine which of these forces—the increase in risk or increase in return—dominate. The analysis centers on two conditions which we call Condition **M** and Condition **V**. Condition **M** is about the complementarily of the two goods (and the implicit risk aversion). Condition **V** is about the precautionary motives to exporting and, hence, it depends upon multivariate prudence or the third partial derivative of the good being imported. Our main result is that how exports change with information is exactly sum of these two conditions.

Finally, we numerically focus on the standard case where home and foreign agents have symmetric CES preferences over both goods. In the CES case, how trade responds to information frictions depends critically on the elasticity of substitution. Consistent with the results developed above, when the home and foreign goods are gross complements, information frictions lead to less trade—consistent with conventional wisdom. The more puzzling/interesting case is when the goods are substitutable (as is typically assumed in the trade literature). This is the situation in which a higher expected terms of trade motivates agents to trade more in spite of the risk associated with trade.

Related papers There is a spate of recent papers modeling and measuring information frictions in trade. The most closely related is Steinwender (2014), where exporters in one country learn about exogenous market prices in another country. More precise information decreases uncertainty and increases the expected profits, trade volume and welfare. Our paper is similar because agents learn about aggregate economic conditions in another country and then choose exports. But instead of one trader facing an exogenous price, our model features equilibrium two-country trade. The fact that both parties know something the other does not creates non-trivial higher-order beliefs that are essential for our surprising results.

Other papers look at different types of information frictions. In Allen (2013), Petropoulou (2011), Rauch and Watson (2004) and Eaton, Eslava, Krizan, Kugler, and Tybout (2011), producers are uncertain about firm- or match-specific variables such as the location of the best trading partner, the quality of their match or local demand for their specific product. These are undoubtedly important information frictions. But if these frictions inhibit foreign trade more than domestic trade, there must be some country component to them that is known at home, but not abroad. As such, our model complements these theories by filling in that missing piece, the role of uncertainty about a foreign economy.

The effect of information globalization on risk sharing is similar to the effect of allowing international borrowing (Brunnermeier and Sannikov (2014)). Both mechanisms undermine risk-sharing, but ours has the opposite predictions for trade volumes.

In financial markets, information also frequently inhibits risk-sharing. The Hirshleifer (1971) effect arises when information precludes trade in assets whose payoffs are contingent on an outcome revealed by the information. Our effect is distinct because 1) our signals are private to each country, not public and 2) it works through an effect of the quantity exported on the international relative price. Our effect does not change the set of securities traded because no financial securities are traded in this model.

1. Benchmark Model

In order to understand how an information asymmetry affects trade, we write down a simple model with two countries, an endowment economy and an information asymmetry. The first two ingredients constitute a standard equilibrium model of trade. The information asymmetry is that agents in each country know their own country's aggregate endowment, but have imperfect information about the other country's endowment. The key feature of the model is that the relative price of goods is not known at the time when exports are chosen. It could be that there is shipping delay. This could be a demand shock. It could be that some fixed costs must be incurred to export before export contracts are written. But what this captures is the idea that exporting is risky. It is the essence of an information asymmetry: Something is known when one sells domestically that is unknown when selling abroad. Our question is what happens when the unknowns in exporting become more known.

This is a repeated static model with the following economic environment.

Preferences: There are 2 countries and a continuum of agents within each country. We denote individual variables with lower case and aggregates with upper case. The problems are symmetric across countries, so we only describe the problem for the domestic country. Agents like to consume two goods, x and y and their utility flow each period is

$$\mathbb{E}\left[U(c_x, c_y)\right].$$

where for now we only restrict U to be increasing and concave in both goods.

Endowments: Each agent in the domestic country has an idiosyncratic endowment of z_x units of good x, where $\ln z_x \sim \mathcal{N}(\mu_x, \sigma_x^2)$. Each agent in the foreign country has an idiosyncratic endowment z_y units of good y, where $\ln z_y \sim \mathcal{N}(\mu_y, \sigma_y^2)$. The means of these distributions are themselves independent random variables: $\mu_x \sim \mathcal{N}(m_x, s_x^2)$ and $\mu_y \sim \mathcal{N}(m_y, s_y^2)$. Because they are average endowment around which all individual endowments are distributed, μ_x and μ_y are aggregate shocks. If we let Φ denote the cumulative density (cdf) of a standard normal variable, then the conditional cdf's of z_x and z_y are $F(ln(z_x)|\mu_x) = \Phi((ln(z_x) - \mu_x)/\sigma_x)$ and $F(ln(z_y)|\mu_y) = \Phi((ln(z_y) - \mu_y)/\sigma_y)$.

Notice that there are no financial markets that agents could use to hedge their productivity shocks or those of the foreign country. Instead, the equilibrium movements in international prices are an important source of risk-sharing Cole and Obstfeld (1991). One of the key insights of this model will be that the information countries have about each others' productivity changes how they share risk. Therefore, we first consider a model with no risk-sharing instruments, so as not to obscure the information effect. Then, we re-introduce instruments to share risk in section **??**.

Information: At the beginning of the period, firms in country x observe their own endowment z_x and the mean of their country's endowment μ_x. Likewise, agents in country y observe z_y and μ_y. Furthermore, we assume that both countries know the distribution from which mean productivities are drawn and the cross-sectional variance of firm outcomes. In other words, $m_x, m_y, s_x, s_y, \sigma_x$ and σ_y are common knowledge.

In addition, agents in country x observe a signal about the y-endowment $\tilde{m}_y = \mu_y + \eta_y$ where $\eta_y \sim N(0, \tilde{s}_y^2)$. Similarly, agents in country y observe a signal about the x-endowment $\tilde{m}_x = \mu_x + \eta_x$ where $\eta_x \sim N(0, \tilde{s}_x^2)$. Let $\mathcal{I}_x$ denote the information set of an agent in the home country and let $\mathcal{I}_y$ denote the information set of a foreign agent. All home country choices will be a function of the three random variables in the home agents' information set: $\mathcal{I}_x = \{z_x, \mu_x, \tilde{m}_y\}$. Likewise, foreign choices depend on $\mathcal{I}_y = \{z_y, \mu_y, \tilde{m}_x\}$.

Bayesian updating Agents in each country combine their prior knowledge of the distribution of the others' productivity and their signal to form posterior beliefs. By Bayes' law, the posterior probability distribution is normal with mean $\hat{m}$ and variance $\hat{s}^2$ given by

$$F(\mu_y|\mathcal{I}_x) = \Phi\left(\frac{\mu_y - \hat{m}_y}{\hat{s}_y}\right) \quad \text{where} \quad \hat{m}_y = \frac{s_y^{-2} m_y + \tilde{s}_y^{-2}\tilde{m}_y}{s_y^{-2} + \tilde{s}_y^{-2}}, \quad \hat{s}_y^2 = \frac{1}{s_y^{-2} + \tilde{s}_y^{-2}} \tag{1}$$

$$F(\mu_x|\mathcal{I}_y) = \Phi\left(\frac{\mu_x - \hat{m}_x}{\hat{s}_x}\right) \quad \text{where} \quad \hat{m}_x = \frac{s_x^{-2} m_x + \tilde{s}_x^{-2}\tilde{m}_x}{s_x^{-2} + \tilde{s}_x^{-2}}, \quad \hat{s}_x^2 = \frac{1}{s_x^{-2} + \tilde{s}_x^{-2}} \tag{2}$$

In order to forecast prices, agents will need to forecast the other country's export choices. Since others' export choices depend on their forecasts of one's own endowment, actions will also depend on beliefs about the beliefs of others. According to Bayes' law, these second-order beliefs are

$$F(\hat{m}_x|\mathcal{I}_x) = \Phi\left(\frac{\hat{m}_x - \hat{\hat{m}}_x}{\hat{\hat{s}}_x}\right) \quad \text{where} \quad \hat{\hat{m}}_x = \frac{s_x^{-2} m_x + \tilde{s}_x^{-2}\mu_x}{s_x^{-2} + \tilde{s}_x^{-2}}, \quad \hat{\hat{s}}_x^2 = \frac{\tilde{s}_x^{-2}}{(s_x^{-2} + \tilde{s}_x^{-2})^2} \tag{3}$$

$$F(\hat{m}_y|\mathcal{I}_y) = \Phi\left(\frac{\hat{m}_y - \hat{\hat{m}}_y}{\hat{\hat{s}}_y}\right) \quad \text{where} \quad \hat{\hat{m}}_y = \frac{s_y^{-2} m_y + \tilde{s}_y^{-2}\mu_y}{s_y^{-2} + \tilde{s}_y^{-2}}, \quad \hat{\hat{s}}_y^2 = \frac{\tilde{s}_y^{-2}}{(s_y^{-2} + \tilde{s}_y^{-2})^2} \tag{4}$$

Note that there is a one-to-one mapping between signals $\tilde{m}$ and posterior beliefs $\hat{m}$. Instead of using signals as a state variable, we will use posterior beliefs, for simplicity and without loss of generality. Thus, we write $\mathcal{I}_x = \{z_x, \mu_x, \hat{m}_y\}$ and $\mathcal{I}_y = \{z_y, \mu_y, \hat{m}_x\}$.[1]

Price and budget set: Each agent chooses how much to export, t_x or t_y. The relative price q

[1] In fact, all higher orders of beliefs can matter for export choices. But, because there are only two shocks observed by each country, the first two orders of beliefs are sufficient to characterize the entire hierarchy.

is the number of units of x good required to purchase one unit of y good on the international market. In equilibrium, this price clears the market. An agent who exports t_x units of x goods receives $\frac{t_x}{q}$ units of y, for immediate consumption (there is no secondary resale market). We restrict exports and consumption to be non-negative. Therefore the country x budget set is:

$$c_x \quad \epsilon \quad [0, z_x - t_x] \tag{5}$$

$$c_y \quad \epsilon \quad \left[0, \frac{t_x}{q}\right] \tag{6}$$

and the country y budget set is:

$$c_x \quad \epsilon \quad [0, t_y q] \tag{7}$$

$$c_y \quad \epsilon \quad [0, z_y - t_y] \tag{8}$$

When the export decision is made, firms do not know the price q. It is a random variable whose realization depends on their own (known) aggregate state, on the foreign (unknown) aggregate state, on home beliefs about the foreign state and on foreign beliefs about the home state.

Equilibrium An equilibrium is given by export policy functions for domestic $t_x(z_x, \mu_x, \hat{m}_y)$ and foreign $t_y(z_y, \mu_y, \hat{m}_x)$ countries, aggregate exports $T_x(\mu_x, \hat{m}_y), T_y(\mu_y, \hat{m}_x)$, perceived price functions $\tilde{q}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)$ for each country and an actual price function $q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)$ such that:

1. Given perceived price functions $\tilde{q}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)$, export policies maximize expected consumption of every firm in each country. Substituting the budget sets (5) to (8) into $E[(c_x^\theta + c_y^\theta)^{1/\theta}]$, we can write this problem as

$$t_x(z_x, \mu_x, \hat{m}_y) = \arg\max E\left[\left((z_x - t_x)^\theta + \left(\frac{t_x}{\tilde{q}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)}\right)^\theta\right)^{1/\theta} \middle| \mathcal{I}_x\right] \tag{9}$$

$$t_y(z_y, \mu_y, \hat{m}_x) = \arg\max E\left[\left((t_y \tilde{q}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y))^\theta + (z_y - t_y)^\theta\right)^{1/\theta} \middle| \mathcal{I}_y\right] \tag{10}$$

Using the conditional densities (1), (2), (4) and (3), we can compute expectations as

$$t_x(z_x, \mu_x, \hat{m}_y) = \arg\max \int\int \left((z_x - t_x)^\theta + \left(\frac{t_x}{\tilde{q}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)}\right)^\theta\right)^{1/\theta} dF(\mu_y|\mathcal{I}_x) dF(\hat{m}_x|\mathcal{I}_x) \tag{11}$$

$$t_y(z_y, \mu_y, \hat{m}_x) = \arg\max \int \int \left((t_y \tilde{q}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y))^\theta + (z_y - t_y)^\theta \right)^{1/\theta} dF(\mu_x|\mathcal{I}_y) dF(\hat{m}_y|\mathcal{I}_y) \tag{12}$$

2. The relative price q clears the international market. Since every unit of x-good exported must be sold and paid for with y exports, and conversely, every unit of y exports must be sold and paid for with x exports, the only price that clears the international market is the ratio of aggregate exports:

$$q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) = \frac{T_x(\mu_x, \hat{m}_y)}{T_y(\mu_y, \hat{m}_x)} \tag{13}$$

where aggregate exports are

$$T_x(\mu_x, \hat{m}_y) = \int t_x(z_x, \mu_x, \hat{m}_y) dF(z_x|\mu_x) \tag{14}$$

$$T_y(\mu_y, \hat{m}_x) = \int t_y(z_y, \mu_y, \hat{m}_x) dF(z_y|\mu_y). \tag{15}$$

3. The perceived and actual price functions coincide:

$$\tilde{q}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) = q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) \qquad \forall (\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)$$

2. Terms of trade average and volatility are decreasing in export covariance.

The following section focus on the effects that information frictions have on the terms of trade. The main results are summarized in the following propositions. The key result is that—in general equilibrium—information affects both the volatility *and* the expected terms of trade. Moreover, these results do not depend upon any specification about the utility function.

Proposition 1 (Larger covariance between exports decreases the expected terms of trade.) *If the terms of trade are given by the ratio of aggregate exports, $\frac{p_x}{p_y} = \frac{T_y}{T_x}$ and the expectation and variances of aggregate exports $\mathbb{E}[T_x], \mathbb{E}[T_y]$, $\mathbb{V}[T_x]$, $\mathbb{V}[T_y]$ are kept constant, an increase in the covariance between exports $\mathbb{C}[T_x, T_y]$ decreases the expected terms of trade for both countries: $\mathbb{E}\left[\frac{p_x}{p_y}\right]$ and $\mathbb{E}\left[\frac{p_y}{p_x}\right]$.*

Proof. A second order approximation of $p = \frac{T_y}{T_x}$ around the expectation of aggregate exports $(\mathbb{E}[T_x], \mathbb{E}[T_y])$ yields:

$$\frac{T_y}{T_x} \approx \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]} + \frac{1}{\mathbb{E}[T_x]}(T_y - \mathbb{E}[T_y]) - \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]^2}(T_x - \mathbb{E}[T_x]) + \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]^3}(T_x - \mathbb{E}[T_x])^2 - \frac{1}{\mathbb{E}[T_x]^2}(T_x - \mathbb{E}[T_x])(T_y - \mathbb{E}[T_y])$$

Taking expectations on both sides, which makes the first order terms equal to zero, yields the result:

$$\mathbb{E}\left[\frac{T_y}{T_x}\right] \approx \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]} + \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]^3}\mathbb{V}[T_x] - \frac{1}{\mathbb{E}[T_x]^2}\mathbb{C}[T_x, T_y]$$

The proof is analogous from the perspective of the foreign country, with an approximation of $\frac{1}{p}$. ■

Proposition 2 (Larger covariance between exports decreases terms of trade volatility.) *If the variances of aggregate exports is kept constant $\mathbb{V}[T_x]$, $\mathbb{V}[T_y]$, larger covariance between exports $\mathbb{C}[T_x, T_y]$ decreases the volatility of the terms of trade.*

Proof. A first order approximation of $p = \frac{T_y}{T_x}$ around the expectation of aggregate exports $(\mathbb{E}[T_x], \mathbb{E}[T_y])$:

$$\frac{T_y}{T_x} \approx \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]} + \frac{1}{\mathbb{E}[T_x]}(T_y - \mathbb{E}[T_y]) - \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]^2}(T_x - \mathbb{E}[T_x])$$

Now take expectation on both sides and cancel the first order terms:

$$\mathbb{E}\left[\frac{T_y}{T_x}\right] \approx \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]}$$

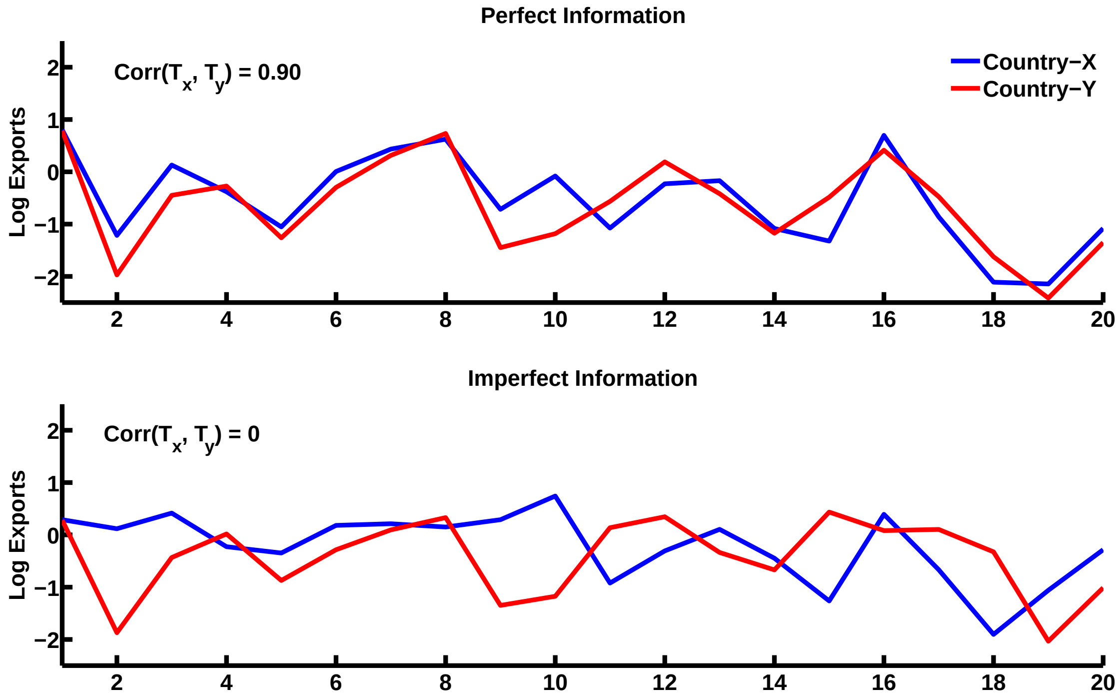


Figure 1: Information and Export Covariance

Subtract the two expressions to compute the variance:

$$\begin{aligned}\mathbb{V}\left[\frac{T_y}{T_x}\right] &= \mathbb{E}\left[\left(\frac{T_y}{T_x}-\mathbb{E}\left[\frac{T_y}{T_x}\right]\right)^2\right]=\mathbb{E}\left[\left(\frac{1}{\mathbb{E}[T_x]}(T_y-\mathbb{E}[T_y])-\frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]^2}(T_x-\mathbb{E}[T_x])\right)^2\right]\\ &= \frac{1}{\mathbb{E}[T_x]^2}\left[\mathbb{V}[T_y]+\frac{\mathbb{E}[T_y]^2}{\mathbb{E}[T_x]^2}\mathbb{V}[T_x]-2\frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]}\mathbb{C}[T_x,T_y]\right]\end{aligned}$$

Keeping the variance of exports constant, the larger covariance decreases the variance of the terms of trade. ■

Proposition 3 (As export covariance increases, variance changes proportionally to average.) *If the expectation and variances of aggregate exports $\mathbb{E}[T_x], \mathbb{E}[T_y]$, $\mathbb{V}[T_x]$, $\mathbb{V}[T_y]$ are kept constant, then to a first order, the variance of the terms of trade can be written an affine function of the expected terms of trade.*

Proof. Using the approximations above, we have that the partial derivative of the expected terms of trade and their variance with respect to the covariance is as follows:

$$\begin{aligned}\frac{\partial \mathbb{E}\left[p\right]}{\partial \mathbb{C}[T_x, T_y]} &\approx -\frac{1}{\mathbb{E}[T_x]^2}\mathbb{C}[T_x, T_y] \\ \frac{\partial \mathbb{V}\left[p\right]}{\partial \mathbb{C}[T_x, T_y]} &\approx -2\frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]^3}\mathbb{C}[T_x, T_y]\end{aligned}$$

Combining both expressions, we can express one derivative in terms of the other as:

$$\frac{\partial \mathbb{V}\left[p\right]}{\partial \mathbb{C}[T_x, T_y]} \approx 2\frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]}\frac{\partial \mathbb{E}\left[p\right]}{\partial \mathbb{C}[T_x, T_y]}$$

Assuming symmetry across countries, we have that $\frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]} = 1$ and thus we can write the variance of the terms of trade as a linear function of the expected terms of trade, with a slope of 2.

$$\mathbb{V}\left[p\right] = \sigma_p + 2\mathbb{E}\left[p\right]$$

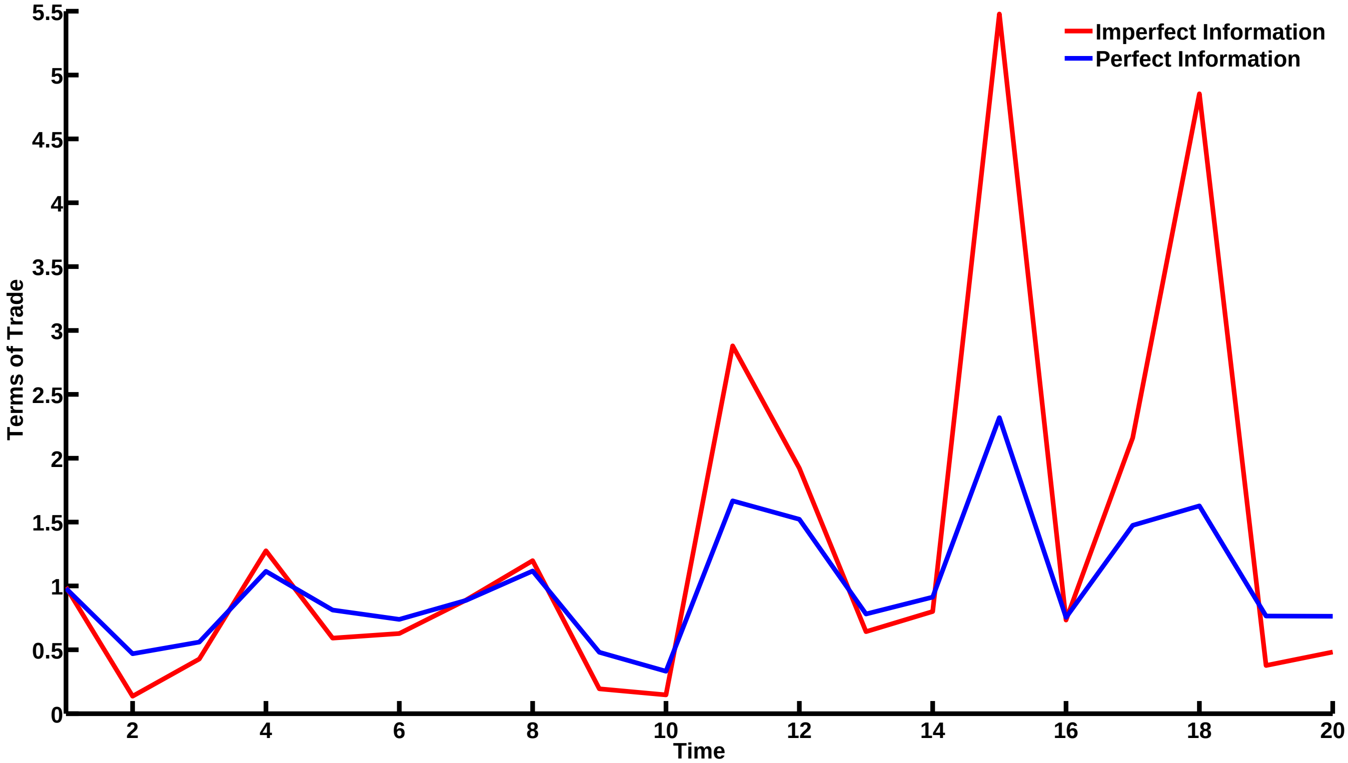
■

Figure 2, 2(a) and 2(b) illustrate how these results matter. Given restrictions on utility functions (which we discuss next), changes in information affect the coordination of exports across countries. In the example in Figure 2, less information results in exports between the two countries becoming less correlated. The intuition is simple, because agents in the home country can not condition their exporting behavior on the state of the world realized in the foreign country, their exports will not covary with the exports of the foreign country (which similarly no nothing about home).

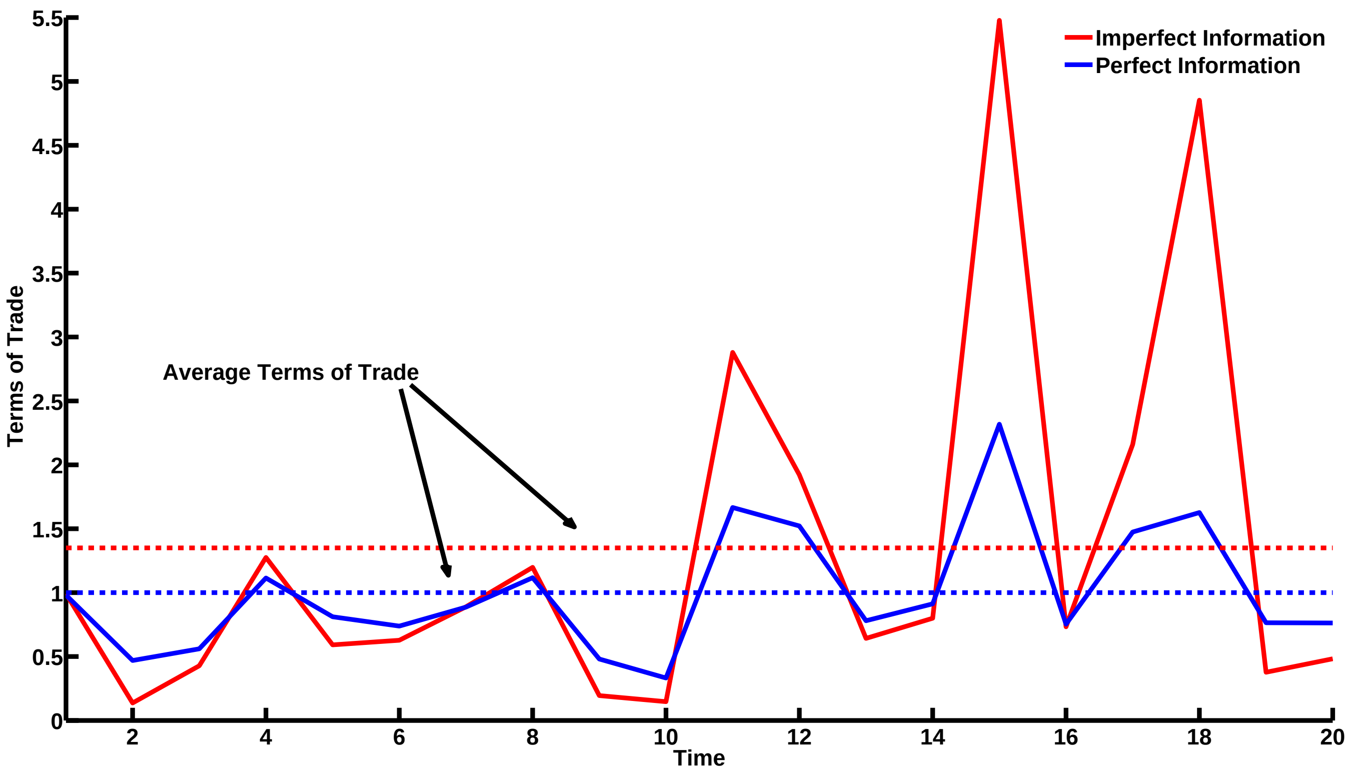
The consequence of the reduction in export covariance is its impact on volatility in the terms of trade and the average terms of trade. This are the arguments that Proposition 1-3 discussed. Less coordination or covariance in exports results in volatile terms of trade. The comparison of the blue line relative to the red line in 2(a) illustrates this point. As the coordination decreases, the volatility of the terms of trade increases.

A final point is that the covariance of exports affects the average terms of trade as well. Figure 2(b) reproduces Figure 2(a), but plots the average terms of trade. The key issue is that the expected terms of trade also increase.

These observations have the following implications: How the level of trade responds to changes in information will depend on preferences and how agents respond to change in the mean and changes in the variance of the terms of trade. The next section derives restrictions on preferences to isolate this tradeoff.



(a) **Information Reduces Terms of Trade Volatility**



(b) **And Reduces the Mean**

Figure 2: Information Reduces Terms of Trade Volatility and Reduces the Mean

3. Preliminaries on Utility

The second step of the paper derives general conditions on preferences to determine which of these forces—the increase in risk or increase in return—dominate. The analysis first discusses some ways to express the utility function and its dependence upon the terms of trade. Then we derive Condition **M** and Condition **V** and discuss their interpretation.

- Assume a general utility function of representative agent $\mathbb{E}\left[U(C_x, C_y)\right]$ where U is increasing and concave in both goods: $U_x, U_y > 0$, U_{xx} and $U_{yy} < 0$. We allow for U_{xy} to have any sign.

- Let the terms of trade be denoted by $p = \frac{p_x}{p_y}$, which in equilibrium at equal to $\frac{T_y}{T_x}$.

- Substitute the budget constraints $(C_x = f_x - T_x,\ C_y = pT_x)$ and take FOC with respect to T_x:

$$\mathbb{E}\left[-U_x\left(f_x - T_x, pT_x\right) + pU_y\left(f_x - T_x, pT_x\right)\right] = 0$$

  and define a function g as follows: $\quad g(T_x, p) \equiv pU_y\left(f_x - T_x, pT_x\right) - U_x\left(f_x - T_x, pT_x\right)$

- With perfect information, g is the marginal utility of exports, and the FOC reads $g(T_x, p) = 0 \quad \forall p$.

3.1. Condition M $\Longrightarrow$ Marginal utility of exports increasing in terms of trade.

Definition 1 *Let $\rho_y \equiv -\frac{C_y U_{yy}}{U_y}$ be the coefficient of relative risk aversion for risky good y. We say that utility $U(c_x, c_y)$ satisfies Condition M if*

$$\rho_y\left(1 - \frac{U_{xy}/p}{U_{yy}}\right) \leq 1$$

Proposition 4 *If utility satisfies Condition M, then with perfect information, the marginal utility of exports g is increasing in the terms of trade.*

Proof. Using the envelope theorem (only direct effect), the partial derivative of g with respect to p is

$$\frac{\partial g}{\partial p} = U_y + pU_{yy}T_x - U_{xy}T_x \qquad (16)$$

This derivative is non-negative if and only if $\frac{U_{xy}}{U_y}T_x - \frac{U_{yy}}{U_y}T_y \leq 1$. Substitute the equilibrium condition $T_y = C_y$ and write the previous condition in terms of the risk aversion coefficient $\rho_y\left(1 - \frac{U_{xy}/p}{U_{yy}}\right) \leq 1$. ■

Corollary 1 (Exports are increasing in terms of trade) *If utility satisfies Condition M, then with perfect information optimal exports T_x are increasing in the terms of trade p (i.e. $\frac{dT_x}{dp} \geq 0$).*

Proof. To sign the relationship of exports T_x and relative prices p, we use the implicit function theorem:

$$\frac{dT_x}{dp} = -\frac{\partial g / \partial p}{\partial g / \partial T_x}$$

The partial derivative of g with respect to T_x is always negative by assumptions on the utility function (i.e. marginal utility of exports is decreasing in exports):

$$\frac{\partial g}{\partial T_x} = U_{xx} - 2pU_{xy} + p^2 U_{yy} < 0 \tag{17}$$

The partial derivative of g with respect to p is non-negative by Condition M. Combining results, a negative denominator and a positive numerator (under Condition M), we have that optimal exports T_x also increase with terms of trade p. ∎

Interpretation Condition M can be thought both in terms of low risk aversion but also in terms of the low complementarity between the goods. When the terms of trade increase, there is an income and a substitution effect. Condition M says implies that the substitution effect dominates and thus exports will increase to get the benefits of stronger terms of trade, but for that, we need goods to be largely substitutes.

- For a given complementarity (U_{xy}), relative risk aversion must be low enough for Condition M to hold.
- For a given level of risk aversion, complementarity must be low enough for Condition M to hold.
- If U_{xy} is positive (complements), then $\left(1 - \frac{U_{xy}/p}{U_{yy}}\right)$ is larger than one, and thus Condition M requires risk aversion to be very small.
- If U_{xy} is negative (substitutes), then $\left(1 - \frac{U_{xy}/p}{U_{yy}}\right)$ is lower than one, and thus risk aversion can be larger and still Condition M still hold.

3.2. Condition V $\Longrightarrow$ Marginal utility of exports convex in terms of trade.

Definition 2 *Let $\pi_y \equiv -\frac{C_y U_{yyy}}{U_{yy}}$ be the coefficient of relative prudence of risky good y. We say that utility $U(c_x, c_y)$ satisfies Condition V if*

$$\pi_y \left(1 - \frac{U_{xyy}/p}{U_{yyy}}\right) > 2$$

Proposition 5 *If utility satisfies Condition V, then with perfect information, the marginal utility of exports g is is strictly convex on the terms of trade.*

Proof. The second derivative of g with respect to p:

$$\frac{\partial^2 g}{\partial p^2} = 2T_x U_{yy} + pT_x^2 U_{yyy} - T_x^2 U_{xyy} \tag{18}$$

g is strictly convex in p if following condition holds $2 \leq -\frac{C_y U_{yyy}}{U_{yy}} + \frac{T_x U_{xyy}}{U_{yy}}$ which can be rewritten in terms of the coefficient of relative prudence $\pi_y \left(1 - \frac{U_{xyy}/p}{U_{yyy}}\right) > 2$. ■

Interpretation Condition V is a condition on the existence of precautionary savings in a multivariate setting (precautionary savings refers to a situation where savings increase when the future becomes more uncertain).

- In a univariate setting, Rotschild and Stiglitz (1970) established the result of existence of precautionary savings when relative prudence is larger than 2. Sandmo (1970) also established that if the third derivative of utility is positive (if marginal utility is convex), then precautionary savings arise as well.
- In our bivariate setting, the conditions are in terms of relative prudence and also in terms of the second cross-derivative, also called cross-prudence.

3.3. With Condition M, higher expected terms of trade increase trade.

Proposition 6 *Assume utility satisfies Condition M. Then a FOSD shift in the distribution of the terms of trade increases the volume of exports.*

Note: This is a stronger condition than just increasing the mean, because the variance could be also changing but as long as we have a FOSD shift then the proposition works.

Proof.

Let F be the distribution of term of trade p and consider a FOSD shift of F given by H. This means that there exists some gamble ϵ such that $p' = p + \epsilon \sim H$ where $\epsilon \geq 0$ in all possible states (and strictly positive in at least one state). Note that $\mathbb{E}[p'] \geq \mathbb{E}[p]$. We will show that under distribution H there is more trade.

With imperfect information, the FOC under distribution H is equal to

$$\mathbb{E}_H[g(T, p')] = 0$$

We can rewrite this FOC under F by substituting p' for $p + \epsilon$:

$$\mathbb{E}_F[g(T, p + \epsilon)] = 0$$

Taking total derivative with respect to the constant ϵ and applying the Implicit Function Theorem to compute $\frac{dT}{d\epsilon}$, we obtain:

$$\frac{dT}{d\epsilon} = -\frac{\partial \mathbb{E}[g]/\partial \epsilon}{\partial \mathbb{E}[g]/\partial T}$$

- The denominator is always negative: exchange the derivative and expectations operations, $\frac{\partial \mathbb{E}[g]}{\partial T} = \mathbb{E}\left[\frac{\partial g}{\partial T}\right]$ and recall from (17) that the derivative of g wrt T is negative for every p; therefore the expectation across all p is also negative.

$$\mathbb{E}\left[\frac{\partial g}{\partial T}\right] \leq 0 \tag{19}$$

- The numerator is non-negative if condition M is satisfied: exchange the derivative and expectations operations and note that the derivative wrt ϵ is the same as wrt p:

$$\frac{\partial \mathbb{E}[g]}{\partial \epsilon} = \mathbb{E}\left[\frac{\partial g}{\partial \epsilon}\right] = \mathbb{E}\left[\frac{\partial g}{\partial p}\right]$$

  If condition M holds $\forall p$, then the function g is increasing in the terms of trade: $\frac{\partial g}{\partial p} \geq 0$ and therefore the expectations is non-positive as well:

$$\mathbb{E}\left[\frac{\partial g}{\partial p}\right] \geq 0 \tag{20}$$

- Together, we have that exports are increasing in ϵ, and thus exports are larger under the distribution H than F.

■

3.4. With Condition V, higher volatility of terms of trade increases trade.

Proposition 7 *Assume utility satisfies Condition V. Then a mean preserving spread in the terms of trade increases the volume of exports.*

Proof.

Let F be the distribution of term of trade p and consider a mean preserving spread (MPS) called H. This means that there exists a random variable η, with mean zero $\mathbb{E}[\eta|p] = 0$, such that

$p' = p + \eta \sim H$. Note that $\mathbb{V}[p'] \geq \mathbb{V}[p]$. Now let g be any convex function of the terms of trade p. For any T, the expectation of a function g with respect to distribution H is

$$\mathbb{E}_H[g(T, p')] = \int g(T, p')dH(p') \qquad \forall T$$

Using the definition of p' and the law of iterated expectations, we get:

$$\int g(T, p')dH(p') = \int \mathbb{E}[g(T, p + \eta)|p]dF(p)$$

Now since g is convex in the terms of trade p (because condition V is satisfied), we have that

$$\int \mathbb{E}[g(T, p + \eta)|p]dF(p) \geq \int g(T, \mathbb{E}[p + \eta])|p]dF(p) = \int g(T, p)dF(p)$$

where the equality comes from the assumption of zero mean of η. Thus the first result is that if H is a MPS of F and g is convex in p, then:

$$\mathbb{E}_H[g(T, p)] \geq \mathbb{E}_F[g(T, p)] \qquad \forall T$$

Let T^F and T^H be the optimal policy under distribution F and H respectively, then by the FOC:

$$0 = \mathbb{E}_F[g(T^F, p)], \qquad 0 = \mathbb{E}_H[g(T^H, p)]$$

where in this case the function g is defined in (3). Combining the FOC and the result above we have:

$$\mathbb{E}_H[g(T^H, p)] = 0 = \mathbb{E}_F[g(T^F, p)] \leq \mathbb{E}_H[g(T^F, p)]$$

or simply

$$\mathbb{E}_H[g(T^H, p)] \leq \mathbb{E}_H[g(T^F, p)]$$

Since $\mathbb{E}[g]$ is strictly decreasing in T (as we showed above in (19)), we have that

$$T^H \geq T^F$$

Therefore, if H is a MPS of F and g is convex in p, then $T^H \geq T^F$. As information decreases and the lack of coordination increases the volatility of the terms of trade, exports would increase (there are precautionary savings).

■

3.5. Effect of a change in mean and a change in variance

Proposition 8 (Decomposition of effects) *Assume the covariance between export decreases and both the mean and variance of the terms of trade increase. Also assume the variance is a linear function of the mean terms of trade. Then the change in aggregate exports coming from the change in covariance can be decomposed as the sum of Condition M and Condition V.*

$$\frac{\partial T}{\partial \mathbb{C}[T_x, T_y]} = Condition\ M + Condition\ V$$

Proof. Let η be a random variable with mean $\mathbb{E}[\eta] = \bar{\eta}$ and variance $\mathbb{V}[\eta] = \sigma_\eta + 2\mathbb{E}[\eta]$ (this linear relationship was established above coming from changes in the covariance). Recall the function $g(T, p)$ defined in (3) and create a new function $m(\eta) \equiv \mathbb{E}[g(T, p + \eta)]$, where the expectation is taken over the distribution of the terms of trade p. From the Implicit Function Theorem and the definition of m, we have that

$$\frac{dT}{dp} = -\frac{\partial \mathbb{E}[g]/\partial p}{\partial \mathbb{E}[g]/\partial T} = -\frac{\partial m(\eta)/\partial \eta\Big|_{\eta=0}}{\partial m(\eta)/\partial T\Big|_{\eta=0}}$$

Since the denominator is always negative ($\mathbb{E}[g]$ is strictly decreasing in T as we showed above in (19)), we will characterize the sign of the numerator $\partial m(\eta)/\partial \eta\Big|_{\eta=0}$.

A second order approximation of $m(\eta)$ around $\eta = \bar{\eta}$ yields:

$$m(\eta) \approx \mathbb{E}[m(\bar{\eta})] + \frac{1}{2}m''(\bar{\eta})\mathbb{V}[\eta] = \mathbb{E}[g(T, p + \bar{\eta})] + \left(\frac{\sigma_\eta}{2} + \bar{\eta}\right)\frac{\partial^2 \mathbb{E}[g(T, p + \bar{\eta})]}{\partial p^2}$$

where we have used that the variance of terms of trade is proportional to its mean. Take the derivative with respect to $\bar{\eta}$:

$$\frac{\partial m(\eta)}{\partial \bar{\eta}} = \frac{\partial \mathbb{E}[g(T, p + \bar{\eta})]}{\partial p} + \frac{\partial^2 \mathbb{E}[g(T, p + \bar{\eta})]}{\partial p^2} + \left(\frac{\sigma_\eta}{2} + \bar{\eta}\right)\frac{\partial^3 \mathbb{E}[g(T, p + \bar{\eta})]}{\partial p^3}$$

Evaluate at $\bar{\eta} = 0$ and $\sigma_\eta = 0$ to get:

$$\frac{\partial m(\eta)}{\partial \bar{\eta}}\Big|_{\bar{\eta}=0, \sigma_\eta=0} = \underbrace{\frac{\partial \mathbb{E}[g(T, p)]}{\partial p}}_{\geq 0 \text{ with Condition M}} + \underbrace{\frac{\partial^2 \mathbb{E}[g(T, p)]}{\partial p^2}}_{\geq 0 \text{ with Condition V}}$$

Now we get an exact expression for the derivative:

$$\begin{aligned}\left.\frac{\partial m(\eta)}{\partial \bar{\eta}}\right|_{\bar{\eta}=0,\sigma_\eta=0} &= \mathbb{E}\left[U_y + pU_{yy}T_x - U_{xy}T_x\right] + \mathbb{E}\left[2T_xU_{yy} + pT_x^2U_{yyy} - T_x^2U_{xyy}\right] \\ &= \mathbb{E}\left[-\frac{T_yU_{yy}}{\rho_y}\left[1-\left(1-\frac{U_{xy}/p}{U_{yy}}\right)\rho_y\right]\right] + \mathbb{E}\left[\frac{T_xT_yU_{yyy}}{\pi_y}\left[\left(1-\frac{U_{xyy}/p}{U_{yyy}}\right)\pi_y - 2\right]\right] \\ &= \mathbb{E}\left[U_y\underbrace{\left[1-\left(1-\frac{U_{xy}/p}{U_{yy}}\right)\rho_y\right]}_{\geq 0 \text{ with Condition M}}\right] + \mathbb{E}\left[\frac{U_y\rho_y}{p}\underbrace{\left[\left(1-\frac{U_{xyy}/p}{U_{yyy}}\right)\pi_y - 2\right]}_{\geq 0 \text{ with Condition V}}\right]\end{aligned}$$

Therefore, if Condition M and V are satisfied (high substitutability + precautionary savings), exports decrease when the mean and the variance of terms of trade do decrease. Condition M ensures high substitutability across goods: by facing lower returns from exports agents substitute away from the foreign goods. Condition V implies precautionary savings: by facing lower variance from exports, agents reduce their precautionary exports. ■

Example with Cobb-Douglas Consider the following utility function

$$\mathbb{E}[C] = \mathbb{E}[C_x^\alpha C_y^\beta]$$

The FOC is $\frac{C_y}{C_x} = \frac{\beta}{\alpha p}$ and in equilibrium $C_y = pT_x$, therefore we have the following relationship:

$$\frac{T_x}{C_x} = \frac{\beta}{\alpha}$$

Let's compute the appropriate derivatives:

$$\begin{aligned} U_y &= \frac{\beta C}{C_y}, \qquad U_{yy} = -\frac{\beta(1-\beta)C}{C_y^2}, \qquad U_{yyy} = \frac{\beta(1-\beta)(2-\beta)C}{C_y^3} \\ U_{xy} &= \frac{\alpha\beta C}{C_xC_y}, \qquad U_{xyy} = -\frac{\alpha\beta(1-\beta)C}{C_xC_y^2}\end{aligned}$$

The coefficients of relative risk aversion and relative prudence

$$\begin{aligned}\rho_y &= -\frac{C_yU_{yy}}{U_y} = 1-\beta \\ \pi_y &= -\frac{C_yU_{yyy}}{U_{yy}} = 2-\beta\end{aligned}$$

Condition M:

$$1-\rho_y\left(1-\frac{U_{xy}/p}{U_{yy}}\right)=1-(1-\beta)\left(1-\frac{\frac{\alpha\beta C}{C_xC_y}/p}{-\frac{\beta(1-\beta)C}{C_y^2}}\right)=1-(1-\beta)\left(1+\frac{\frac{\alpha T_x}{C_x}}{1-\beta}\right)=-\beta+\frac{\alpha T_x}{C_x}=-\beta+\beta=0$$

Condition V:

$$\pi_y\left(1-\frac{U_{xyy}/p}{U_{yyy}}\right)-2=(2-\beta)\left(1-\frac{-\frac{\alpha\beta(1-\beta)C}{C_xC_y^2}/p}{\frac{\beta(1-\beta)(2-\beta)C}{C_y^3}}\right)-2=(2-\beta)\left(1+\frac{\frac{\alpha T_x}{C_x}}{2-\beta}\right)-2=-\beta+\frac{\alpha T_x}{C_x}=0$$

Therefore, with Cobb-Douglas preferences, increases in the mean and the variance of the terms of trade do not have any effect on trade, as both conditions are equal to zero.

3.6. If utility satisfies condition M, then information increases covariance between aggregate exports.

Proposition 9 *If utility satisfies condition M, then more information increases the covariance between aggregate exports (i.e. coordination).*

Proof. With no information about T_y, domestic exports cannot be a function of foreign exports because $T_x(T_y)$ is not a measurable strategy given the information set. Therefore, $\frac{dT_x}{dT_y}=0$. With perfect information, if condition M being satisfied by the utility function, then $\frac{dT_x}{dT_y}>0$. To prove that covariance of exports increases in information, we establish two additional conditions: (i) the sign of the derivative determines the sign of the covariance and (ii) the covariance operator is continuous.

(i) From derivative to covariance The first step is to connect the derivative $\frac{dT_x}{dT_y}$ with the covariance $\mathbb{C}[T_x,T_y|\mathcal{I}_x]$. Note that T_y and μ_x are the only random variables for the agent in country x. A first order approximation of the policy function $T_x(T_y,\mu_x)$ yields

$$T_x(\mu_x,T_y)\approx T_x(\mu_x,\mathbb{E}[T_y|\mathcal{I}_x])+\beta\left(T_y-\mathbb{E}[T_y|\mathcal{I}_x]\right)+\gamma\left(\mu_x-m_x\right)=\alpha+\beta T_y+\gamma\mu_x$$

where α gathers all the constants. From an ex-ante perspective, T_x is a random variable. With this approximation, the covariance with T_y is given by:

$$\mathbb{C}[T_x,T_y]\approx\mathbb{C}(\alpha+\beta T_y+\gamma\mu_x,T_y)=\beta\mathbb{V}(T_y)$$

i.e. the own aggregate shock does not induce covariance with other countries exports. Therefore, the slope is given by

$$\beta = \frac{\mathbb{C}[T_x, T_y]}{\mathbb{V}(T_y)} = \frac{dT_x}{dT_y}\Big|_{T_y=\mathbb{E}[T_y]}$$

With no information $\beta = 0$. With perfect information and condition M, $\frac{dT_x}{dT_y} > 0 \ \forall T_y$, and therefore, $\beta > 0$. We have established that

$$sign\left(\frac{dT_x}{dT_y}\right) = sign\left(\mathbb{C}(T_x, T_y)\right) = sign(\beta)$$

(ii) Continuity of the covariance in the amount of information. If the conditional distribution of terms of trade p is a continuous function of the signal and its precision, then the continuity of Bayesian updating together with the continuity of the integral operator ensure that any conditional expectation is continuous as well. Since the covariance is an expectation, it is also a continuos function of the signal precision.

By (i) for no information (zero precision) the covariance is zero, and for perfect information (infinity precision) the covariance is positive. By the continuity established in (ii), there exists an interval for precision between 0 and infinity for which the covariance is increasing in precision. Therefore, more information increases the covariance of aggregate exports. ■

3.7. Summary

- If M is satisfied (low complementarity), then more information increases covariance (becomes positive), which reduces both mean and variance of terms of trade. **Lower mean reduces exports** (since M is satisfied, substitution effect wins).
  - If V is satisfied, lower variance reduces exports, amplifying mean effect.
  - If V is not satisfied, lower variance increases exports, against mean effect.
- If M is not satisfied (high complementarity), then more information decreases covariance (becomes negative), which increases both mean and variance of terms of trade. **Higher mean reduces exports** (since M is not satisfied, wealth effect wins).
  - If V is satisfied, higher variance increases exports, against mean effect.
  - If V is not satisfied, higher variance decreases exports, amplifying mean effect.

Therefore, if condition M is satisfied or not, the resulting changes in the mean reduce exports. The change in the variance will amplify or offset the mean effect depending if there are precautionary savings or not.

Trade response to information

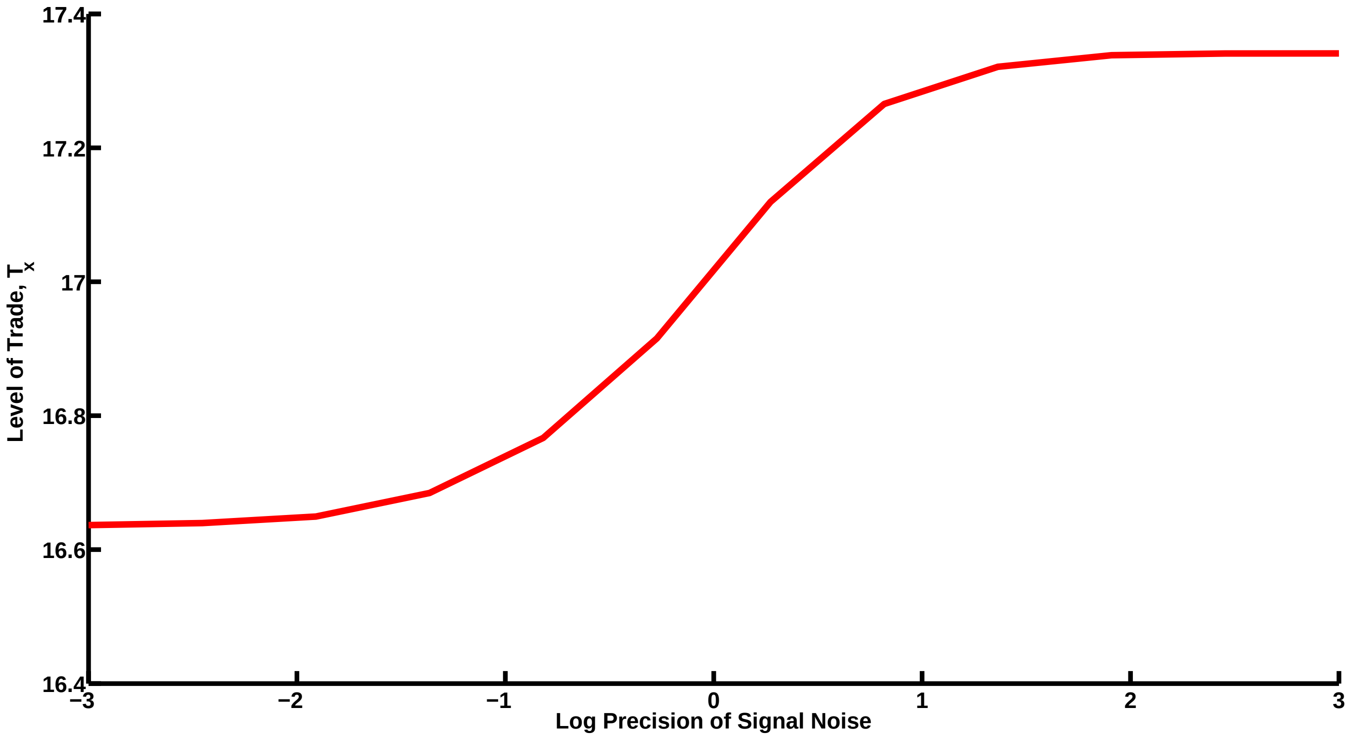
	Terms of trade	V ✓ (prudent)	V × (not prudent)
M ✓ (substitutes)	$\mathbb{E}\downarrow$ $\mathbb{V}\downarrow$	$T\downarrow$	?
M × (complements)	$\mathbb{E}\uparrow$ $\mathbb{V}\uparrow$	?	$T\downarrow$

4. Numerical Examples with CES Utility

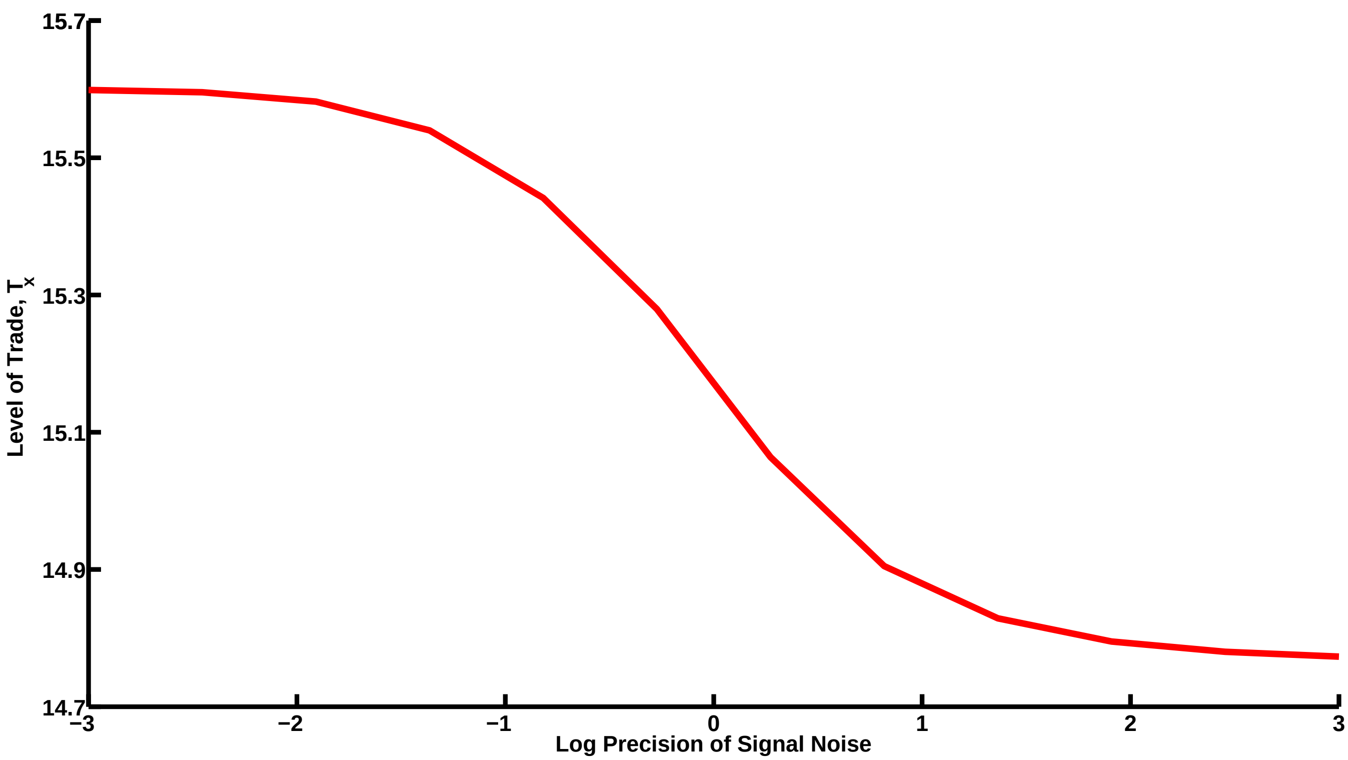
Consider utility of the form

$$U(c_x, c_y) = (c_x^\theta + c_y^\theta)^{\frac{1}{\theta}}.$$

With the elasticity of substitution $= \frac{1}{1-\theta}$. This is complicated because of the non-separability. Next pictures illustrate results numerically...its all about θ.



(a) CES, $\theta < 0$: **Information Increases Trade**



(b) CES, $\theta = 0.25$: **Information Decreases Trade**

Figure 3: Information Reduces Terms of Trade Volatility and Reduces the Mean

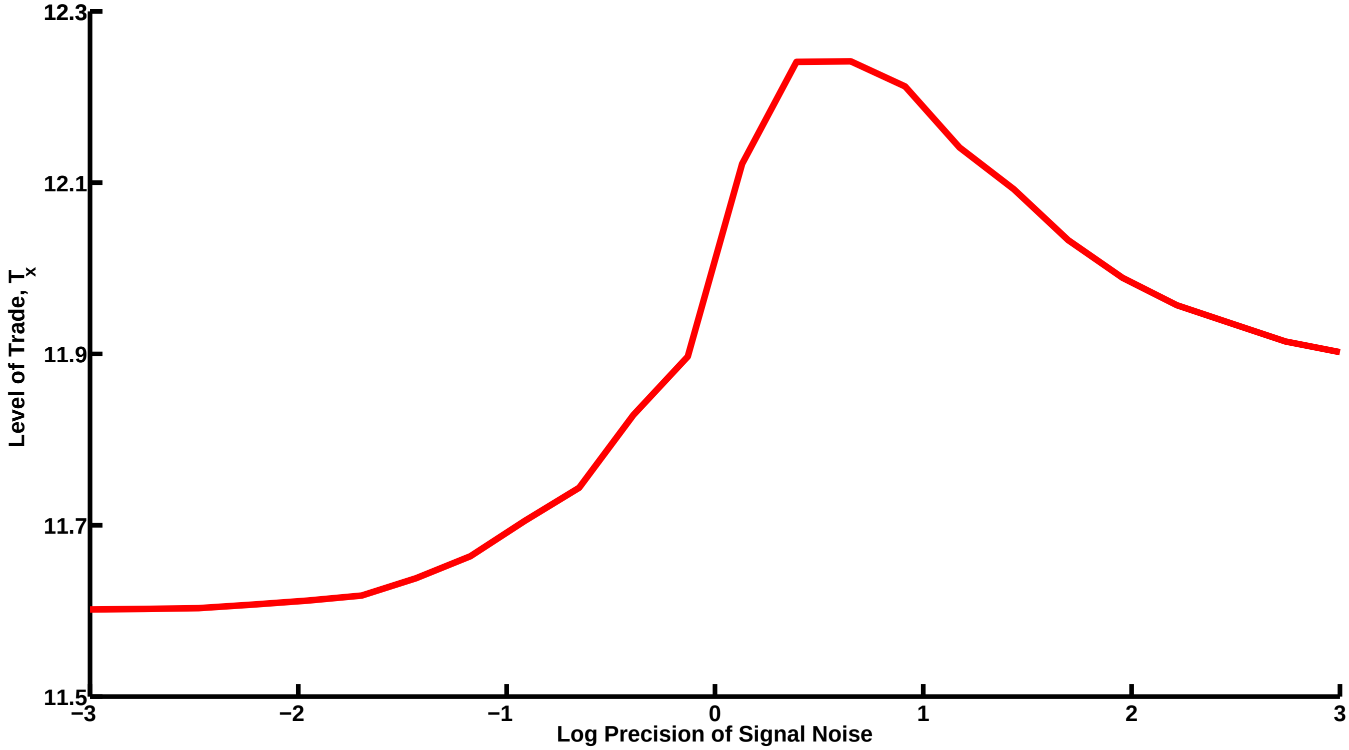


Figure 4: CES, $\theta = 0.75$: Information Increases and Decreases Trade

5. Conclusions…

If information frictions are responsible for lower levels of foreign trade than domestic trade, the the information friction must take the form of an information asymmetry: home firms must know something about other home firms that foreigners do not. Otherwise, the information friction would inhibit home and domestic trade equally. This paper articulates a simple benchmark model of international trade with asymmetric information.

This benchmark model tells us that asymmetric information is not a barrier to trade. Instead, pairs of countries with more asymmetric information should trade more. The results quantify and explain the following logical steps. If another country exports more, my country would like to also export more because when others' goods are abundant, my goods fetch a higher price. Better information about the foreign endowment facilitates this export coordination. Better coordinated trade means that the ratio of home exports to foreign exports fluctuates less. But this ratio of home to foreign exports is the relative price of foreign goods. Movements in the relative price are the key instrument for sharing international endowment risk. When that relative price is less flexible, risk-sharing breaks down. Finally, since optimal exports are a convex function of the expected relative price, when the relative price varies less, the positive Jensen inequality term gets smaller and the average amount of trade falls.

These results do not imply that information frictions cannot rationalize low trade. It teaches us that the relationship between asymmetric information and trade share is not as simple as previously thought. The results should prompt researchers to look for ways to interact asymmetric information with other frictions in a way that might reduce trade. This model provides the foundation for such further exploration.

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