Turbulence and Unemployment in Matching Models*

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Abstract

Ljungqvist and Sargent (2007) show that increases in turbulence, in the sense of worse skill transition probabilities for workers who suffer involuntary layoffs, generate higher unemployment in a welfare state. den Haan, Haefke and Ramey (2005) challenge this finding and argue that if turbulence also exposes voluntary quits to a tiny risk of skill loss, then higher turbulence leads to a reduction in unemployment. In this paper we explore the source of these disparate results within the two adopted matching models. We find the explanation to be the latter authors' auxiliary model features that suppress workers' bargaining power and lower returns to labor reallocation, as further accentuated by the authors' parameterization of productivity distributions. The resulting small incentives for labor mobility cause voluntary separations to shut down in response to any small cost to mobility. For example, the imposition of small government mandated layoff costs in tranquil times has counterfactually large effects of suppressing unemployment. Once the auxiliary model features are dropped and the parameterization is adjusted to account for historical observations on unemployment and layoff costs, the positive relationship between turbulence and unemployment reemerges.

JEL: E24, J63, J64

Keywords: matching, skills, turbulence, unemployment, labor mobility, involuntary layoffs, voluntary quits, bargaining, employment protection, layoff taxes

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1 Introduction

Ljungqvist and Sargent (1998) investigate the causes of high and persistent unemployment in European welfare states after 1980, and propose the interaction of economic turbulence—immediate skill loss after a layoff—and a generous welfare state, as a force that can explain the unemployment dynamics. They find that in a pure search model, increased turbulence in the economic environment within welfare states is the cause of persistently higher unemployment. The welfare state adversely affects the response to economic shocks, making unemployment more persistent, and the response to changes in the economic environment (turbulence) which explains the higher unemployment rate at steady state. The key force is that turbulence decreases unemployed workers' willingness to accept new jobs as their unemployment benefits are linked to past jobs where their skills were high. We call this the *job rejection channel*. In a follow-up paper, Ljungqvist and Sargent (2007) [LS from now on] show that the turbulence story is robust to introducing a matching framework.

In contrast to the LS's rationale for decreased willingness of unemployed workers to accept new jobs, den Haan, Haefke and Ramey (2005) [DHHR from now on] focus on the response by employed workers. Since displacement induces costly skill obsolescence in a turbulent environment, workers ought to be much more reluctant to part with their existing jobs, instead offering wage concessions to avoid layoffs, i.e. as the speed of skill obsolescence rises, workers become more reluctant to separate, and job destruction falls. We call this the *job retention channel*. Indeed, this intuition became commonly-held among economic observers. Under this argument, an increase in turbulence should have induced a reduction in the rate of job destruction, exerting downward pressure on unemployment and working against the job rejection channel highlighted by LS.

The puzzle The main finding of DHHR is that "if turbulence has only a tiny effect on the skills of workers experiencing endogenous separation, then the results of Ljungqvist and Sargent (1998, 2004a) are reversed, and higher turbulence leads to a *reduction* in unemployment". Specifically,

> *"allowing for a skill loss probability following endogenous separation that is only 3% of the probability following exogenous separation eliminates the positive turbulence-unemployment relationship. Increasing this proportion to 5% gives rise to a strong negative relationship between turbulence and unemployment."* (DHHR, p. 1362)

It is surprising that so small probabilities can have such large effects on equilibrium outcomes. In contrast, when skill loss at endogenous separations is introduced in the LS model, turbulence

has only small effects on outcomes: turbulence following voluntary separations has to be more than 40% of exogenous turbulence before it can suppress unemployment relative to tranquil economic times (when turbulence is zero). This is the puzzling difference.

In this paper, we provide an answer to the puzzle and show the robustness of the turbulence mechanism to the enhancement of job retention incentives. Given our results, we believe the DHHR caveat to the LS turbulence story discussed by Hornstein, Krusell and Violante (2005) in the 2005 Handbook of Growth should be revisited. More generally, our results shed light on hidden "forces" behind matching models that researchers in this area should be aware of.

Three potential explanations The LS and DHHR frameworks differ in three dimensions, two in terms of the model structure, and one regarding the calibration of incentives for labor mobility. Our strategy to uncover the resolution of the puzzle is to start with the LS model and make one perturbation at a time. In the Appendix, we start from the DHHR model and work through the perturbations in reverse.

1. **Labor market tightness.** LS adopt the standard assumption of free entry of firms and a zero-profit condition for posting vacancies, which together determine labor market tightness; while in DHHR the measures of both firms and workers are assumed to be fixed and equal, which in turn delivers a constant value of market tightness and an exogenous matching probability.

   Verdict: Not guilty. With endogenous matching, higher turbulence decreases market tightness as the "invisible hand" struggles to restore the profitability of firms; lower tightness means lower job finding rates and unemployment increases. In contrast, this force cannot be accommodated with exogenous matching (vacancy posting does not respond, even if the profitability of vacancies plummets in response to turbulence), and thus unemployment does not increase coming from this mechanism. Nevertheless, this force is not strong enough and unemployment dynamics are not significantly altered when we move from endogenous to exogenous tightness.

2. **Timing of the consummation of skill upgrade.** LS assume that skill upgrades are immediately consummated and, upon skill upgrades, workers immediately become entitled to high unemployment benefits. In contrast, DHHR assume that a worker who receives a positive skill shock at work must remain with the same firm for one period in order to consummate the higher skill level.

   Verdict: Not guilty. The alternative assumptions affect workers' bargaining power vis-à-vis firms. Delayed consummation effectively erodes the bargaining position of high-skilled workers. When studying the employment effects of layoffs costs, Ljungqvist (2002) found

that different assumptions about relative bargaining positions could have large impact on equilibrium outcomes. Indeed, under delayed consummation, wages of upgraded workers become negative (firms are extracting rents from the workers). Still, this force does not happen to be strong enough quantitatively to alter other equilibrium outcomes.

3. **Productivity distributions.** LS postulate truncated Normal distributions with a wide support, whereas DHHR assume Uniform productivity distributions with narrow support.

   Verdict: Guilty! While the shape of the distribution is not crucial, their support range is of overriding importance. The DHHR parameterization is associated with a much less robust incentive for high-skilled workers to transit between jobs, even in tranquil times. Hence, any small cost to mobility would cause voluntary separations to shut down. Once the parameterization is adjusted to account for the unemployment dynamics in data, the positive relationship between turbulence and unemployment reemerges.

Key: Incentives for labor mobility To further analyze the forces at play and strengthen our point regarding labor mobility, we conduct a layoff cost analysis. Introducing a small layoff cost closes down unemployment in the DHHR framework, which illustrate the fickleness of its parameterization of productivity distributions. We find that mobility of high-skilled employed shuts down with a labor tax equivalent to 14% of the average yearly output per worker, which is indeed a small number. Therefore, a tiny government mandated layoff cost (or any small mobility cost, such as turbulence following voluntary quits) has counterfactually large effects of suppressing unemployment by shutting down voluntary separations.

Moreover, the DHHR framework features an additional force that reduces incentives for labor mobility. They assume that high-skilled unemployed workers are also subject to turbulence risk following an unsuccessful meeting with a firm. DHHR seem to have slipped this additional exposure to endogenous turbulence into their framework as a simplification that allows to reduce the number of workers to keep track off and it looks entirely unintended. However, this exposure radically changes the incentives for labor mobility, as high-skilled workers hold onto their jobs given the risk that they face skill obsolescence now also affects them in case they are unemployed. In the main text, we abstract from this additional risk and refer the interested reader to a very detailed Appendix on this subject.

Organization Section 2 develops a matching framework with turbulence which builds on the models of LS and DHHR. Section 3 documents the puzzle, dissects the analysis, and finds the culprit. Section 4 conducts the layoff tax analysis. Finally, Section 5 concludes.

2 A Matching Framework with Turbulence

We develop a modified version of the matching model in LS that includes both exogenous turbulence—worse skill transition probabilities for workers who suffer involuntary layoffs—, as well as endogenous turbulence—worse skill transition probabilities for workers who suffer voluntary quits.

2.1 Environment

Workers Consider a unit mass population consisting of workers, which are either employed or unemployed. Workers are risk neutral and value consumption, with preferences ordered according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_t \tag{1}$$

They discount future utilities at a rate $\beta \equiv \hat{\beta}(1-\rho^r)$, which includes the subjective time discount factor $\hat{\beta} \in (0,1)$ and the constant probability of retirement $\rho^r \in (0,1)$. In order to keep the mass of workers constant, it is assumed that a retired worker exits the labor force permanently and is substituted by a newborn worker.

Heterogeneity Besides employment status, workers differ in two dimensions: their current skill level i, which can be either low (l) or high (h), and their skill level during their last employment spell j, which in turn determines their entitlement to unemployment benefits. Workers experience accumulation or deterioration of skills, conditional on employment status and instances of exogenous and endogenous job terminations. We denote variables with two indices (i,j), the first for current skill and the second for benefit entitlement. We assume that all newborn workers enter the labor force with low skills and a low benefit entitlement.

Firms and employment relationships In order to create jobs, firms post vacancies at a cost μ. We think of a job opportunity as a productivity draw z from a stationary distribution $v_i(z)$, which is indexed by the worker's skill level i. Then an employment relationship produces output z per period. It is assumed that the high-skilled distribution first-order stochastically dominates the low-skilled distribution: $v_h(z) \leq v_l(z)$.

Sources of uncertainty Besides retirement, there are five additional sources of uncertainty. There are three types of productivity shocks—upgrade, switch and exogenous termination—and two types of turbulence shocks, following exogenous and endogenous separations, respectively.

(i) During employment, skills can be accumulated. Conditional on no exogenous job termination, a worker's skills can get upgraded from low to high with probability γ^u.

(ii) Idiosyncratic shocks within a worker-firm match affect employed workers productivities in the job. With probability γ^s, a new productivity is drawn from a distribution that depends on the current skill level. As a consequence of the new draw, a relationship may continue or may be endogenously terminated, i.e. a voluntary quit.

(iii) An employed worker faces a probability ρ^x of having her job terminated for exogenous reasons, i.e. a layoff.

(iv) Upon an exogenous job termination, a high skilled worker is subject to a potential risk of losing her skills with probability $\gamma^{d,x}$. We label this risk as *exogenous turbulence*.

(v) Upon an endogenous job termination, high skilled workers are subject to the risk of losing their skills with probability γ^d. We label this risk as *endogenous turbulence*. Following DHHR, we parametrize endogenous turbulence as a fraction ϵ of exogenous turbulence, and we vary it from zero—only exogenous turbulence— to one—both types of turbulence are equal—as follows: $\gamma^d = \epsilon\gamma^{d,x}$.

Timing of turbulence shocks We make the following assumption regarding the timing of turbulence shocks. In the beginning of the period, exogenous separations from jobs happen and exogenous turbulence $\gamma^{d,x}$ hits. These workers become part of the high-skilled unemployed, and will sit in the matching function one period. Then employed workers upgrade their skills and get new productivity draws. If they reject the draws, they are subject to endogenous turbulence that happens at the end of the period. The high-skilled employed workers that are hit with endogenous turbulence become part of the low-skilled unemployed at the beginning of next period, joining the workers that were hit with exogenous turbulence.

2.2 Joint surplus from a match

When an unemployed worker with skill i and benefit entitlement j meets a firm with a vacancy, the firm-worker pair draws productivity z from the stationary distribution $v_i(z)$. The firm and the worker will stay together and produce if the match is successful, which happens if the surplus is positive. Then wages are determined through Nash bargaining, with π and $1-\pi$ the bargaining weights of workers and firms, respectively.

The joint surplus for a new $s^o_{ij}(z)$ or continuing $s_{ij}(z)$ relationship is given by the after-tax productivity $(1-\tau)z$ plus the future joint continuation value $g_i(z)$, minus the outside options of the match: ω_{ij}, the worker's future value from entering the unemployment pool in the current period when he receives an unemployment benefit of b_j; and ω^f, the firm's value from entering the vacancy pool in the current period. For simplicity, we define the joint future value ω_{ij} to be equal to the sum of the worker's future value of unemployment ω^w_{ij} and the firm's future value of posting a vacancy ω^f, i.e. $\omega_{ij} \equiv \omega^w_{ij} + \omega^f$.

The joint surplus for a new $s^o_{lj}(z)$ or continuing $s_{lj}(z)$ relationship with a low-skilled worker with benefits j, is given by[1]

$$s^o_{lj}(z) \;=\; s_{lj}(z) \;=\; (1-\tau)z + g_l(z) - [b_j + \omega_{lj}], \qquad j = l, h \tag{2}$$

To compute the surplus for relationships with high-skilled workers, we must differentiate between new and continuing relationships. The surplus for a new relationship with a currently unemployed high-skilled worker, denoted by s^o_{hh}, considers outside options that are unaffected by turbulence:

$$s^o_{hh}(z) \;=\; (1-\tau)z + g_h(z) - [b_h + \omega_{hh}] \tag{3}$$

In contrast, the surplus for a continuing relationship with a high-skilled worker $s_{hh}(z)$, or a new relationship with a low-skilled worker that receives a skill upgrade that is immediately consummated, is affected by turbulence at rate γ^d, therefore

$$s_{hh}(z) \;=\; (1-\tau)z + g_h(z) - [b_h + (1-\gamma^d)\omega_{hh} + \gamma^d\omega_{lh}] \tag{4}$$

Productivity thresholds and rejection rates The worker and firm split the match surplus through Nash bargaining and outside values as threat points. Since they both want a positive surplus, there is mutual agreement whether to form a match or not. The reservation productivity $\underline{z}_{ij}$ is the lowest productivity that makes a match profitable and satisfies

$$s_{ij}(\underline{z}_{ij}) \;=\; 0 \tag{5}$$

for new or continuing matches, respectively. Given the productivity threshold $\underline{z}_{ij}$, let ν_{ij} denote the rejection probability, which is given by the mass of all productivity draws from distribution $v_i(y)$ that fall below the threshold:

$$\nu_{ij} \;=\; \int_{-\infty}^{\underline{z}_{ij}} dv_i(y) \tag{6}$$

[1]The equality between surpluses for new and continuing relationships holds in the absence of layoff taxes. In Section 4 we introduce layoff taxes and the expressions for the match surplus are no longer equal for employed and unemployed low-skilled workers.

Also denote with E_{ij} the expected value of a match following a new productivity draw from distribution $v_i(y)$, conditional on acceptance ($y \geq \underline{z}_{ij}$):

$$E_{ij} = \int_{\underline{z}_{ij}}^{\infty} [(1-\tau)y + g_j(y)] \, dv_i(y) \tag{7}$$

2.3 Match continuation values

Consider a match between a firm and a worker with skill level i. Given the match-specific productivity z which determines current output, let $g_i(z)$ be the joint continuation value of such match. We now characterize this value for low- and high-skilled workers.

High-skilled worker The joint continuation value of a match with a high-skilled worker with current productivity z, denoted by $g_h(z)$, is affected by exogenous turbulence after a high-skilled worker is laid-off and by endogenous turbulence after a productivity switch is rejected

$$\begin{aligned}
\text{Exogenous separation:} \quad g_h(z) &= \beta\Big[\rho^x(b_h + \underbrace{\gamma^{d,x}\omega_{lh} + (1-\gamma^{d,x})\omega_{hh}}_{\text{exogenous turbulence}}) \\
\text{No changes:} \quad &+ (1-\rho^x)(1-\gamma^s)((1-\tau)z + g_h(z)) \\
\text{Productivity switch:} \quad &+ (1-\rho^x)\gamma^s(E_{hh} + \nu_{hh}(b_h + \underbrace{(1-\gamma^d)\omega_{hh} + \gamma^d\omega_{lh}}_{\text{endogenous turbulence}}))\Big]
\end{aligned} \tag{8}$$

Low-skilled worker The joint continuation value for a match with a low-skilled worker takes into account the following contingencies: no changes in productivity or skills, an exogenous separation, a productivity switch, or a skill upgrade. Regarding this last contingency, we assume that skill upgrades are immediately consummated, and it is assumed that upon skill upgrades workers immediately become entitled to high unemployment benefits, even if matches break up at skill upgrades. Furthermore, a skill upgrade comes with a new draw from the high-skilled productivity distribution v_h.

These assumptions limit the number of worker categories to be considered, and it keeps the productivity distribution's support exogenous. In this case, low-skilled workers face the risk of endogenous turbulence when their human capital is upgraded and they reject the new productivity draw that comes with it (see last line in Equation (9) below).

Thus the joint continuation value of a match with a low-skilled worker with current productivity z, which is subject to immediate skill upgrade, is given by:

(Continuation value for a match with low-skilled worker in LS)

$$\begin{aligned}
\text{Exogenous separation:} \quad g_l(z) &= \beta\Big[\rho^x(b_l+\omega_{ll}) \\
\text{No changes:} &+ (1-\rho^x)(1-\gamma^u)(1-\gamma^s)((1-\tau)z+g_l(z)) \\
\text{Productivity switch:} &+ (1-\rho^x)(1-\gamma^u)\gamma^s(E_{ll}+\nu_{ll}(b_l+\omega_{ll})) \\
\text{Immediate skill upgrade:} &+ (1-\rho^x)\gamma^u(E_{hh}+\nu_{hh}(b_h+\underbrace{(1-\gamma^d)\omega_{hh}+\gamma^d\omega_{lh}}_{\text{endogenous turbulence}}))\Big]
\end{aligned} \tag{9}$$

In contrast, DHHR assume that a worker who received skill upgrade shock must remain with the same firm for one period in order to consummate the higher skill level. Moreover, in order to guarantee that such continued relationships are privately optimal, it is assumed that a positive human capital shock is accompanied with a new productivity draw from a rescaled distribution $v_u(z) = v_h(z)/(1-v_h(\underline{z}_{hh}))$ with a lower support given by the endogenous reservation productivity of a high-skilled worker in an ongoing employment relationship. The rationale is to reduce the number of worker categories to be considered, as the assumptions imply that relationships that experience an upgrade will always choose to continue.[2]

Under this alternative, the joint continuation value of a match with a low-skilled worker, which is subject to delayed skill upgrade, is given by:

(Continuation value for a match with low-skilled worker in DHHR)

$$\begin{aligned}
\text{Exogenous separation:} \quad g_l(z) &= \beta\Big[\rho^x(b_l+\omega_{ll}) \\
\text{No changes:} &+ (1-\rho^x)(1-\gamma^u)(1-\gamma^s)((1-\tau)z+g_l(z)) \\
\text{Productivity switch:} &+ (1-\rho^x)(1-\gamma^u)\gamma^s(E_{ll}+\nu_{ll}(b_l+\omega_{ll})) \\
\text{Delayed skill upgrade:} &+ (1-\rho^x)\gamma^u\int_{\underline{z}_{hh}}^{\infty}[(1-\tau)y+g_h(y)]\,dv_u(y)
\end{aligned} \tag{10}$$

Section 3.2 analyzes the differences in the timing of consummation of skill upgrades.

[2]The alternative assumption of immediate upgrades enables us to discard DHHR's simplifying, but debatable, assumption that a skill upgrade is accompanied with a new productivity draw from $v_u(z)$. In the LS model, the distributions from which productivities are drawn do not change with endogenous reservation productivities.

2.4 Outside options

Value of unemployment / Outside value for employed workers A worker that enters the unemployment pool obtains benefits b_j according to the skill level at its last job—benefits are determined by a replacement rate ϕ on the average labor income within the worker's skill category when last employed—and a continuation value of ω^w_{ij}. The value of unemployment also corresponds to a worker's outside option if he has a job offer at hand, which is taken in case the match is rejected. Let $\lambda^w(\theta)$ be the probability that a worker type (i, j) meets a vacancy; we specify this rate below and show that it does not depend on the type, given the assumptions.

A low-skilled unemployed worker with benefit entitlement j obtains $b_j + \omega^w_{lj}$, where

$$\omega^w_{lj} \;=\; \beta\Big[\underbrace{\lambda^w(\theta)\int_{\underline{z}_{lj}}^{\infty} \pi s^o_{lj}(y)\; dv_l(y)}_{\text{match + accept}} \;+\; \underbrace{b_j + \omega^w_{lj}}_{\text{match + reject / no match}}\Big] \qquad j = l, h \tag{11}$$

A high-skilled unemployed worker with benefit entitlement h, obtains $b_h + \omega^w_{hh}$, where

$$\omega^w_{hh} \;=\; \beta\Big[\underbrace{\lambda^w(\theta)\int_{\underline{z}^o_{hh}}^{\infty} \pi s^o_{hh}(y)\; dv_h(y)}_{\text{match + accept}} \;+\; \underbrace{(b_h + \omega^w_{hh})}_{\text{match + reject / no match}}\Big] \tag{12}$$

Value of a vacancy / Firms' outside value A firm that searches for a worker enters the vacancy pool by paying an upfront cost μ and obtains a fraction $(1 - \pi)$ of the surplus if the match is consolidated. Let $\lambda^f_{ij}(\theta)$ be the probability of filling the vacancy with an unemployed worker with type (i, j); this rate will be specified below. Then a firm's value of entering the vacancy pool, denoted by ω^f, is given by:

$$\omega^f \;=\; -\mu + \beta\left[\sum_{(i,j)} \lambda^f_{ij}(\theta)\int_{\underline{z}^o_{ij}}^{\infty} (1-\pi) s^o_{ij}(y)\; dv_i(y) + \omega^f\right] \tag{13}$$

where $s^o_{ij}(y)$ is the match specific surplus from a new relationship.

2.5 Labor market and matching technology

Let u_{ij} be the number of unemployed workers with current skill i and benefit entitlement j, then the total number of unemployed workers is $u = \sum_{i,j} u_{ij}$. Let v be the number of vacancies posted by firms. We assume a single matching function for all unemployed workers and vacancies. Matches are created according to an increasing, concave and linearly homogenous matching function $M(v, u)$ that takes as inputs the aggregate unemployment u and endogenously posted

vacancies v. Let $\theta \equiv \frac{v}{u}$ be the vacancy/unemployment ratio, or market tightness, and define $m(\theta) \equiv M(\frac{v}{u}, 1) = M(\theta, 1)$.

Equilibrium market tightness We assume free entry by firms, which ensures that in equilibrium firms expect to break even when posting a vacancy, and the expected value of a firm is always equal to zero. Free entry, together with equation (13), pins down the equilibrium value of market tightness. Thus the probability that a worker of type (i, j) meets a vacancy, $\lambda^w(\theta)$, is an increasing function of market tightness independent of (i, j); and the probability that a firm meets a worker of type (i, j), $\lambda^f_{ij}(\theta)$ is a decreasing function of market tightness and does depend on (i, j):

(Firm value, market tightness, job finding rate, and vacancy filling rate in LS)

$$\omega^f = 0 \tag{14}$$

$$\mu = \beta(1-\pi) \sum_{(i,j)} \lambda^f_{ij}(\theta) \int_{\underline{z}^o_{ij}}^{\infty} s^o_{ij}(y)\ dv_i(y) \tag{15}$$

$$\lambda^w(\theta) = m(\theta) \tag{16}$$

$$\lambda^f_{ij}(\theta) = \frac{m(\theta)}{\theta} \frac{u_{ij}}{u} \tag{17}$$

In contrast, DHHR's assumption of exogenous matching with a fixed and equal mass of workers and firms. It follows that market tightness is always equal to one $\theta = 1$, and the matching probabilities are constant. In this case, the value of the firm is endogenous and generally different from zero:[3]

(Firm value, market tightness, job finding rate, and vacancy filling rate in DHHR)

$$\omega^f = \frac{1}{1-\beta} \left[-\mu + \beta(1-\pi) \sum_{(i,j)} \lambda^f_{ij} \int_{\underline{z}^o_{ij}}^{\infty} s^o_{ij}(y)\ dv_i(y) \right] \tag{18}$$

$$\theta = 1 \tag{19}$$

$$\lambda^w = m(1) \tag{20}$$

$$\lambda^f_{ij} = m(1) \frac{u_{ij}}{u} \tag{21}$$

Differences in matching protocol and their consequences are analyzed in Section 3.1.

[3] DHHR assume $\mu = 0$; for positive vacancy creation costs, the firm value may be negative for high turbulence.

2.6 Wages

Wages are determined through Nash bargaining. Since there are no layoff costs in our baseline model, there is only one wage function that applies for both an initial round of negotiations between a newly matched firm and worker and for renegotiations in an ongoing match.[4] To simplify the exposition, we only write the equations for the case with immediate skill upgrades.

Wage determination For given current productivity z, the wage of low-skilled worker $p_{lj}(z)$, and the wage of a high-skilled worker $p_{hh}(z)$, solves the following maximization problems:

$$\max_{p_{lj}(z)} \quad \left[(1-\tau)z - p_{lj}(z) + g^f(z) - \omega^f\right]^{1-\pi} \left[p_{lj}(z) + g_l^w(z) - b_l - \omega_{lj}^w\right]^{\pi} \tag{22}$$

$$\max_{p_{hh}(z)} \quad \left[(1-\tau)z - p_{hh}(z) + g^f(z) - \omega^f\right]^{1-\pi} \left[p_{hh}(z) + g_h^w(z) - b_h - (1-\gamma^d)\omega_{hh}^w - \gamma^d\omega_{lh}^w\right]^{\pi}$$

where $g^f(z)$ and $g_i^w(z)$ are the individual continuation values for the firm and the worker from continuing the relationship (the joint continuation values defined in (8) and (9) are equal to the sum $g_i \equiv g^f + g_i^w$), which are given, respectively, by

$$\begin{aligned}
g_l^w(z) &= \beta\rho^x(b_l + \omega_{ll}^w) \\
&+ \beta\pi(1-\rho^x)\Big\{(1-\gamma^u)\Big[(1-\gamma^s)s_{ll}(z) + \gamma^s \int_{\underline{z}_{ll}} s_{ll}(y)\ dv_l(y)\Big] + \gamma^u \int_{\underline{z}_{hh}} s_{hh}(y)\ dv_h(y)\Big\} \\
g_h^w(z) &= \beta\rho^x(b_l + \omega_{ll}^w) \\
&+ \beta\pi(1-\rho^x)\Big[(1-\gamma^s)s_{hh}(z) + \gamma^s \int_{\underline{z}_{hh}} s_{hh}(y)\ dv_h(y)\Big] \\
g^f(z) &= \beta\rho^x\omega^f \\
&+ \beta(1-\pi)(1-\rho^x)\Big\{(1-\gamma^u)\Big[(1-\gamma^s)s_{ll}(z) + \gamma^s \int_{\underline{z}_{ll}} s_{ll}(y)\ dv_l(y)\Big] + \gamma^u \int_{\underline{z}_{hh}} s_{hh}(y)\ dv_h(y)\Big\}
\end{aligned} \tag{23}$$

and ω^f and ω_{ij}^w their outside options, defined in (11), (12), and (13). The solution to the wage determination problems in (22) yields the following wage functions

$$\begin{aligned}
p_{lj}(z) + g_l^w(z) &= \pi s_{lj}(z) + b_l + \omega_{lj}^w \qquad\qquad j = l, h \\
p_{hh}(z) + g_h^w(z) &= \pi s_{hh}(z) + b_h + (1-\gamma^d)\omega_{hh}^w + \gamma^d\omega_{lh}^w
\end{aligned} \tag{24}$$

which set the current payment—the wage—and the future payment—the continuation value—equal to worker's outside value plus a fraction π of the joint surplus. For new relationships, the wages $p_{lj}^o(z), p_{hh}^o(z)$ satisfy the previous equations using $s_{lj}^o(z)$ and $s_{hh}^o(z)$.

[4] In Section 4 we introduce layoff taxes and a two-tier wage structure that takes into account the different threat points that a firm has in a negotiation with a new worker versus a renegotiation with an incumbent worker.

Average wages Let $\hat{e}_j \equiv \sum_i e_{ij}$ denote the total number of workers with entitlement level j, then the average wage of workers with entitlement j is

$$\bar{p}_j = \sum_i \frac{e_{ij}}{\hat{e}_j} \int_{\underline{z}_{ij}}^{\infty} p_{ij}(y) \, \frac{dv_i(y)}{1 - v_i(\underline{z}_{ij})} \tag{25}$$

2.7 Government Policies

Unemployment Benefits Unemployment benefits are a constant fraction ϕ of the average wage of her entitlement group $\bar{p}_j$, computed in (25). Thus, average benefits by entitlement level are $b_j = \phi \bar{p}_j$, and total government expenditure amounts to

$$b_l u_{ll} + b_h(u_{lh} + u_{hh}) = \phi(\bar{p}_l u_{ll} + \bar{p}_h(u_{lh} + u_{hh})) \tag{26}$$

Income Taxes Income taxes are a constant fraction τ of productivity z, and average taxes by skill level are $t_i = \tau \bar{z}_i$. To compute average productivity $\bar{z}_i$, let $e_i \equiv \sum_j e_{ij}$ denote the total number of workers in skill level i, then the average productivity $\bar{z}_i$ of skill group i is

$$\bar{z}_i = \sum_j \frac{e_{ij}}{e_i} \int_{\underline{z}_{ij}}^{\infty} y \, \frac{dv_i(y)}{1 - v_i(\underline{z}_{ij})} \tag{27}$$

Therefore, total government revenue equals

$$t_l e_l + t_h e_h = \tau(\bar{z}_l e_l + \bar{z}_h e_h) \tag{28}$$

Government budget The government satisfies its budget constraint period by period. It adjusts its tax rate to equalize the total value of expenditures in (26) to its revenues (28):

$$\underbrace{\phi(\bar{p}_l u_{ll} + \bar{p}_h(u_{lh} + u_{hh}))}_{\text{gov. expenditure}} = \underbrace{\tau(\bar{z}_l e_l + \bar{z}_h e_h)}_{\text{gov. revenues}} \tag{29}$$

2.8 Worker flows

Workers flow between employment and unemployment status, skills, and benefit entitlement. Within all the flows, we are interested in highlighting those where turbulence has an effect: the low-skilled unemployed with high benefits u_{lh} and the high-skilled employed e_{hh}. See the Appendix for more details on the flow expressions for other groups of workers.

Low-skilled unemployed with high benefits The first group is the low-skilled unemployed with high benefits u_{lh}, which is at the core of the LS mechanism that generates a positive unemployment-turbulence relationship. Outflows occur with retirement and successful job market encounters; the acceptance rate $1-\nu_{lh}$ decreases with turbulence and thus decreases the outflow. Inflows occur in the following instances. Exogenous turbulence affects exclusively high-skilled workers e_{hh} that get laid off; with probability $\gamma^{d,x}$, they become part of the low-skilled workers with high benefit entitlement u_{lh}. Endogenous turbulence for the employed kicks-in with probability γ^d when a high-skilled worker e_{hh} rejects a productivity switch, and when a low-skilled worker gets a human capital upgrade and rejects the new offer (recall that benefits are upgraded immediately).

$$\begin{aligned}\Delta u_{lh} &= -\rho^r u_{lh} - (1-\rho^r)\underbrace{\lambda^w(\theta)(1-\nu_{lh})u_{lh}}_{\text{1. successful match}} \\ &+ (1-\rho^r)\Big\{\underbrace{\rho^x\gamma^{d,x}e_{hh}}_{\text{2. exogenous turbulence}} + \underbrace{(1-\rho^x)\gamma^d\nu_{hh}[\gamma^s e_{hh}+\gamma^u(e_{ll}+e_{lh})]}_{\text{3. endogenous turbulence}}\Big\}\end{aligned} \tag{30}$$

Besides the direct positive effect of turbulence in the inflows to unemployment (LS force), turbulence decreases the high-skilled workers' rejection rate ν_{hh} (DHHR force), and therefore the final effect is ambiguous and has to be determined quantitatively.

High-skilled employed The second group is the high-skilled employed e_{hh}. These workers are at the core of the DHHR mechanism that generates a negative unemployment-turbulence relationship by reducing endogenous separations. High-skilled employed workers have very strong incentives to hold on to their jobs when they face a positive probability of skill loss; this lowers their rejection rate ν_{hh}. The same happens to low-skilled workers that get a skill-upgrade and to high-skilled unemployed that get an offer; they have stronger incentives to accept given a higher turbulence risk. These forces are captured in lower rejection rates, which in turn increase the inflow to and decrease the outflow from high-skilled employed:

$$\Delta e_{hh} = -\rho^r e_{hh} - (1-\rho^r)\left[\rho^x e_{hh} + \underbrace{(1-\rho^x)\gamma^s\nu_{hh}e_{hh}}_{\text{1. unsuccessful switch}}\right] \tag{31}$$

$$+ (1-\rho^r)\left[\underbrace{\lambda^w(\theta)(1-\nu^o_{hh})u_{hh}}_{\text{2. succesful match}} + \underbrace{(1-\rho^x)\gamma^u(1-\nu_{hh})(e_{ll}+e_{lh})}_{\text{3. succesful upgrade}}\right] \tag{32}$$

2.9 Steady state equilibrium

A steady state equilibrium consists of the following objects: labor market tightness θ, job finding rate $\lambda^w(\theta)$ and vacancy filling grate $\lambda^f_{ij}(\theta)$; match surpluses $s_{ij}(z)$, productivity thresholds $\underline{z}_{ij}$, rejection rates ν_{ij}, and average productivity $\bar{z}_i$; wages $p_{ij}(z)$, average wages $\bar{p}_i$, and benefits b_j; future values for unemployment ω^w_{ij}, vacancies ω^f, and matches $g_i(z)$; and a tax rate τ such that:

a) Zero match surplus condition (5) determines reservation productivities

b) Free entry condition (15) pins down tightness in LS; market tightness equal to unity in DHHR;

c) Nash Bargaining (24) sets wages,

d) The tax rate balances the government's budget (29),

e) All net worker flows are equal to zero:

$$\Delta u_{ij} = \Delta e_{ij} = 0 \quad \forall\, i, j$$

2.10 Calibration

We consider the semi-quarterly calibration of LS, which constitutes a serious attempt to calibrate to Trans-Atlantic employment observations.

Preference parameters We set the model period equal to half a quarter, and specify a discount factor $\hat{\beta} = 0.99425$ and a retirement probability $\rho^r = 0.0031$, which together imply an adjusted discount (entering utility) of $\beta = \hat{\beta}(1 - \rho^r) = 0.991$.

Risk sources We set transition probabilities to make the average durations of skill acquisition and skill deterioration agree with data on wage-experience profiles. Exogenous layoffs occur with probability $\rho^x = 0.005$, this is, on average once every 25 years. We set a semiquarterly probability of upgrading skills $\gamma^u = 0.0125$ so that it takes on average 10 years to move from low to high skill, conditional on no job loss. The probability of a productivity switch on the job equals $\gamma^s = 0.05$, so that a worker expects to retain a given level of productivity for 2.5 years.

Labor market institutions We set the worker's bargaining power to be $\pi = 0.5$. The replacement rate is set at $\phi = 0.7$, or 70% of the wage in the last job. Layoff taxes are set to zero in the baseline to make the calibration compatible with that in DHHR. Section 4 studies the effects of layoff taxes in equilibrium allocations.

Matching We assume that the matching technology is Cobb-Douglas, $M(v,u) = Au^{\alpha}v^{1-\alpha}$, where A is matching efficiency and α stands for the elasticity of matches with respect to unemployment. Given this technology, the job finding rate and the vacancy filling rate in LS's endogenous matching framework are given by:

$$\lambda^{w}(\theta) = A\theta^{1-\alpha} \qquad \lambda^{f}_{ij}(\theta) = A\theta^{-\alpha}\frac{u_{ij}}{u} \tag{33}$$

and in the DHHR's exogenous matching framework, they become:

$$\lambda^{w} = A \qquad \lambda^{f}_{ij} = A\frac{u_{ij}}{u} \tag{34}$$

The elasticity of job-finding rate to turbulence is set to $1-\alpha = 0.5$. The matching function's efficiency and the vacancy creation cost are jointly set to deliver a market tightness of 1 in tranquil times (no turbulence), so that it matches the exogenous market tightness in DHHR. For this purpose, we set $A = 0.441$ and $\mu = 0.481$.[5]

Productivity distributions LS assumes productivities are drawn from truncated normals with wide support, $z_l \sim \mathcal{N}(1,1)$ for low skilled workers over the support $[-1,3]$ and $z_h \sim \mathcal{N}(2,1)$ for high skilled workers over the support $[0,4]$. In contrast, DHHR assumes uniform distributions with small support $z_l \sim \mathcal{U}([0.5,1.5])$ and $z_h \sim \mathcal{U}([1.5,2.5])$. Average productivities are 1 for low skilled and 2 for high skilled; the dispersion is 1 for the Normal and $1/\sqrt{12}$ for the Uniform.

The support of each distribution $[z_i^{\min}, z_i^{\max}]$ is a key measure of the incentives for labor mobility embedded in the economy. A wide support means that, conditional on a level of skill, there are large benefits from endogenous separations for workers with low levels of productivity as it is likely that they will draw a relatively more productive job. As discussed in Section 3.3, the LS and DHHR calibrations greatly differ in this respect and are the main explanation for the disparate outcomes.[6]

[5]Under the original LS calibration $(A,\mu) = (0.45, 0.5)$, no layoff taxes and no turbulence, the equilibrium market tightness is equal to $\theta = 0.9618$. We adjust the calibration in order to deliver a market tightness of 1, the same job finding rate for workers as in old calibration and a vacancy filling rate equal to worker job finding rate. Let $(\hat{A},\hat{\mu})$ be a new calibration given by: $\hat{A} = \kappa^{1-\alpha}A, \quad \hat{\mu} = \kappa\mu$. By setting κ equal to the market tightness under the old calibration $\kappa = \theta = 0.9618$, the new calibration achieves the desired outcomes.

[6]Regarding the productivity distributions, LS do not correctly implement the quadrature method to compute expectations; nevertheless, the distributions are still proper and the only difference is a level effect. There are advantages of pursuing a resolution to the turbulence puzzle in terms of the published model specification.

Table 1: Calibration

Parameter	Definition	DHHR	LS
Frequency		quarterly	semiquarterly
Preferences			
$\hat{\beta}$	discount factor	0.995	0.99425
ρ^r	retirement probability	0.005	0.0031
$\beta = \hat{\beta}(1-\rho^r)$	adjusted discount	0.990	0.991
Risk sources			
ρ^x	exogenous break probability	0.01	0.005
γ^s	switch probability	0.1	0.05
γ^u	upgrade probability	0.025	0.0125
$\gamma^{d,x}$	turbulence following layoffs	$\in [0,1]$	$\in [0,1]$
γ^d	turbulence following quits	$\in [0,\gamma^{d,x}]$	$\in [0,\gamma^{d,x}]$
Labor market institutions			
π	worker bargaining	0.5	0.5
ϕ	replacement rate	0.7	0.7
Matching			
A	matching efficiency	0.3	0.441
$1-\alpha$	elasticity of job finding	–	0.5
μ	flow cost of a vacancy	–	0.481
Productivity distributions			
Functional form		Uniform	Truncated Normal
$\mathbb{E}[z_l]$	low-skilled mean	1	1
$\mathbb{E}[z_h]$	high-skilled mean	2	2
$supp[z_l]$	low-skilled support	[0.5, 1.5]	[-1, 3]
$supp[z_h]$	high-skilled support	[1.5, 2.5]	[0, 4]
$std[z_l]$	low-skilled dispersion	$\frac{1}{\sqrt{12}}$	1
$std[z_h]$	high-skilled dispersion	$\frac{1}{\sqrt{12}}$	1

3 The Puzzle

Now we document the puzzle: if voluntary quits are exposed to a tiny risk of skill loss, then higher turbulence leads to a reduction in unemployment in the DHHR framework, while it does not alter the positive relationship in the LS framework.

Summary of differences across frameworks Let us recall the differences across frameworks. LS's framework features (a) endogenous matching, (b) immediate consummation of skill upgrades, and (c) Normal productivity distributions with wide support. DHHR's framework features (a) exogenous matching, (b) delayed consummation of skill upgrades, and (c) Uniform productivity distributions with narrow support.

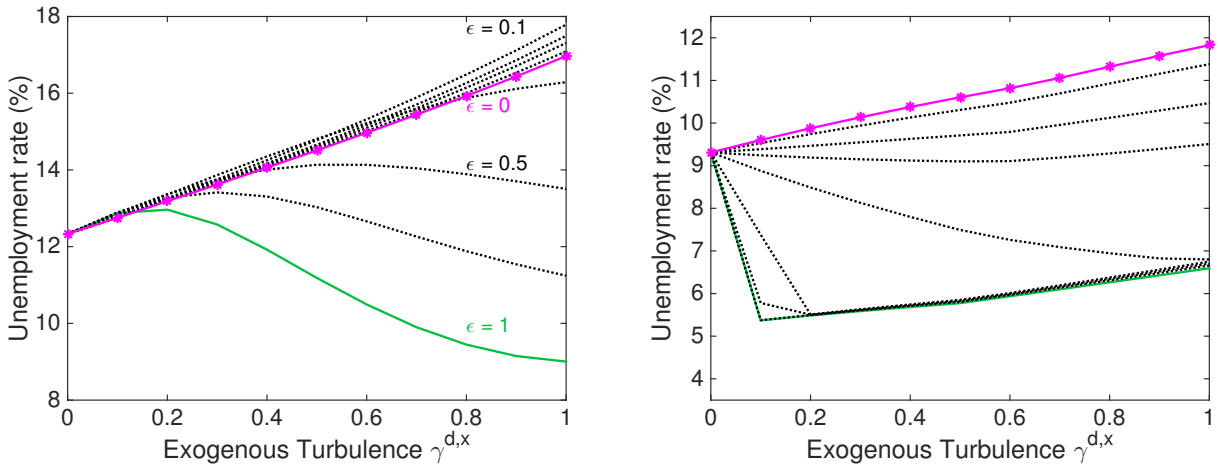
Table 2: Differences across models / calibrations

	LS	**DHHR**
Labor Market		
Tightness	Endogenous	Exogenous
Value of a Firm	Zero (free entry)	Non-zero
Skill Upgrades		
Timing	Immediate	Delayed
Acceptance	Endogenous	Always accept
Productivity Distribution		
Shape	Normal	Uniform
Support	Wide	Narrow

Two frameworks, two opposite conclusions The following figure shows the effect of turbulence, both exogenous and endogenous, on the unemployment rate for the calibration for the welfare state economy in LS and DHHR. The x-axis shows exogenous turbulence $\gamma^{d,x}$ and the y-axis the unemployment rate in %. Each line has a different endogenous turbulence γ^d, represented as a fraction ϵ of exogenous turbulence, i.e. $\gamma^d = \epsilon\gamma^{d,x}$ where $\epsilon \in \{0, 0.01, 0.03, 0.05, 0.1, 0.3, 0.5, 0.7, 1\}$.

Figure 1a shows results for the LS framework. We observe that endogenous turbulence needs to be very high, about 50% of exogenous turbulence, before the aggregate unemployment rate starts varying negatively with turbulence, and in such case, this negative effect only occurs at low levels of exogenous turbulence.

Figure 1: Turbulence and unemployment in LS and DHHR frameworks



(a) Baseline LS

(b) Baseline DHHR

The x-axis shows turbulence following exogenous separations. Each line represents different turbulence following endogenous separations, as a fraction ϵ of turbulence following exogenous separations.

Figure 1b shows the effect of turbulence on the unemployment rate for our baseline DHHR model. This baseline includes two changes to the original DHHR setup that do not alter their results significantly, but allow us to simplify the decomposition of the puzzle. For the first modification, we let new entrants in the labor force be eligible for unemployment benefits equivalent to those of low skilled workers, instead of zero benefits as in the original setup. This minor modification keeps down the number of classes of agents but has hardly any effect on aggregate outcomes. The second modification concerns the risk of losing skills following job market encounters. DHHR assume that after an encounter between a firm and an unemployed worker which does not result in a match being formed, the worker faces the same risk of losing human capital as if she is experiencing an endogenous separation from an ongoing employment relationship. Thus high-skilled unemployed workers are made equal to the high-skilled employed workers. This auxiliary assumption, although made for analytical convenience, does have a quantitatively significant effect. Still, the DHHR puzzle remains intact after this second modification—it just takes some more endogenous turbulence to generate their key findings. See Appendix B for an in-depth discussion of this assumption.

The amount of endogenous turbulence needed in the DHHR framework to revert the unemployment -turbulence relationship is very small. In their original model, the turbulence-unemployment relationship becomes negative at 5% of endogenous turbulence ($\epsilon = 0.05$), and under our baseline model, this risk needs to be 7% ($\epsilon = 0.07$).

3.1 First Candidate: Market Tightness

The first candidate concerns differences in the matching framework. In LS, market tightness is endogenous and determined by the relative number of vacancies posted by firms and the mass of unemployed workers. In contrast, DHHR assumes a fixed and equal mass of workers and firms such that tightness always equals one.

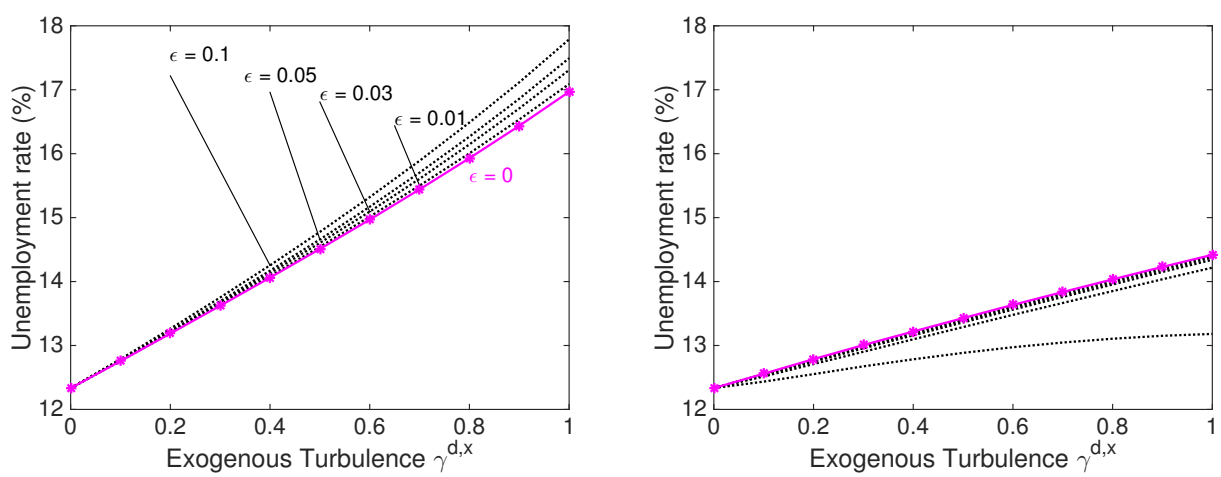
Experiment To implement an exogenous matching protocol within the LS framework, we fix the vacancy creation cost at $\mu = A(1-\pi)\bar{s}(0)$ and fix firms' outside value to zero ($\omega^f = 0$).[7] This experiment could be thought of as a *subsidy to vacancy creation* so that firms break even when maintaining a constant market tightness (or levying a tax if the outside value turns positive for some initial range of turbulence). The subsidy is defined by the following expression

$$1 - subsidy(\gamma^{d,x}) = \frac{A(1-\pi)\bar{s}(\gamma^{d,x})}{\mu} = \frac{\bar{s}(\gamma^{d,x})}{\bar{s}(0)} \tag{35}$$

where $\bar{s}(\gamma^{d,x}) \equiv \sum_{(i,j)} \frac{u_{ij}}{u} \int_{\underline{z}^o_{ij}}^{\infty} s^o_{ij}(y)\ dv_i(y)$ is the expected surplus when turbulence is $\gamma^{d,x}$.

Results We observe an overall suppression of unemployment rates in Figure 2b as compared to Figure 2a, however, the underlying patterns of remain intact and hence, exogenous matching does not explain the DHHR puzzle.

Figure 2: Endogenous vs. exogenous matching in LS



(a) Baseline LS (Endogenous matching)

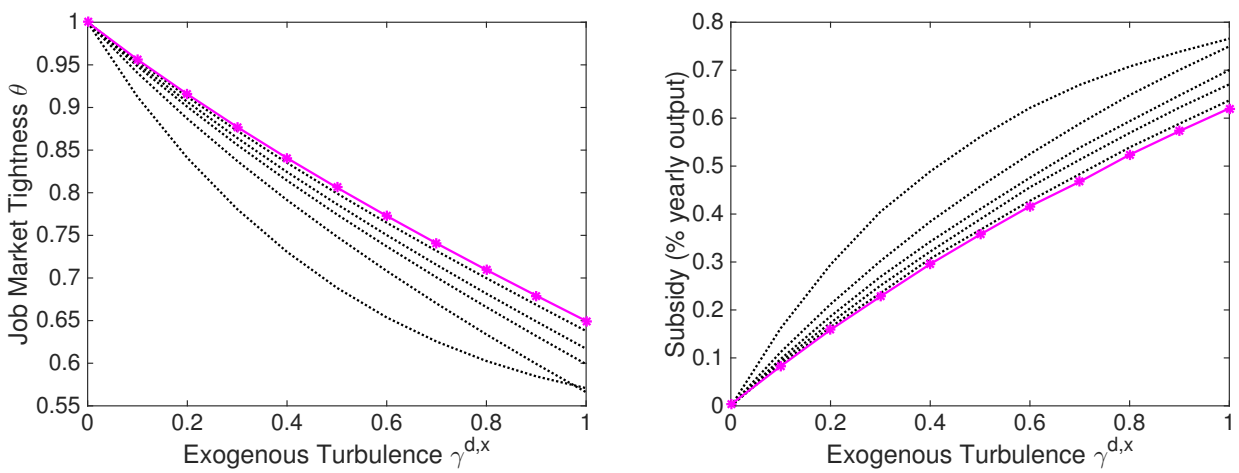
(b) LS + Exogenous Matching

[7] Alternatively, we could allow for endogenous determination of firms' value ω^f. Since turbulence reduces surpluses, ω^f becomes negative for high values of turbulence; thus this experiment could be thought of a *subsidy to firm's profits*. This alternative experiment yields the same conclusions as fixing the firm value to zero.

Discussion: Subsidies to vacancy creation With endogenous matching, there is a dramatic decline in market tightness in response to turbulence as the "invisible hand" struggles to restore the profitability of firms (see Figure 3a), thus vacancy posting and tightness decrease in order to restore firm profitability back to zero. This lower tightness decreases the job finding rate and increases unemployment.

Under the alternative of exogenous matching, the "invisible hand" does not operate as market tightness is constant; thus the profitability of vacancies plummets in response to turbulence. The subsidy to vacancy creation embedded in the exogenous matching framework contributes to lower unemployment rates, and this subsidy grows with turbulence. For this reason, it was not a foregone conclusion whether or not exogenous matching could be written off as the culprit behind the DHHR puzzle. In Figure 3b, the subsidy to vacancy creation is plotted as a fraction of yearly average output per worker in the laissez-faire economy.[8] In all cases shown, the subsidy is very small, lower than 1% of yearly output per worker. This force is not enough for arriving at an explanation to the DHHR puzzle.

Figure 3: Tightness Decrease vs. Subsidy for vacancy creation



(a) Baseline LS (Endogenous matching)

(b) LS + Exogenous Matching

[8]In the laissez-faire economy, average output per worker is 2.3 goods in a semi-quarter or 18.4 goods a year.

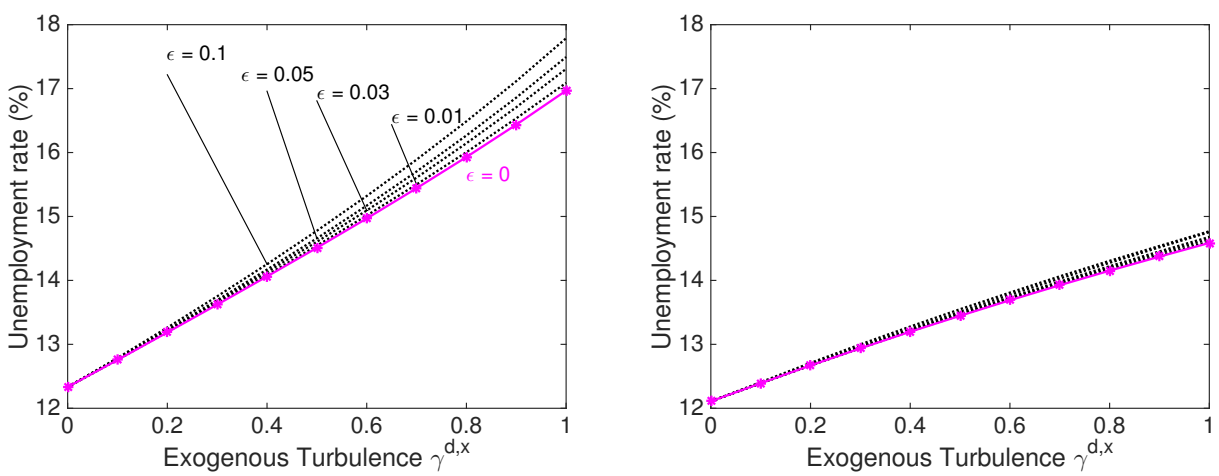
3.2 Second Candidate: Timing of Consummation of Skill Upgrades

The second candidate concerns differences in the timing of consummation of skill upgrades. The LS framework assumes that the skill upgrades are immediately consummated while DHHR assume that a worker who received a positive skill shock at work must remain with the same firm for one period in order to consummate the higher skill level.

Experiment Moving from immediate to delayed consummation of benefits generates new worker types and affects the flow equations between workers. In particular, it has consequential effects for wages. Even though wage determination under Nash bargaining is not relevant for the equilibrium determination, they are a useful tool to shed light on the mechanisms that affect the turbulence-unemployment relationship.

Results Figure 4 show the unemployment response to turbulence for the two cases. The left panel shows the LS original framework. The right panel shows that the delayed timing actually works against the DHHR story, increasing the correlation between unemployment with turbulence. The quantitative outcome in Figure 4b is similar to that of the preceding perturbation in Figure 2b, i.e., an overall suppression in unemployment rates without altering the underlying patterns and hence, cannot be an explanation to the puzzle.

Figure 4: Immediate vs. delayed skill upgrade in LS



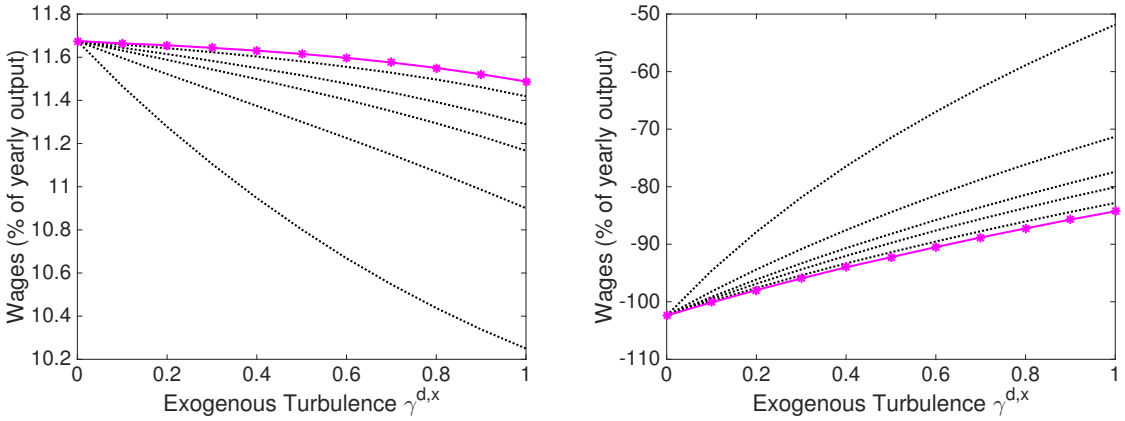
(a) Baseline LS (Immediate Upgrade)

(b) LS + Delayed Upgrade

Discussion: Delayed upgrades require "ransoms" Firms under DHHR's timing assumption are able to "rip off" workers whenever they transition from low to high skill at work, because the consummation of that higher skill is conditional upon a worker remaining with his current employer for at least one more period, in which the firm can "steal" parts of the worker's capital gain.

We compare wages across immediate (Figure 5a) and delayed upgrade of benefits (Figure 5b). It is not only a question of the employer holding the higher benefit hostage but also, and even more importantly, the higher skill level itself with its associated higher labor earnings. It is for those two reasons that the worker is forced to pay the "ransom" in terms of a negative Nash-bargained wage in the period of consummation of a skill upgrade, equivalent to 100% of average yearly output per worker. This "ransom" becomes smaller with higher turbulence since the capital value of a skill upgrade is worth less when it is not expected to last long.

Figure 5: Wages after skill upgrade shock



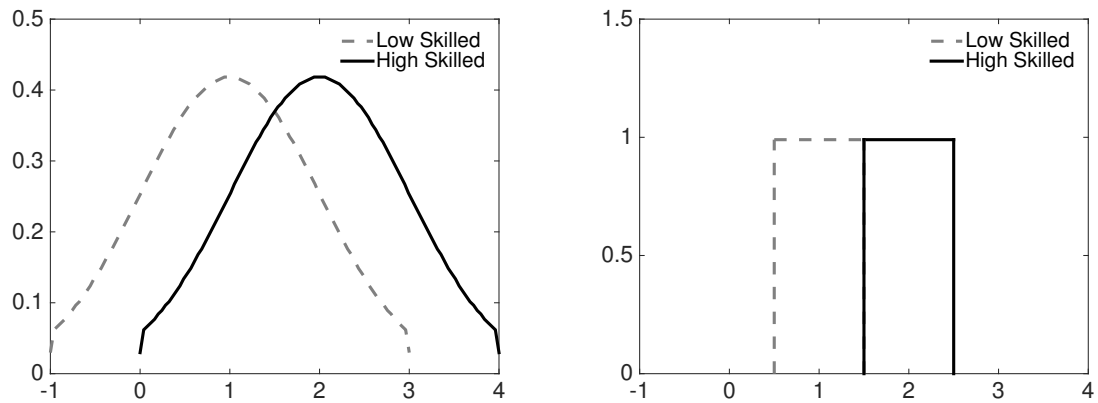
(a) Wages for immediate upgrade

(b) Wages for delayed upgrade

3.3 Third candidate: Productivity Distributions

Experiment We explore differences in the shape and support of the productivity distribution, from LS's truncated Normal distribution with wide support over $[-1, 3]$ and $[0, 4]$ to DHHR's Uniform distribution with narrow support over $[0.5, 1.5]$ and $[1.5, 2.5]$.

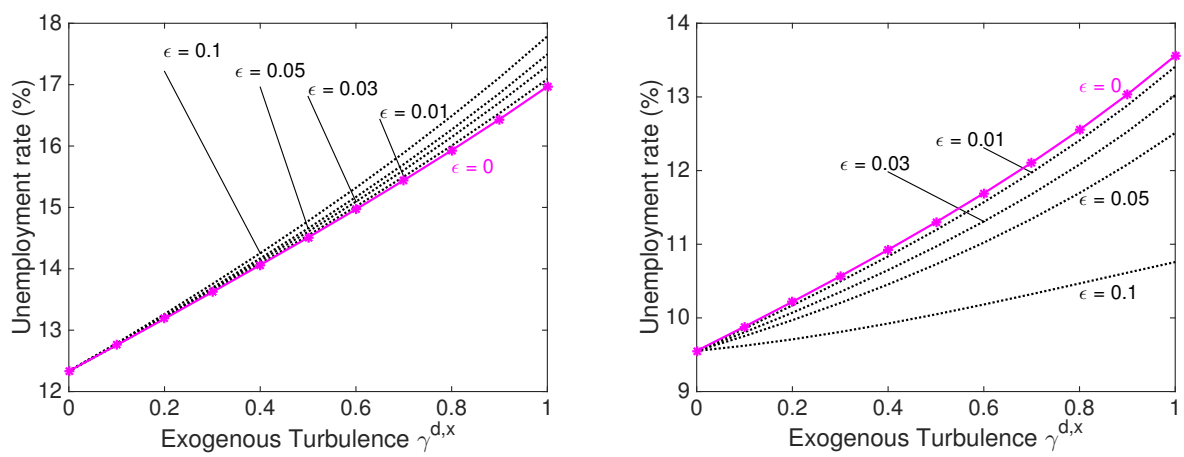
Figure 6: Two alternative productivity distributions



(a) LS (Normal with wide support) (b) DHHR (Uniform with narrow support)

Results Figure 7 shows results for the two alternative productivity distribution. The right panel shows that under the uniform distribution with narrow support, the DHHR force kicks in. In the DHHR calibration, there is a very small implied value of labor mobility in tranquil times, which makes turbulence and unemployment to be negatively related as in DHHR.

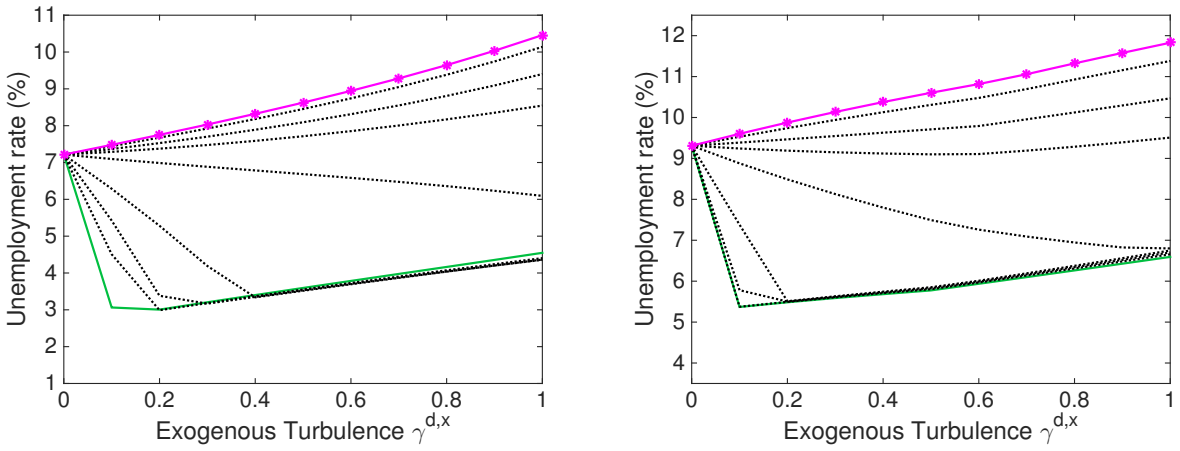
Figure 7: Narrow vs. Wide Support of Productivity Distribution in LS



(a) Baseline LS (Wide Normal) (b) LS + Narrow Uniform

Discussion: Returns to labor mobility Productivity draws on the job create reasons for worker reallocation in order to reproduce observed inflow rates into unemployment, while all prospects for long-run earnings growth are tied to human capital accumulation through learning-by-doing that takes place regardless of firm and current productivity. Our previous results point towards an implied value of mobility under the narrow support that is very low. To further confirm the importance of a narrow productivity support as an explanation of the DHHR puzzle, we offer an additional perturbation and further shrink the support of the Uniform distribution. Figure 8a reproduces the outcomes for the LS framework assuming a Uniform distribution with an even narrowed support of length 0.75. Such a shrinkage of the support takes us very close to the outcomes in our baseline DHHR results, which are reproduced here in Figure 8b for convenience.

Figure 8: Narrow vs. Wide Support of Productivity Distribution in LS



(a) LS + Shrunken Support

(b) Baseline DHHR

4 Layoff tax analysis

We show the fickleness of the DHHR calibration by introducing layoff taxes into both frameworks. We introduce a fixed layoff tax Ω that affects currently employed workers of all skill levels. The layoff tax applies to every endogenous and exogenous separation, except for retirement. When computing wages we will assume that once a worker is hired, firms are the only ones liable for the layoff cost. This generates a two-tier wage system à la Mortensen and Pissarides (1999)[9].

4.1 Layoff taxes in matching models

Ljungqvist (2002) studies the relationship between layoff taxes and unemployment across various frameworks. Regardless of the bargaining protocol, an increase in layoff taxes generates lower reservation productivity and less layoffs, which reduces output per worker; less layoffs but larger total layoff costs as fraction of output are increased; and a reduction in the probability of finding a job. This effect is extremely large in matching models with firm-liable tax.

The effect of layoff taxes on unemployment depends crucially on how the layoff tax enters the bargaining protocol between firms and workers. If layoff costs do not affect the relative share of the match surplus, then layoff taxes decrease unemployment. This is due to a slower reallocation of labor and increase in job tenure. On the contrary, in matching models where the taxes affect bargaining and the relative share of the match surplus, layoff taxes increase unemployment. The main force is the zero profit condition for vacancy creation. Firms surplus is eroded by the layoff tax, and in order to finance vacancy and layoff costs, unemployment increases to push up the vacancy filling rate and lower the bargaining position of workers.

It has been shown in other setups that the timing of the payment of the layoff tax is innocuous. The conjecture is that the wage profile is not important in this setup either, but given that the benefits are computed according to the average wages, the wage profile *might* have an effect on equilibrium outcomes.

[9]The risk neutral firm and worker would be indifferent between adhering to this two-tier wage system or one in which workers receive a fraction π of the match surplus s_{ij} in every period (which would have the worker paying a share π of any future layoff tax). As emphasized by Ljungqvist (2002), the wage profile, not the allocation, is affected by the two-tier wage system. Optimal reservation productivities remain the same. Under the two-tier wage system, a newly hired worker in effect posts a bond that equals his share of the future layoff tax

4.2 Incorporating layoff taxes

The introduction of layoff taxes, denoted with Ω, generates a wedge between the joint surplus of a continuing match and a new match. However, this wedge only affects the reservation productivities for ongoing matches, as these most take into account the layoff tax:

$$s_{ij}(\underline{z}_{ij}) = -\Omega \tag{36}$$

Two-tier wage scheme To implement the layoff tax, we set the firm's threat point to $-\Omega$ in future Nash Bargaining protocols. This introduces a two-tier wage scheme for new entrants and incumbent workers, where wages solve the following problem:

$$\max_{p_{ij}(z)} \left((1-\tau)z - p_{ij}(z) + g^f(z) - \mu - \omega^f + \Omega \mathbb{1}_{\{\text{already working}\}}\right)^{1-\pi} \left(p + g_i^w(z) - b_j - \omega_{ij}^w\right)^{\pi} \tag{37}$$

where the indicator function equals 1 for the currently employed and 0 for newly employed.

Government revenue With the introduction layoff tax, the government's revenue now includes the layoff taxes collected. Let *seps* be total separations—exogenous layoffs and voluntary quits—excluding retirement, which are equal to

$$seps = (1-\rho^r)\Big[\rho^x(e_l + e_h) + (1-\rho^x)[(1-\gamma^u)\gamma^s\nu_{ll} + \gamma^u\nu_{hh}]e_l + (1-\rho^x)\gamma^s\nu_{hh}e_h \tag{38}$$

Then government revenue now equals income taxes plus layoff taxes

$$revenue = \tau(\bar{z}_l e_l + \bar{z}_h e_h) + seps\ \Omega \tag{39}$$

The government adjusts the income tax rate τ to set revenues equal to benefits.[10]

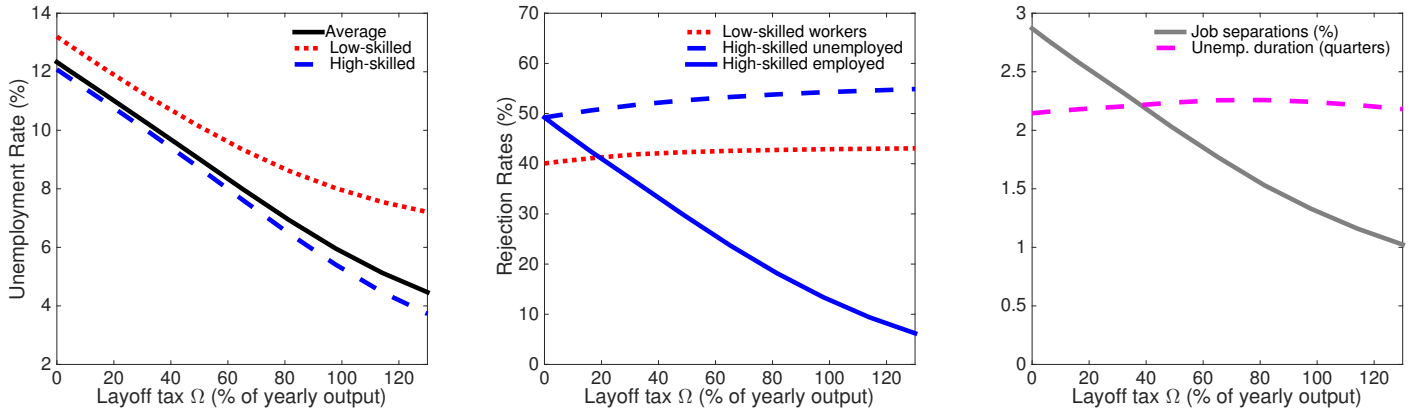
4.3 Layoff taxes in LS

Figure 9 shows the unemployment and rejection rates by type of worker, as well as aggregate labor flows as a function of layoff taxes Ω. Layoff taxes are expressed as a fraction of the average yearly output per worker in the laissez-faire economy. The unemployment rate falls as the layoff tax increases, but the decrease is stronger for the high-skilled unemployed. High-skilled employed workers are particularly affected by the layoff tax as their rejection rates fall dramatically. However, the mobility of these workers remains operative even for very large layoff taxes. For instance, if the layoff tax reaches 100% of their yearly output—an implausible

[10] If layoff costs cover the government expenditure in unemployment benefits, i.e. $seps\ \Omega \geq b_l u_{ll} + b_h(u_{lh} + u_{hh})$, then there is no need for income tax—we do not consider subsidies—and we set $\tau = 0$.

number—high-skill workers still reject about 12% of their job offers. Lastly, the job separation rate falls but unemployment duration remains relatively constant across different layoff taxes.

Figure 9: Layoff Taxes in LS

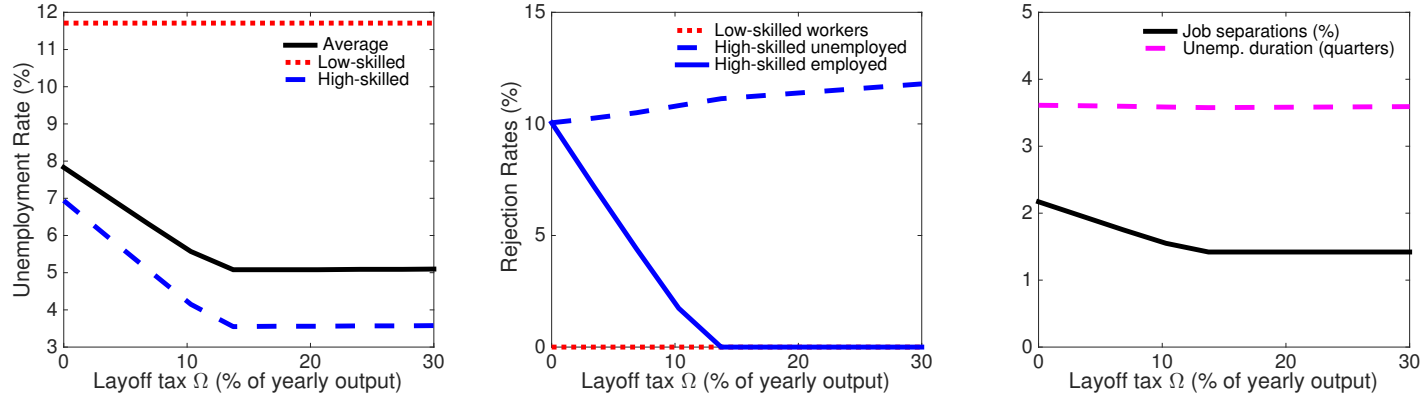


4.4 Layoff taxes in DHHR

We introduce a layoff cost Ω in our baseline DHHR framework with the additional inconsequential change that benefit status is updated immediately (see Appendix B.2 for the irrelevance of this assumption in explaining the disparate results). Figure 10 shows the effect of increasing layoff costs on equilibrium outcomes for the DHHR framework.

Mobility of high-skilled employed completely shuts down at a layoff tax equivalent to 14% of the average yearly output per worker in the laissez-faire economy.[11] After this tiny level of layoff taxes, the rejection rate of these workers becomes zero and separation rates become constant. Any benefits from labor mobility disappear upon adding the small layoff tax.

Figure 10: Layoff Taxes in DHHR



[11] In this economy, the average output is 1.8 goods per worker in a quarter or 7.2 goods per year.

With this exercise we confirm that the DHHR parameterization implies minuscule incentives for labor mobility in tranquil times. A tiny government mandated layoff cost has counterfactually large effects of suppressing unemployment by shutting down voluntary separations. Given this result, it is no surprise that a small cost to mobility, such as a tiny exposure to turbulence following endogenous separations, generated lower unemployment and reversed the LS mechanism.

5 Conclusion

We have shown that the negative turbulence-unemployment relationship is robust to introducing the risk of losing human capital for workers that quit.

Qualitatively speaking, candidate one (exogenous matching) and candidate two (delayed consummation of skill upgrades) are similar in the sense of exerting a general suppression of unemployment, but without changing the general pattern of the turbulence-unemployment relationship. Therefore, it comes down to a rather "plain vanilla" explanation that the DHHR calibration of productivity distributions lacks sufficient returns for high-skilled workers to reallocate. As a result, any further small deterioration in those returns threatens to close down all mobility of high-skilled workers such as endogenous turbulence that subjects job movers to small probabilities of losing skills, or alternatively, the imposition of rather small layoff taxes.

We conclude that, once the parameterization is adjusted to account for historical observations on unemployment and layoff costs, the positive relationship between turbulence and unemployment reemerges. The job rejection channel emphasized by LS is not outweighed quantitatively by the rise in incentives to preserve jobs, even if workers face a large probability of skill loss following an endogenous separation. The LS turbulence story indeed holds up when the plausible responses of employed workers are taken into account.

References

den Haan, W., Haefke, C. and Ramey, G. (2005). Turbulence and unemployment in a job matching model. *Journal of the European Economic Association*, **3** (6), 1360–1385.

Hall, R. E. and Milgrom, P. R. (2008). The limited influence of unemployment on the wage bargain. *The American Economic Review*, **98** (4), 1653–1674.

Hornstein, A., Krusell, P. and Violante, G. L. (2005). The effects of technical change on labor market inequalities. *Handbook of Economic Growth*, **1**, 1275 – 1370.

Ljungqvist, L. (2002). How do lay-off costs affect employment? *The Economic Journal*, **112** (482), 829–853.

— and Sargent, T. J. (1998). The european unemployment dilemma. *Journal of political Economy*, **106** (3), 514–550.

— and — (2007). Understanding european unemployment with matching and search-island models. *Journal of Monetary Economics*, **54** (8), 2139–2179.

— and — (2017). The fundamental surplus. *American Economic Review*.

A Computation of equilibrium

A.1 General algorithm structure

The algorithm computed the equilibrium of the model. It approximates continuation values of joint surpluses $g_i(z)$ using a linear projection on a productivity grid.

1. Fix a calibration and construct productivity distributions over a grid of size N_z.

2. Guess initial values for:

   - c_i^k: coefficients for g linear approximation $\hat{g}_i(z) = c_i^0 + c_i^1 z$
   - b_j: unemployment benefits
   - ω_{ij}^w: outside value of workers after benefits
   - ω^f: firms' outside option (in LS, $\omega^f = 0$)
   - τ: tax rate
   - u_{ij}, e_{ij}: masses of unemployed and employed workers

3. Using g's linear approximation, compute thresholds $\underline{z}_{ij}$ such that $s_{ij}(\underline{z}_{ij}) = 0$ for continuing relationships, $s_{ij}^0(\underline{z}_{ij}) = 0$ for new relationships, or $s_{ij}(\underline{z}_{ij}) = -\Omega$ with layoff taxes.

$$\underline{z}_{ij} = \frac{b_j + \omega_{ij}^w - c_j^0}{(1 - \tau + c_j^1)}$$

   - Set $\tau = 0$ if $b_j + \omega_{ij}^w - c_j^0 < 0$.
   - When the cutoff falls out of the productivity support ($\underline{z}_{ij} < z_{min}$ or $\underline{z}_{ij} > z_{max}$), then evaluate $\hat{g}(z_{min})$ or $\hat{g}(z_{max})$ respectively and solve as:

$$\underline{z}_{ij} = \frac{b_j + \omega_{ij}^w - c_j^0 - c_j^1 z_{min}}{(1 - \tau)} \qquad \text{or} \qquad \underline{z}_{ij} = \frac{b_j + \omega_{ij}^w - c_j^0 - c_j^1 z_{max}}{(1 - \tau)}$$

   and set $\tau = 0$ if $b_j + \omega_{ij}^w - c_j^0 - c_j^1 z_{min} < 0$ or $b_j + \omega_{ij}^w - c_j^0 - c_j^1 z_{max} < 0$

4. Given cutoffs $\underline{z}_{ij}$, compute rejection probabilities ν_{ij} using (6) and expected continuation values E_{ij} using (7).

5. Compute expected surpluses

$$\bar{s} \equiv \sum_{(i,j)} \left[\frac{u_{ij}}{u} \int_{\underline{z}_{ij}}^{\infty} s_{ij}^o(y) \; dv_i(y) \right]$$

6. Compute exact joint future values $g_i(z)$ using formulas (8) and (9). Then update coefficients c_i by regressing $g_i(z)$ on $[1 \;\; z \;\; z^2]$. See Appendix A.3 for more details on g and its approximation.

7. Update firm value, market tightness and finding rates

   - In LS's endogenous matching

$$w^f = 0, \qquad \theta = \left(\frac{A(1-\pi)\bar{s}}{\mu}\right)^{1/\alpha}, \qquad \lambda^w(\theta) = A\theta^{1-\alpha}, \qquad \lambda^f_{ij}(\theta) = A\theta^{-\alpha}\frac{u_{ij}}{u}$$

   - In DHHR's exogenous matching

$$\omega^f = \frac{\beta}{1-\beta}A(1-\pi)\bar{s}, \qquad \theta = 1, \qquad \lambda^w = A, \qquad \lambda^f_{ij} = A\frac{u_{ij}}{u}$$

8. Update unemployment values ω^w_{ij} using formulas in (11) and (12).

9. Compute net changes in worker flows (all must be zero in steady state)

$$\begin{aligned}
\Delta u_{ll} &= \rho^r + (1-\rho^r)\left\{\rho^x + (1-\rho^x)(1-\gamma^u)\gamma^s\nu_{ll}\right\} e_{ll} \\
&- \rho^r u_{ll} - (1-\rho^r)\lambda(1-\nu_{ll})u_{ll}
\end{aligned}$$

$$\begin{aligned}
\Delta u_{lh} &= (1-\rho^r)\left\{\rho^x\gamma^{d,x}e_{hh} + (1-\rho^x)\nu_{hh}\gamma^d(\gamma^s e_{hh} + \gamma^u e_{ll}) + \lambda\nu^o_{hh}\gamma^e u_{hh}\right\} \\
&- \rho^r u_{lh} - (1-\rho^r)\lambda(1-\nu_{lh})u_{lh}
\end{aligned}$$

$$\begin{aligned}
\Delta u_{hh} &= (1-\rho^r)\left\{\rho^x(1-\gamma^{d,x})e_{hh} + (1-\rho^x)\nu_{hh}(1-\gamma^d)(\gamma^s e_{hh} + \gamma^u e_{ll})\right\} \\
&- \rho^r u_{hh} - (1-\rho^r)\lambda(1-\nu^o_{hh} + \nu^o_{hh}\gamma^e)u_{hh}
\end{aligned}$$

$$\begin{aligned}
\Delta e_{ll} &= (1-\rho^r)\lambda\left\{(1-\nu_{ll})u_{ll} + (1-\nu_{lh})u_{lh}\right\} \\
&- \rho^r e_{ll} - (1-\rho^r)[\rho^x + (1-\rho^x)(\gamma^u + (1-\gamma^u)\gamma^s\nu_{ll}]e_{ll}
\end{aligned}$$

$$\begin{aligned}
\Delta e_{hh} &= (1-\rho^r)\left\{\lambda(1-\nu^o_{hh})u_{hh} + (1-\rho^x)\gamma^u(1-\nu_{hh})e_{ll}\right\} \\
&- \rho^r e_{hh} - (1-\rho^r)[\rho^x + (1-\rho^x)\gamma^s\nu_{hh}]e_{hh}
\end{aligned}$$

10. Given masses of workers, compute average wages for each skill level j using (25). With layoff taxes, we take into account the two-tier wage schedule that distinguishes between wages of new or continuing workers.

11. Compute benefits b_j as a fraction ϕ of average wages within skill level j at the past job; this determines government expenditures.

12. Compute average productivity for each skill level i using (27) to determine government's tax income. With layoff taxes, also include severance payments from separations.

13. Adjust tax rate τ to balance budget in (29).

14. Check convergence of a set of moments. If convergence if achieved, stop. If convergence is not achieved, go to 2 and use as guesses the last values computed.

A.2 Wage groups

Table A.1: Wage groups

Immediate	**Delayed**	**Skill**	**Benefits**	**Description**
e_{ll}	e_{ll}	Low skill	Low benefits	Old workers and Newly hired from u_{ll}
e^0_{lh}	e^*_{ll}	Low skill	Depends	Newly hired from u_{lh}
e_{lh}	e_{lh}	Low skill	Low benefits	Have not switched productivity
e^0_{hh}	e_{hh}	High skill	High benefits	Newly hired from u_{hh}
e^1_{hh}	e^1_{hh}	High skill	High benefits	Have not switched productivity
e_{hh}	e^*_{hh}	High skill	Depends	Upgraded
e_{hh}	e_{hh}	High skill	High benefits	Old workers

For computing wages in DHHR, we must take into account their timing assumption in which workers need to work one period with the firm in order to upgrade their benefits. We add two new groups of workers for wage computation: high-skilled unemployed that are newly hired e^0_{hh}, and high-skilled employed workers who have not yet switched productivity e^1_{hh} and thus their reservation productivity is still that of a newly hired. Recall that in this setup we already considered two special groups of employees for wage calculations: e^*_{hh}, low skilled workers that had an upgrade but must stay with the firm one period to consolidate their new skills; and e^*_{ll}, low skilled workers with high benefits that just got hired, and their benefits have not been downgraded.

A.3 More details on g and its approximation

The joint continuation value of a relationship with a low skilled worker is given by[12]:

$$
\begin{aligned}
g_l(z) &= \underbrace{\left[\frac{\beta(1-\rho^x)(1-\gamma^u)(1-\gamma^s)}{1-\beta(1-\rho^x)(1-\gamma^u)(1-\gamma^s)}\right](1-\tau)\,z}_{\text{slope}_l} \\
&+ \frac{\beta(1-\rho^x)\left[(1-\gamma^u)\gamma^s(E_{ll}+\nu_{ll}(b_l+w_{ll})+\gamma^u(E_{hh}+\nu_{hh}(b_h+(1-\gamma^d)w_{hh}+\gamma^d w_{lh}))\right]}{1-\beta(1-\rho^x)(1-\gamma^u)(1-\gamma^s)} \\
&+ \frac{\beta\rho^x(b_l+w_{ll})}{1-\beta(1-\rho^x)(1-\gamma^u)(1-\gamma^s)}
\end{aligned}
$$

where $E_{ll} = \int_{\underline{z}_{ll}}^{\infty}(1-\tau)y + g_l(y)\ dv_l(y)$ and $E_{hh} = \int_{\underline{z}_{hh}}^{\infty}(1-\tau)y + g_h(y)\ dv_h(y)$.

The joint continuation value of a relationship with a high skilled worker is given by:

$$
\begin{aligned}
g_h(z) &= \underbrace{\left[\frac{\beta(1-\rho^x)(1-\gamma^s)}{1-\beta(1-\rho^x)(1-\gamma^s)}\right](1-\tau)\,z}_{\text{slope}_h} \\
&+ \frac{\beta(1-\rho^x)\gamma^s\left[E_{hh}+\nu_{hh}(b_h+(1-\gamma^d)w_{hh}+\gamma^d w_{lh})\right]}{1-\beta(1-\rho^x)(1-\gamma^s)} + \frac{\beta\rho^x(b_h+(1-\gamma^{d,x})w_{hh}+\gamma^{d,x}w_{lh})}{1-\beta(1-\rho^x)(1-\gamma^s)}
\end{aligned}
$$

Some observations:

1. The slope coefficient on productivity represents the extra value that is to be had from accepting the present productivity that is expected to last for some given duration (that depends on the probability of drawing a new productivity at work γ^s, the probability of upgrade γ^u as well as the probability for being exogenously laid off ρ^x).

2. In the calibration used, the difference in slopes between skills is only given by the upgrade probability: the slope for high skilled is equal to the slope of low skilled setting $\gamma^u = 0$. Since $\frac{\partial slope_l}{\partial \gamma^u} < 0$, then $\forall\ \gamma^u > 0$ we have $slope_h > slope_l$.

3. Turbulence does not affect the slope directly, only through its effect on the tax rate.

4. The intercept represents the capital value of a worker of a particular skill, not including the described extra value associated with the present productivity draw.

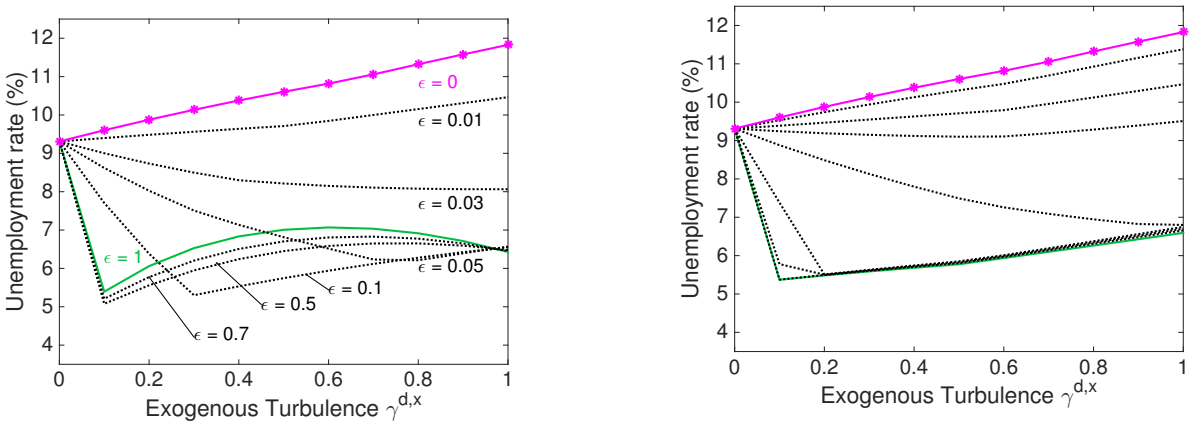
[12] $\beta = \hat{\beta}(1-\rho^r)$ includes the retirement probability.

B Starting from DHHR framework

We now work in reverse the perturbations, starting from the DHHR framework. The original DHHR framework features (i) exogenous labor market tightness, (ii) delayed consummation of skill upgrade, a (iii) uniform productivity distribution with narrow support.

Simplification: Eliminate turbulence for unemployed DHHR assume that after an encounter between a firm and an unemployed worker which does not result in a match being formed, the worker faces the same risk of losing human capital as if she is experiencing an endogenous separation from an ongoing employment relationship. They claim that this assumption is only for simplification as it allows to reduce the number of workers to keep track off. Indeed, within their model and given their parametrization, this assumption is innocuous. In order to clearly convey the message, we first modify the original DHHR framework and explore the alternative in which mere encounters of firms and workers in the labor market would not expose the latter ones the risk of losing skills. Figure B.1 presents results for the original DHHR framework with $\gamma^e = \gamma^d$ and our modified version with $\gamma^e = 0$.

Figure B.1: Turbulence risk for the unemployed in DHHR



(a) Original DHHR (Turbulence for unempl.)

(b) Baseline DHHR (No turbulence for unempl.)

Clearly, DHHR unintended inclusion of additional exposure to endogenous turbulence by assuming that mere encounters between vacancies and unemployed workers are associated with the risk of losing human capital, will dampen rewards to labor mobility of high-skilled workers. But coupled with the compressed productivity distributions in DHHR, there is not much added harm from such a suppression. In contrast, the significant reasons for labor mobility coming from LS's choice of productivity distributions, are indeed severely impacted and suppressed by that auxiliary assumption of DHHR, as discussed in detail in Appendix C.

B.1 First candidate: Market Tightness

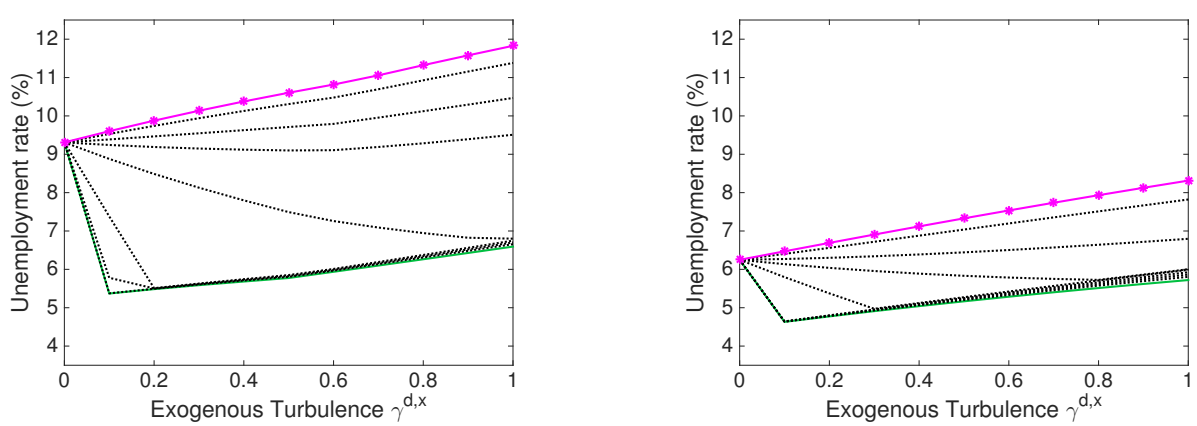
Experiment In the DHHR framework, the value of the firm does not take into account the vacancy creation cost μ and thus its value is always non-negative. In order to implement the endogenous matching framework in which firms post vacancies, we first change the value of the firm to include a vacancy creation cost.

$$w^f = -\mu + A\beta(1-\pi)\bar{s}(\gamma^{d,x}) + \beta w^f$$

Secondly, we calibrate μ such that under exogenous matching, the value of the firm is equal to zero at zero turbulence, i.e. $w^f = 0$ if $\gamma^{d,x} = 0$, which yields $\mu = A\beta(1-\pi)\bar{s}(0) = 0.617$ and fix this value of vacancy creation costs. In this way, the value of the firm is zero and other matching outcomes are equal under both matching assumptions at zero turbulence.

Results Figure B.2 shows the unemployment-turbulence relationship for the DHHR framework under the two alternative matching frameworks. In both cases, a tiny exposure to endogenous turbulence reduces unemployment significantly. Even more, we find that the effect is stronger under endogenous matching.

Figure B.2: Exogenous vs. Endogenous Matching in DHHR



(a) Baseline DHHR (Exogenous matching)

(b) DHHR + Endogenous matching

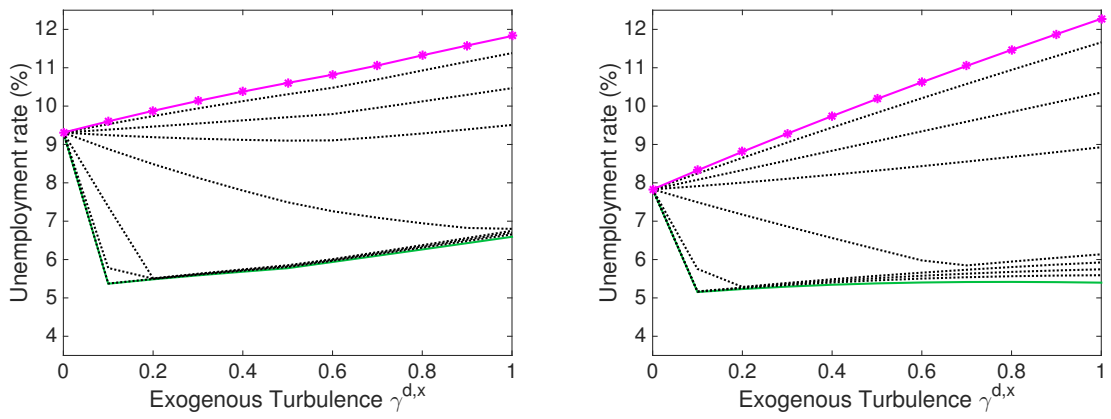
Note that the level of the unemployment rate differs at zero turbulence across the two matching frameworks. The reason is that with endogenous matching the value of the firm is equal to zero, whereas in the exogenous matching the value of the firm is large and positive, reducing the surplus, changing productivity thresholds, rejection rates, and other endogenous objects. We conclude, as before, that the type of matching is irrelevant for the disparate outcomes across frameworks.

B.2 Second candidate: Timing of Consummation of Skill Upgrades

DHHR assume that after a skill upgrade, one period has to pass before workers are entitled to high benefits. In this section we introduce immediate benefit upgrade as in LS. This change reduces the number of workers that need to be tracked.

Results Figure B.3 shows that there is no big difference between the timing in the DHHR. Recall the ransom interpretation in Section 3.2, that showed how wages became negative for recently upgraded high-skilled workers as the firms extract the surplus. In this case, such a force is not present. Still, it is not quantitatively important to revert the turbulence-unemployment relationship.

Figure B.3: Immediate vs. Delayed Consummation of Skills in DHHR



(a) Baseline DHHR (Delayed upgrade)

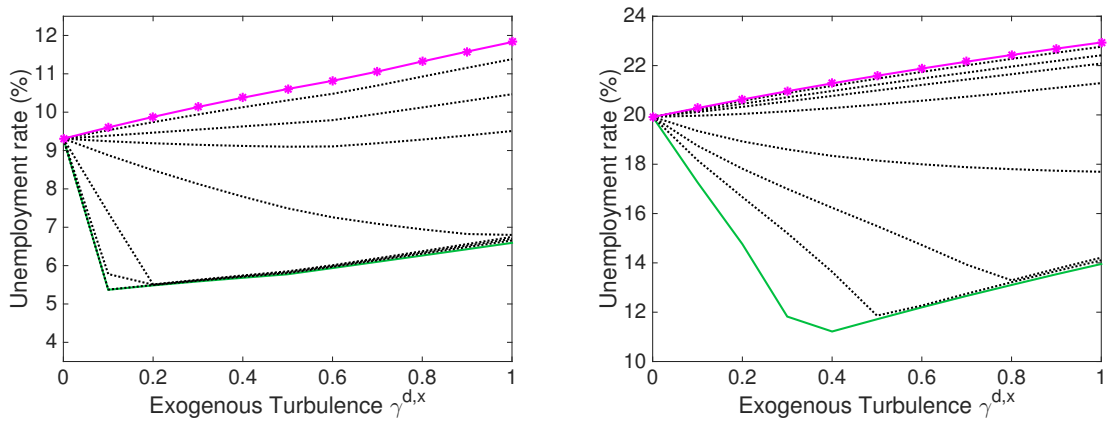
(b) DHHR + Immediate upgrade

B.3 Third candidate: Productivity distributions

DHHR assumes uniform distributions with narrow support $z_l \sim \mathcal{U}([0.5, 1.5])$ and $z_h \sim \mathcal{U}([1.5, 2.5])$. In contrast, LS assume productivities are drawn from truncated normals $z_l \sim \mathcal{N}(1, 1)$ for low skilled workers over the support $[-1, 3]$ and $z_h \sim \mathcal{N}(2, 1)$ for high skilled workers over the support $[0, 4]$. In both frameworks, the means are 1 for low skilled and 2 for high skilled; the dispersion is 1 in the Normal case and $1/\sqrt{12}$ in the Uniform case.

Results Figure B.4 shows the results for the original calibration of the productivity distribution and the alternative with wider support. Note that the unemployment rate is much higher under the new calibration. Under the wider support, it takes much higher turbulence to generate a negative relationship compared to the narrower support. Turbulence and unemployment are positive related until endogenous turbulence reaches about 30% of exogenous turbulence when the relationship is slightly negative for small values of turbulence.

Figure B.4: Turbulence in the DHHR model with alternative productivity distributions



(a) Baseline DHHR (Narrow Uniform)

(b) DHHR + Wide Normal

C Turbulence affecting job market encounters

DHHR assume that after an encounter between a firm and an unemployed worker which does not result in a match being formed, the worker faces the same risk of losing human capital as if she is experiencing an endogenous separation from an ongoing employment relationship. They claim that this assumption is only for simplification as it allows to reduce the number of workers to keep track off. This assumption works on their favor. We explore the alternative in which mere encounters of firms and workers in the labor market would expose the latter ones to a different (zero) risk of losing skills. This modified model considers differences in the rates at which the high-skilled employed and the high-skilled unemployed suffer skill obsolescence, either after voluntary quits γ^d or mere job market encounters γ^e. During unemployment, high-skilled workers that reject an offer after a job market encounter with a firm, are subject to the risk of losing their skills with probability γ^e.

C.1 Introducing turbulence risk for the unemployed in LS

To compute the surplus for relationships with high-skilled workers, we must differentiate between new and continuing relationships, because of the different probabilities of facing endogenous turbulence risk. The surplus for a new relationship with a currently unemployed high-skilled worker, denoted by s^o_{hh}, considers outside options that are affected by turbulence upon labor market encounters at rate γ^e, as follows

$$s^o_{hh}(z) = (1-\tau)z + g_h(z) - [b_h - \mu + (1-\gamma^e)\omega_{hh} + \gamma^e\omega_{lh}] \tag{C.1}$$

In contrast, the surplus for a continuing relationship with a high-skilled worker $s_{hh}(z)$, or a new relationship with a low-skilled worker that receives a skill upgrade that is immediately consummated, is affected by turbulence at rate γ^d, therefore

$$s_{hh}(z) = (1-\tau)z + g_h(z) - [b_h - \mu + (1-\gamma^d)\omega_{hh} + \gamma^d\omega_{lh}] \tag{C.2}$$

Endogenous turbulence for high-skilled unemployed u_{ll} activates with probability γ^e after an unsuccessful job market encounter (we denote with ν^o_{hh} the rejection rate of high-skilled unemployed workers).

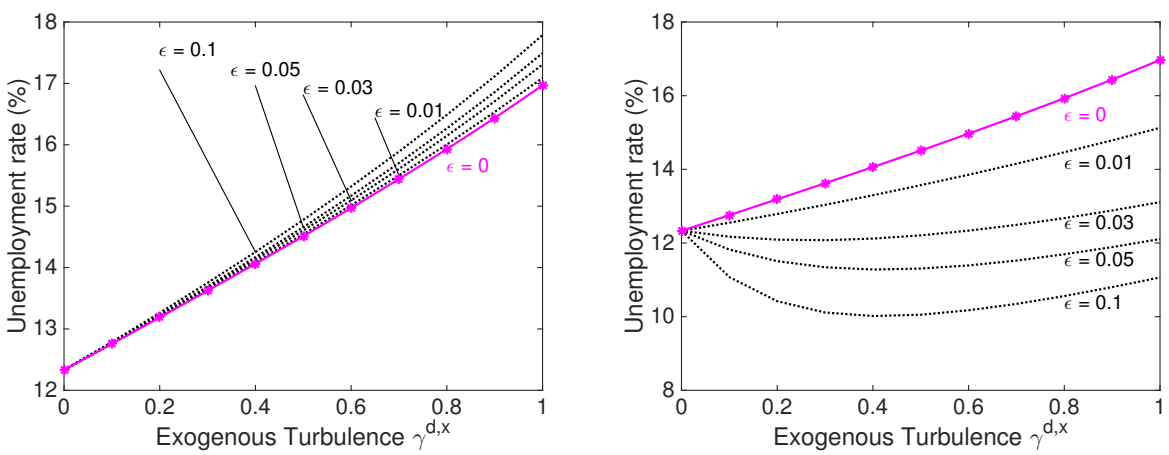
$$\begin{aligned}\Delta u_{lh} &= -\rho^r u_{lh} - (1-\rho^r)\underbrace{\lambda^w(\theta)(1-\nu_{lh})u_{lh}}_{\text{1. succesful match}} \\ &+ (1-\rho^r)\Big\{\underbrace{\rho^x\gamma^{d,x}e_{hh}}_{\text{2. exogenous turbulence}} + \underbrace{(1-\rho^x)\gamma^d\nu_{hh}[\gamma^s e_{hh} + \gamma^u(e_{ll}+e_{lh})]}_{\text{3. end. turb. employed}} + \underbrace{\lambda^w(\theta)\gamma^e\nu^o_{hh}u_{hh}}_{\text{4. end. turb. unemployed}}\Big\}\end{aligned} \tag{C.3}$$

The following results show different outcomes for two cases: $\gamma^e = 0$, which means that job market encounters do not pose any risk for unemployed workers with high skills; and $\gamma^e = \gamma^d$, which means that high-skilled unemployed workers that reject a job offer face the same risk of skill loss that an employed worker with high skills.

Results Figure C.5 plots the unemployment-turbulence relationship in the LS framework for the two extreme cases, $\gamma^e = 0$ and $\gamma^e = \gamma^d$. We find that adding the exposure of the unemployed to turbulence through job market encounters has a sizeable effect on unemployment. Therefore, the "simplifying" assumption by DHHR is actually not innocuous. An unemployed high-skilled worker faces additional significant risk of skill loss after each period of unemployment.

Using the metric of equilibrium outcomes at zero turbulence in the LS analysis, an unemployed worker has a match probability of around 0.5 at a semi-quarterly frequency and a rejection rate of around 0.5 (implying an average unemployment duration of 4 model periods, i.e., two quarters). Thus, an unemployed high-skilled worker faces a probability $0.5 * 0.5 = 0.25$ of being exposed to endogenous turbulence after each period of unemployment. And this skill-loss risk is in addition to the exposure to endogenous turbulence at the separation itself. No wonder that unemployment falls-off sharply in the second panel below.

Figure C.5: Turbulence and unemployment in LS with turbulence for the unemployed



(a) Baseline LS (No turbulence for unempl.)

(b) LS + Turbulence for unempl.

How come that tiny risk of skill loss at voluntary separations—endogenous turbulence—does not overturn LS's positive relationship between turbulence and unemployment, but that extending those tiny risks of skill losses to encounters between vacancies and the unemployed, has dramatic effects of reducing unemployment? As long as the average number of encounters needed to form a match is pretty small, one would expect that those tiny risks of skill loss

would not overturn LS's relationship. The next section decomposes the effect of turbulence during job market encounters into an allocation and a bargaining channel to shed light on the forces behind this striking result.

C.2 Allocation vs. Bargaining Channels

We seek to decompose two forces from DHHR's less than innocuous assumption that unemployed high-skilled workers are subject to the risk of losing skills after unsuccessful job market encounters:

(i) allocation channel: the mere risk of losing skills incurs a cost on the unemployed and hence, they are more prone to transition into employment, and

(ii) bargaining channel; that risk associated with unsuccessful job market encounters worsen a worker's bargaining power vis-à-vis a potential employer.

How to decompose those two effects with all the intricate general-equilibrium forces in a matching model? Our attempt below to an illustrative decomposition introduces an exogenous length of time (horizon) under which an unemployed is exposed to the risk of skill loss after unsuccessful job market encounters, and after that horizon, there is no risk of skill loss for the remaining duration of that unemployment spell. When successively increasing that horizon, we find that even after a vast majority of initially high-skilled unemployed have left unemployment, the equilibrium unemployment rate continues to decrease significantly in response to additional increases in that parameterization of the horizon.

Limit skill loss during unemployment We study what happens if turbulence following job market encounters γ^e is limited to the first $\bar{k}$ periods of unemployment i.e., after those first $\bar{k}$ periods there is no longer any risk of skill loss while unemployed. Given a number of unemployment periods $\bar{k}$ where turbulence risk γ^e is active, we consider a vector $\mathbf{u_{hh}}$ consisting of $\bar{k}+1$ categories of high skilled unemployed workers, where each category k contains high-skilled unemployed workers with unemployment duration of $k = 0, \ldots, \bar{k}$ periods. There is an absorbing category $u_{hh}^{\bar{k}}$ which includes all the u_{hh} for which the risk has disappeared, i.e. $k \geq \bar{k}$.

Notation We denote with a superscript k the variables related to high-skilled unemployed workers: masses u_{hh}^k, surpluses $s_{hh}^k(z)$, reservation productivities $\underline{z}_{hh}^k$, rejection rates ν_{hh}^k, unemployment values ω_{hh}^k, and wages $p_{hh}^k(z)$.

Laws of motion The law of motion for the category u_{hh}^k workers is as follows. The first stage u_{hh}^0 receives the new unemployed from the employed pool, and then all these workers leave next

period to the following category u^1_{hh}: 1) either they accept a match and move to e_{hh}; 2) they have a turbulent rejection and move to u_{lh}; or 3) they have a non-turbulence rejection and move to the following category $k+1$. The subsequent categories u^k_{hh}, for $k \leq \bar{k}-1$, receive the inflow of unemployed from the previous category u^{k-1}_{hh} that did not match and those that had non-turbulent rejected matches, and have a complete outflow of workers for the same three reasons above.

$$\Delta u^k_{hh} = \begin{cases} (1-\rho^r)\Big[\underbrace{\rho^x(1-\gamma^{d,x})e_{hh}}_{\text{non-turbulent exo separation}} + \underbrace{(1-\rho^x)\nu_{hh}(1-\gamma^d)(\gamma^s e_{hh}+\gamma^u e_{ll})}_{\text{non-turbulent endo separation}}\Big] - u^k_{hh} & \text{if } k=0 \\ (1-\rho^r)\Big[\underbrace{(1-\lambda)}_{\text{no match}} + \underbrace{\lambda\nu^{k-1}_{hh}(1-\gamma^e)}_{\text{non-turbulent rejected match}}\Big]u^{k-1}_{hh} - u^k_{hh} & \text{if } k \leq \bar{k}-1 \end{cases}$$

The absorbing category $\bar{k}$ gets the inflow from $\bar{k}-1$, but now the outflow is not complete. The only workers that leave are the retirees and those with accepted matches (those with rejected matches are no longer affected by turbulence and thus always remain):

$$\Delta u^{\bar{k}}_{hh} = (1-\rho^r)\Big[(1-\lambda)+\lambda\nu^{\bar{k}-1}_{hh}(1-\gamma^e)\Big]u^{\bar{k}-1}_{hh} - \Big[\rho^r+(1-\rho^r)\lambda(1-\nu^{\bar{k}}_{hh})\Big]u^{\bar{k}}_{hh}$$

The law of motion for the u_{lh} workers is modified to receive the inflow from the different u^k_{hh} categories that suffered turbulent rejections in their first $\bar{k}-1$ periods of unemployment.

$$\begin{aligned} \Delta u_{lh} &= (1-\rho^r)\Big[\underbrace{\rho^x\gamma^{d,x}e_{hh}+(1-\rho^x)\nu_{hh}\gamma^d(\gamma^s e_{hh}+\gamma^u e_{ll})}_{\text{turbulent separations}} + \underbrace{\lambda\gamma^e\sum_{k=0}^{\bar{k}-1}\nu^k_{hh}u^k_{hh}}_{\text{turbulent rejections}}\Big] \\ &- [\rho^r+(1-\rho^r)\lambda(1-\nu_{lh})]\,u_{lh} \end{aligned}$$

The law of motion for the high-skilled employed e_{hh} is adjusted as

$$\begin{aligned} \Delta e_{hh} &= (1-\rho^r)\Big[\underbrace{\lambda\sum_{k=0}^{\bar{k}}(1-\nu^k_{hh})u^k_{hh}}_{\text{accepted new matches}} + \underbrace{(1-\rho^x)\gamma^u(1-\nu_{hh})e_{ll}}_{\text{accepted upgrades}}\Big] \\ &- [\rho^r+(1-\rho^r)(\rho^x+(1-\rho^x)\gamma^s\nu_{hh})]\,e_{hh} \end{aligned}$$

Surplus, value of unemployment, and wages for high-skilled unemployed For all $k \leq \bar{k}$, the surpluses of high-skilled unemployed workers include the outside option with risk

γ^e, whereas after $\bar{k}$ periods the risk has disappeared and no longer affects the outside option.

$$s_{hh}^k(z) = \begin{cases} (1-\tau)z + g_h(z) - \left[b_h + (1-\gamma^e)\omega_{hh}^{w,k} + \gamma^e \omega_{lh}^w - \mu + \omega^f\right] & \text{if } k \leq \bar{k} - 1 \\ \\ (1-\tau)z + g_h(z) - \left[b_h + \omega_{hh}^{w,k} - \mu + \omega^f\right] & \text{if } k = \bar{k} \end{cases}$$

Reservation productivities, rejection rates, and expected surpluses are computed as:

$$\underline{z}_{hh}^k = {s_{hh}^k}^{-1}(0) \qquad \nu_{hh}^k = \int_{-\infty}^{\underline{z}_{hh}^k} dv_h(y) \qquad E_{hh}^k \equiv \int_{\underline{z}_{hh}^k}^{\infty} [(1-\tau)y + g_h(y)]\, dv_h(y)$$

The value of unemployment for a high-skilled unemployed worker with k periods in unemployment, is equal to $b_h + \omega_{hh}^{w,k}$, where

$$\omega_{hh}^{w,k} = \begin{cases} \beta \Bigg[\underbrace{\lambda \int_{\underline{z}_{hh}^k}^{\infty} \pi s_{hh}^k(y)\, dv_h(y)}_{\text{match + accept}} + \underbrace{\lambda(b_h + (1-\gamma^e)\omega_{hh}^{w,k+1} + \gamma^e \omega_{lh}^w)}_{\text{match + reject (possibility of skill loss)}} + \underbrace{(1-\lambda)(b_h + \omega_{hh}^{w,k+1})}_{\text{no match}} \Bigg] & \text{if } k \leq \bar{k} - 1 \\ \\ \beta \Bigg[\underbrace{\lambda \int_{\underline{z}_{hh}^k}^{\infty} \pi s_{hh}^k(y)\, dv_h(y)}_{\text{match + accept}} + \underbrace{b_h + \omega_{hh}^{w,k}}_{\text{match+reject or no match}} \Bigg] & \text{if } k = \bar{k} \end{cases}$$

The wages for high-skilled unemployed $p_{hh}^k(z)$ computed through Nash Bargaining must satisfy the following condition:

$$p_{hh}^k(z) + g_h^w(z) = \begin{cases} \pi s_{hh}^k(z) + b_h + \underbrace{(1-\gamma^e)\omega_{hh}^{w,k} + \gamma^e \omega_{lh}^w}_{\text{possibility of skill loss}} & \text{if } k \leq \bar{k} - 1 \\ \\ \pi s_{hh}^k(z) + b_h + \omega_{hh}^{w,k} & \text{if } k = \bar{k} \end{cases}$$

Surplus, value of unemployment, and wages for high-skilled employed The surpluses of high-skilled employed workers include the risk of exogenous and endogenous separations which may be affected by turbulence. In case they do not have a turbulent separation, they fall into the initial category of high skilled unemployed, u^0_{kk}. We adjust the surpluses, continuation values and outside options of these workers to reflect the new outside option $\omega^{w,0}_{hh}$.

- Surplus of high skilled employees

$$s_{hh}(z) = (1-\tau)z + g_h(z) - [b_h + (1-\gamma^d)\omega^{w,0}_{hh} + \gamma^d\omega^w_{lh} - \mu + \omega^f]$$

- Match continuation values for high and low skilled employed workers (low skilled get affected by the change because of upgrades)

$$\begin{aligned} g_h(z) &= \beta\Big[\rho^x\left(b_h + \gamma^{d,x}\omega^w_{lh} + (1-\gamma^{d,x})\omega^{w,0}_{hh} - \mu + \omega^f\right)\Big] \\ &+ \beta\Big[(1-\rho^x)(1-\gamma^s)((1-\tau)z + g_h(z))\Big] \\ &+ \beta\Big[(1-\rho^x)\gamma^s\left(E_{hh} + \nu_{hh}\left(b_h + (1-\gamma^d)\omega^{w,0}_{hh} + \gamma^d\omega^w_{lh} - \mu + \omega^f\right)\right)\Big] \end{aligned}$$

$$\begin{aligned} g_l(z) &= \beta\Big[\rho^x(b_l + \omega^w_{ll} - \mu + \omega^f)\Big] \\ &+ \beta\Big[(1-\rho^x)(1-\gamma^u)(1-\gamma^s)((1-\tau)z + g_l(z))\Big] \\ &+ \beta\Big[(1-\rho^x)(1-\gamma^u)\gamma^s\left(E_{ll} + \nu_{ll}(b_l + \omega^w_{ll} - \mu + \omega^f)\right)\Big] \\ &+ \beta\Big[(1-\rho^x)\gamma^u\left(E_{hh} + \nu_{hh}\left(b_h + (1-\gamma^d)\omega^{w,0}_{hh} + \gamma^d\omega^w_{lh} - \mu + \omega^f\right)\right)\Big] \end{aligned}$$

- Wages for high skilled employed (and upgraded low-skilled employed)

$$p_{hh}(z) + g^w_h(z) = \pi s_{hh}(z) + b_h + (1-\gamma^d)\omega^{w,0}_{hh} + \gamma^d\omega^w_{lh}$$

- Average surplus The average expected surplus $\bar{s}$ now takes into account the categories of high skilled unemployed:

$$\begin{aligned} \bar{s} &\equiv \frac{u_{ll}}{u}\int_{\underline{z}_{ll}}^{\infty} s^o_{ll}(y)\ dv_l(y) + \frac{u_{lh}}{u}\int_{\underline{z}_{lh}}^{\infty} s^o_{lh}(y)\ dv_l(y) + \sum_{k=0}^{\bar{k}}\frac{u^k_{hh}}{u}\int_{\underline{z}^k_{hh}}^{\infty} s^k_{hh}(y)\ dv_h(y) \\ &= \frac{u_{ll}}{u}\left(E_{ll} + (1-\nu_{ll})[b_l - \mu + \omega_{ll}]\right) + \frac{u_{lh}}{u}\left(E_{lh} + (1-\nu_{lh})[b_h - \mu + \omega_{lh}]\right) \\ &+ \sum_{k=0}^{\bar{k}}\frac{u^k_{hh}}{u}\left(E^k_{hh} + (1-\nu^k_{hh})\left[b_h + (1-\gamma^e)\omega^{w,k}_{hh} + \gamma^e\omega^w_{lh} - \mu + \omega^f\right]\right) \end{aligned}$$

C.3 Fraction of unemployment spells being completed within $\bar{k}$ periods

What is the fraction of high-skilled entrants to unemployment who remain uninterruptedly unemployed when passing the critical horizon $\bar{k}$ periods? Let R_h^k be the fraction of high-skilled unemployed that remain in unemployment after k periods, and R_l^k the fraction of high-skilled unemployed that have lost their skills while unemployed and remain in unemployment after k periods. Lastly, let the total fraction be $R^k = R_h^k + R_l^k$. We will compute these fractions recursively.

- Entrants are given by high-skilled workers that lose their jobs due to non-turbulent exogenous and endogenous separations; they flow into the first category of high-skill unemployed u_{hh}^0:

$$\begin{aligned} R_h^0 &= (1-\rho^r)\Big[\rho^x(1-\gamma^{d,x})e_{hh} + (1-\rho^x)\nu_{hh}(1-\gamma^d)(\gamma^s e_{hh} + \gamma^u e_{ll})\Big] \\ R_l^0 &= 0 \\ R^0 &= R_h^0 + R_h^0 \end{aligned}$$

- Remaining fractions after $k \geq 1$ periods of unemployment:

$$\begin{aligned} R_h^k &= (1-\rho^r)\left(1-\lambda+\lambda\nu_{hh}^{k-1}(1-\gamma^e)\right)R_h^{k-1} \\ R_l^k &= (1-\rho^r)\left((1-\lambda+\lambda\nu_{lh})R_l^{k-1} + \lambda\nu_{hh}^{k-1}\gamma^e R_h^{k-1}\right) \\ R^k &= R_h^k + R_l^k \end{aligned}$$

  with initial conditions R_h^0, R_l^0 and R^0.

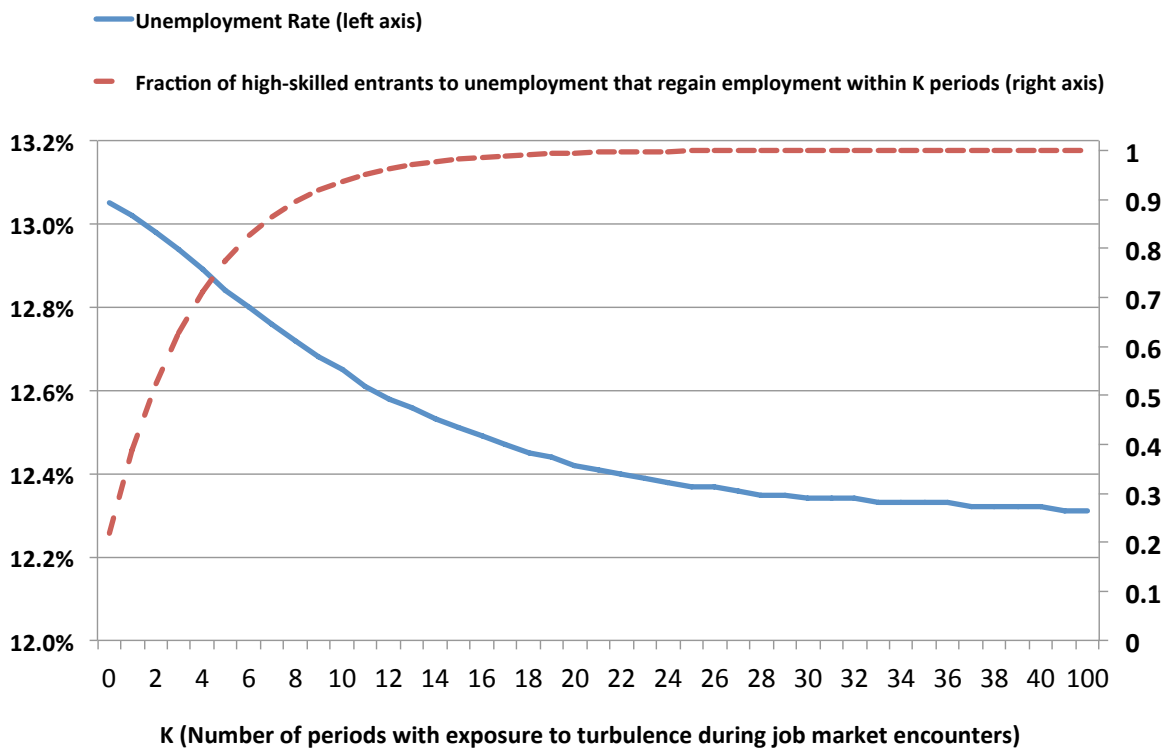
- Therefore, the fraction of high-skilled unemployed agents that leave unemployment within $\bar{k}$ periods are given by:

$$\text{fraction that leaves} = \frac{R_0 - R^{\bar{k}}}{R_0} = 1 - R^{\bar{k}}$$

C.4 Numerical example

As an illustration, the following numerical example (based on a prominent calibration in DHHR's argumentation about 3% endogenous turbulence) shows that only half of the total suppression in unemployment under endogenous turbulence is attained when more than 90 percent of initially high-skilled unemployed have left unemployment at a horizon of 8 semi-quarters, i.e., exposure to skill loss during the first year of an unemployment spell.

Figure C.6: Bargaining vs. Allocation channels



An "innocuous" simplification? Since most of the initially unemployed high-skilled workers have already exited unemployment (at a lower parameterization of the horizon), as well as some of the remaining unemployed workers will have experienced skill loss, it has to be the case that the observed successive increases in the equilibrium unemployment rate (when moving to higher parametrizations of the horizon) must come from force (ii) above, i.e., the worsening bargaining power of workers from the very beginning of their unemployment spell. Hence, DHHR's "simplifying assumption" is far from innocuous—besides the small risk of skill loss after unsuccessful job market encounters, workers lose out with the length of the horizon under which they would be subject to this risk because of weakened bargaining power.

Our finding is reminiscent of the message of the fundamental surplus, as defined in Ljungqvist and Sargent (2017). When computed for the alternating-bargaining assumption of Hall and Milgrom (2008), the fundamental surplus is suppressed by a firm's costs of delay in bargaining and hence, increases the volatility of unemployment in productivity-driven business cycles, even though that cost of delay is never incurred in an equilibrium. In this case, the analogue is that the mere threat under an enlarged horizon of skill loss exposure affects the equilibrium unemployment rate, even though there are hardly any workers who would ever experience such lengthy unemployment spells.